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Member grouping optimization in a multi-objective structural problem of a steel spatial frame

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Abstract. Structural optimization problems arise from a demand for a low-cost and high-performance structure. Thus, the structures are optimized to minimize the weight, volume, or cost concerning their mechanical aspects of strength and displacements. In theory, these optimization problems are searching to find structures with various elements such as columns and beams. On the order hand, in practice, the wide variety of distinct elements can be an inconvenience in the case of real constructions. This paper analyzes the optimum design of steel spatial frames in which these frames are submitted to a multi-objective optimization considering the weight as the first objective function and the number of distinct members in the frame as the second objective function, both to be minimized. Thus, the paper aims to evaluate the behavior of these two conflicting objective functions to provide to the designer the benefits and disadvantages of increasing the number of distinct profiles in a frame. As a result, a Pareto front that summarizes the conflict between the number of distinct members and the structure's weight will be placed. In addition, the paper compares this curve of conflicting objective functions with other existing alternatives, such as those obtained by using cardinality constraints.

Keywords: Multi-objective Optimization, Steel spatial frame, Member Grouping

1 Introduction

In structural optimization problems, it is very common that lighter structures are sought. In minimizing the weight of the structure, a few aspects must be considered, especially those concerning the mechanical constraints, such as strength and displacements. Frequently, the total weight of the structure is the primary objective function to be minimized, leading to a single-objective structural optimization problem. Usually, the solutions considered optimal in the minimization problem lead to structures in which the members were freely chosen without predefined grouping, especially when symmetry does not demand satisfactory aesthetical aspects. In terms of weight, the optimum design will present the best performance. In terms of a building design, the option for different elements chosen without a deep analysis may not be the most suitable design. Considering a steel frame structural system, for example, choosing a wide range of steel profiles can result in some drawbacks [1]. The increase in the variety of profiles in the building can lead to high purchase costs (since retail acquisition is usually more expensive than wholesale); transportation (since the profiles may have different commercial origins); and assembly delays (considering the greatest care when placing the profiles in the right places). Furthermore, without taking into account the risks of inadequate allocation of the profiles. Considering all inconvenience of absence in the standardization in real constructions, alternatives to evaluate a member grouping are proposed [2–4]. As solutions to this problem, there are already some well-established proposals, such as the pre-grouping proposed by experienced designers and the automatic member grouping using cardinality constraints [5, 6]. This paper will evaluate the member grouping of a steel spatial frame using a multi-objective structural optimization problem. Multi-objective structural optimization is a problem in which more than one objective function is considered in the problem formulation. In this paper, the weight and the number of distinct members (W profiles) are the objective functions, both to be minimized. Thus, while the solution to be sought should have the lowest possible weight, it should also have the lowest number of different profiles from each other.

The paper's organization is divided into six sections: Section 2 presents the formulations of the single and multi-objective optimization problems. The multi-objective metaheuristic with iterative parameter distribution (MMIPDE) is the evolutionary algorithm adopted to solve the optimization problems discussed in this paper. Basic information on this algorithm is presented in Section 3. Section 4 presents the numerical experiments. The multi-objective problem is proposed to evaluate the grouping of members in a spatial frame of 78 bars subject to vertical and horizontal loading. Section 5 shows the analysis of results, where the results of the 78-bars are validated through comparison with single-objective problems using cardinality constraints. Finally, the conclusions are reported in Section 6.

2 Single and Multi-objective optimization problem

A structural optimization problem applied to a steel spatial frame consists in finding, in a steel commercial profile search space, an index vector $x = \{I_1, I_2, ..., I_N\}$ (N is the number of design variables), in such a way that each index points to a commercial profile. This vector will be a candidate solution to minimize the objective function $of_1(x)$ satisfying the design constraints. For the case of a single-objective problem in which only the weight should be minimized, the optimization problem is written as:

$$\begin{array}{c} \min \quad of_1(\mathbf{x}); \\ of_1(\mathbf{x}) = W(\mathbf{x}) = \sum_{i=1}^{N} \rho_i A_i L_i \\ s.t. \quad structural \quad constraints \\ x^L \leq \mathbf{x} \leq x^U, \end{array} \tag{1}$$

where L_i, A_i and ρ_i are the length, cross-sectional area, and specific mass of each i profile, respectively.

A multi-objective problem is defined when the minimization of functions other than weight is desired. In this paper, two other objective functions were proposed to be minimized: the number of distinct profiles for columns (m_c) and the number of different profiles for beams (m_b). Equation 2 describes the multi-objective problem that will be analyzed in the paper. The formulation of this multi-objective optmization problem can be written as:

$$\begin{array}{cccc} \min \quad of_1(\mathbf{x}), & \min \quad of_2(\mathbf{x}) & \text{and} & \min \quad of_3(\mathbf{x}); \\ of_1(\mathbf{x}) = W(\mathbf{x}), & of_2(\mathbf{x}) = m_c & \text{and} & of_3(\mathbf{x}) = m_b; \\ s.t. & structural \quad constraints & & \\ & x^L \leq \mathbf{x} \leq x^U, & & \end{array} \tag{2}$$

where m_c and m_b are counters that verify if the indexes that constitute the candidate solution (i.e., the spatial frame) are distinct from each other.

The design constraints of the problem are: the LRFD (Load and Resistance Factor Design) interaction equations for combined axial force and bending moments and the LRDF shearing equation eq. (3); the inter-story drift and the maximum horizontal displacement eq. (4); and geometric constraints referring to column-column connection eq. (5). As a reference for the formulations and the definition limits for strength and displacement, the recommendations of Brazilian ABNT [7] and American ANSI [8] codes were used.

$$\begin{cases} \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \frac{P_r}{P_c} \geq 0.2 \\ \frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \frac{P_r}{P_c} < 0.2 \end{cases}$$
$$\frac{V_r}{V_c} - 1 \leq 0 \quad , \tag{3}$$

where P_r, M_{rx} and M_{ry} are the required axial, flexural about major axis and flexural about minor axis strengths and P_c, M_{cx} and M_{cy} are the available strengths. V_r is the required shearing strength and V_c is the available shearing strength.

$$\frac{d_{max}}{\frac{h}{500}} - 1 \leq 0 \quad and \quad \frac{\delta_{max}}{\frac{H}{400}} - 1 \leq 0 \quad , \tag{4}$$

where d_{max} is the inter-story drift, δ_{max} is the maximum horizontal displacement, h is the height between two consecutive stories and H is the building height.

$$\begin{array}{l} \frac{Cd_{i+n_p}}{Cd_i} - 1 \leq 0; \quad i = 1, [n_p \cdot (N_{pav} - 1)] \\ \frac{Cm_{i+n_p}}{Cm_i} - 1 \leq 0; \quad i = 1, [n_p \cdot (N_{pav} - 1)] \, , \end{array} \tag{5}$$

where Cd is the column profile depth,Cm is the column profile mass, n_p is the number of columns per storey and N_{pav} is the number of storeys in the building.

3 Evolutionary algorithm

This paper solved the structural optimization problems using a meta-heuristic optimization algorithm. Meta-heuristics consist of methods that use iterative processes to find suitable solutions by exploring the search space and interactions between individuals. Concisely, meta-heuristics is an advanced trial-and-error process that well simulates the characteristics of natural evolution [9]. One of the simplest algorithms but much explored in optimization problems is the Differential Evolution (DE). Proposed by Storn and Price in 1995 [10], this algorithm basically consists of an evolution process governed by mutation, crossover operations, and selection. New proposals to improve DE's processes have recently emerged [11]. The present paper used an algorithm based on differential evolution. It is the multi-objective metaheuristic with iterative parameter distribution estimation (MMIPDE). The algorithm proposed by Wansasueb *et al* [12] also consists of mutation, crossover, and selection stages. However, in the MMIPDE the parameters of mutation (or scale factor - F) and crossover rate (C_r) are adapted to accelerate the convergence in turns of the optimal solution. Due to the advantages produced by the adaptation of the parameters, the MMIPDE proved to be an excellent evolution compared to the GDE3 (the first DE algorithm to solve multi-objective optimization problems [13]).

4 Numerical experiment

The steel spatial frame analyzed is composed of 78 steel bars. Thirty-six of these bars act as columns (bars used vertically that are selected from the possibilities of a table of 29 commercial profiles), and 42 act as beams (horizontal bars among the options of 56 W-shapes). Due to the way the bars are connected, the structure features 42 nodes. Regarding the orientation of the columns, they were positioned so that the largest dimension was oriented coincidentally with the largest size of the frame itself, i.e., perpendicular to the windward (since the building receives the frontal wind load to its largest face). Thus, in terms of structure, the stiffness matrix of each frame element has six degrees of freedom, and the global matrix has 252 degrees of freedom. Only the first 36 have their displacements known and equal to zero due to the essential boundary condition given by embedding the bars on the surface where the frame is supported.

The loads imposed on the structure are normative recommendations found in the Brazilian codes NBR 6120 [14] and NBR 6123[15]. According to the deliberations of the recommendations on loading, it is concluded that the vertical load acting on the frame will be given by the expressions eq. (6) and eq. (7):

$$G_{ib} = P_p \cdot 1.25 + 4.5kN/m \cdot 1.4 + 3kN/m \cdot 1.5 = 10.8kN/m + S_w \cdot 1.25 \tag{6}$$

$$G_{ob} = P_p \cdot 1.25 + 2.25kN/m \cdot 1.4 + 1.5kN/m \cdot 1.5 = 5.4kN/m + S_w \cdot 1.25 \tag{7}$$

where G_{ib} is the gravitational loading acting on the inner beams and G_{ob} is the gravitational loading on the outer beams. The values of self-weight (S_w) vary with each choice of the element that makes up the frame.

Regarding the horizontal loading from the wind action, it is noticed that the windward face has three vertical lines of columns, being a central line (denoted with a cc sub-index) and two lateral (lc). Thus, there are the expressions eq. (8) and eq. (9):

$$W_{cc} = 504.92N/m^2 \cdot \frac{[(1.5m + 1.5m) \cdot 18m]}{18m} \cdot 1.4 = 504.92N/m^2 \cdot 3m \cdot 1.4 = 2120.67N/m = 2.12kN/m \quad (8)$$

$$W_{lc} = 504.92N/m^2 \cdot \frac{[(1.5m) \cdot 18m]}{18m} \cdot 1.4 = 504.92N/m^2 \cdot \frac{3}{2}m \cdot 1.4 = 1060.34N/m = 1.06kN/m \quad (9)$$

where W_{cc} is the loading of the wind in the central columns and W_{lc} is the loading of the wind in the lateral columns. Figure 1 depicts the vertical and horizontal loads with the respective values formulated.

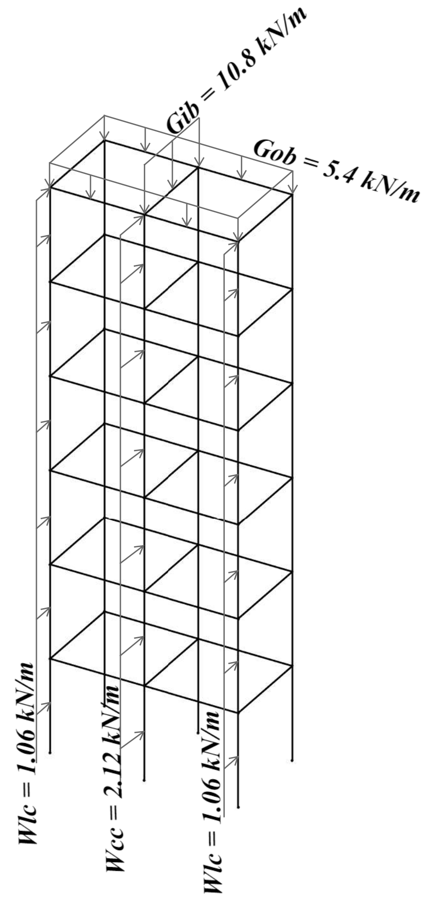

Figure 1. 78 bar spatial frame with horizontal and vertical loading

The experiment itself consists of an evaluation of the multi-objective problem in which the first function to be minimized is the weight of the structure $of_1(x) = W(x)$; the second one, also to be minimized, is the number of distinct profiles used as columns $of_2(x) = m_c$; and the third function is the number of distinct profiles used as beams $of_3(x) = m_b$. Thus, it is a minimization of three objectives. However, considering that the free choice of profiles represents the scenario of greater arbitrariness and freedom in the search for the optimal solution, the minimization of the number of distinct elements conflicts with the minimization of the structure's weight. There is, therefore, a trade-off between weight and standardization of the frame elements. The number of iterations to obtain the weights presented was determined proportionally to experiments already performed [16]. As observed in the single-objective problem of the 78-bar frame, a population of 50 frames and 100 generations efficiently approximates the global minimum of the structure weight function.

Considering that to obtain the same results through the solution of the multi-objective problem would require dozens of equivalent single-objective executions, with hundreds of generations for each problem, it was proposed to use the same population but 10,000 generations. This implies a few dozen iterations more than the proposals for the single-objective problem. The accordance of the results presented below served to ratify the choice of the algorithm parameters. Regarding the results obtained, Tables 1 and 2 provide the final Weights for each member group setting two objective functions.

Table 1. - Structure weight in kg according to the values of the objective functions: number of distinct columns and number of distinct beams

Number of distinct columns \ Number of distinct beams	1	2	3	4	5	6	7	8
1	10,912.6	9,711.5	8,875.5	8,755.9	8,820.2	8,810.5	-	9,096.7
2	10,674.3	7,382.7	7,269.2	7,100.6	6,928.2	6,816.1	7,247.5	
3	-	7,089.7	6,990.3	6,905.6	6,823.6	-		
4	-	7,039.8	7,046.4	7,020.3	6,737.4			
5	-	7,011.5	8,184.3	6,974.3				
6	-	10,774.4	-					
7	-	10,744.2						
8	-							

Regarding the table 1, the values of weights obtained come from a ranking among the results with three objective functions. This ranking works so that, among the thousands of feasible frames found by the evolutionary algorithm, the frames not dominated by anyone are selected. From this, groups with up to nine distinct elements are displayed. The multi-objective analysis gave this limiting value to groups with up to nine different profiles since no solution with a configuration that used ten elements, or more, was found after 10,000 generations.

Table 2. - Weight for each member group setting two objective functions.

Grouping	2	3	4	5	6	7	8	9
Element	1 Col. 1 Beam	2 Col. 1 Beam	2 Col. 2 Beams	3 Col. 2 Beams	3 Col. 3 Beams	3 Col. 4 Beams	2 Col. 6 Beams	4 Col. 5 Beams
Weight (kg)	10,912.6	9,711.5	7,382.7	7,089.7	6,990.3	6,905.6	6,816.1	6,737.4

5 Analysis of results

The analysis of results presented in section 4 is evaluated through two observations. The first one concerns the fact that the behavior obtained by the multi-objective analysis was consistent as expected. From the point of view of structural optimization, it was expected that, as the number of profiles to be used was larger, the structure should be lighter. The reasoning can well explore this hypothesis that the greater the freedom of choice of profiles, the more varied the final frame composition would be, and this would obviously be the most optimized in terms of weight. To allow a comparison with the weight of the optimal structure, the problem was performed without any constraint on the grouping of profiles. In this sense, the result obtained was a structure of 6,225.4 kg (43% lighter than the frame of 2 profiles and 8% lighter than the frame of 9 profiles).

Another verification that was explored has a character of validation of results. It was made an execution of a parallel code that solved a set of single-objective problems (being the weight the only objective function to be minimized), but with cardinality constraints. As it is possible to rely on [5, 6], the cardinality constraint works as follows: among the constraints imposed on the structure, a variable m is designated that will limit how many distinct profiles can be used in the search for the solution. In practice, what happens is the addition of m elements (in the case of this paper, m_c for columns and m_b for beams, where $m = m_c + m_b$) in the candidate vector to the solution. In this case, 78 bars make up the spatial frame. Thus, the candidate vector will have $78 + m$ indexes. The first 78 indexes point to one of the m indexes of the end of the vector. This process of each element receiving a number from 1 to m that points to one of the m groups consists of an automatic grouping process. A more precise understanding of pointer operation and automatic grouping can be found in [17]. It is not the intention of this paper to present the method of cardinality constraints, but it is interesting to understand the difference between this methodology and the multi-objective approach used in section 4.

To obtain an analysis capable of evaluating the results of table 1, 36 single-objective executions were performed. Throughout this process, the values of m_c and m_b ranged from 1 to 8, maintaining the constraint that $m \leq 9$. Thus, a table similar to 1 was obtained. The results of this table were condensed in table 3, which provides a summary of the 36 single-objective analyses showing the evolution of the structure's weight with the increase of cardinality.

Table 3. - Weight for each member group setting one objective functions.

Grouping	2	3	4	5	6	7	8	9
Element	1 Col. 1 Beam	2 Col. 1 Beam	2 Col. 2 Beams	3 Col. 2 Beams	1 Col. 5 Beams	4 Col. 3 Beams	4 Col. 4 Beams	6 Col. 3 Beams
Weight (kg)	9,943.3	8,584.2	7,951.9	7,471.9	7,184.6	6,925.1	6,634.5	6,308.6

To compare the results obtained via multi-objective analysis and the 36 single-objective analyses, figure 2 shows the evolution of the structure's weight as a function of the number of different profiles (groups of bars). It is noteworthy that the minimum possible is the group with 2 bars, as the presence of at least one type of column profile and one type of beam profile is necessary. Figure 2 validates and shows the coherence in the multi-objective analysis. The very close results in the two analyses show that the multi-objective approach can be used to replace a large number of single-objective analyses. Thus, it is important to notice that the proposal to use the number of different profiles as an objective function can be seen as a powerful tool in searching for structures with greater standardization. Furthermore, the efforts to obtain the result of the tables 1 and 2 are far inferior to the execution of dozens of single-objective problems using the restriction of cardinality, for example.

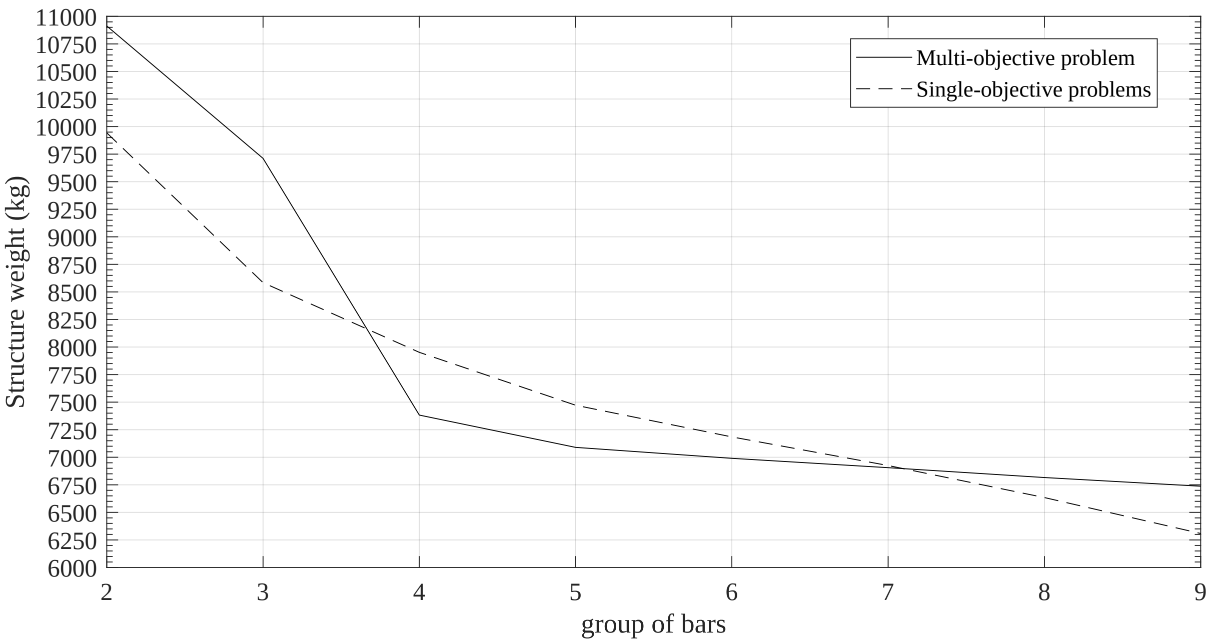

Figure 2. Comparison between the Pareto front from the multi-objective problem and the compilation of the single-objective analyses.

Although slightly different behavior, as can be seen in the differences between the tables 2 and 3, and also in the curves of Figure 2, the results of the multi-objective problem corroborate to the expected. Even with variations in the values of the weights, the greatest difference is not greater than 12% (which occurs for the grouping in 3 types of profiles).

6 Conclusions

This paper aimed to explore the possibility of studying the bets member grouping in a space frame in steel through a multi-objective analysis. The number of distinct profiles of columns and beams were transformed into counters and were minimized in a meta-heuristic evaluation using the MMIPDE algorithm, an evolution of GDE3. From the point of view of the results, feasible frames were obtained with groupings ranging from 2 to 9 distinct bars in different configurations (arrangements of columns and beams). To evaluate the proposal, the paper compared the results obtained with an already consolidated methodology known as cardinality constraints. In conclusion, it is possible to verify that the multi-objective analysis was consistent with the expected values and, in a certain way, more robust due to the execution of a single multi-objective problem. Therefore, future work must be carried out to evaluate the proposal's advantages concerning the computational costs and employability in more complex problems.

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References

[1] L. Pavlovčič, A. Krajnc, and D. Beg. Cost function analysis in the structural optimization of steel frames. *Structural and Multidisciplinary Optimization*, vol. 28, pp. 286–295, 2004.
[2] A. C. Lemonge, H. J. Barbosa, A. L. Coutinho, and C. C. Borges. Multiple cardinality constraints and automatic member grouping in the optimal design of steel framed structures. *Engineering Structures*, vol. 33, n. 2, pp. 433–444, 2011.
[3] J. E. Herencia and R. T. Haftka. Structural optimization with limited number of element properties. *Structural and Multidisciplinary Optimization*, vol. 41, n. 5, pp. 817–820, 2010.
[4] J. E. Herencia, R. T. Haftka, and V. Balabanov. Structural optimization of composite structures with limited number of element properties. *Structural and Multidisciplinary Optimization*, vol. 47, n. 2, pp. 233–245, 2013.
[5] H. Barbosa and A. Lemonge. A genetic algorithm encoding for a class of cardinality constraints. pp. 1193–1200, 2005.
[6] H. Barbosa, A. Lemonge, and C. Borges. A genetic algorithm encoding for cardinality constraints and automatic variable linking in structural optimization. *Engineering Structures - ENG STRUCT*, vol. 30, pp. 3708–3723, 2008.
[7] ABNT. Associação brasileira de normas técnicas - projeto de estruturas de aço e de estruturas mistas de aço e concreto de edifícios. Technical report, ABNT, Rio de Janeiro - Brasil, 2008.
[8] ANSI. Aisc 360-16 specification for structural steel buildings. Technical report, ANSI, AISC, Chicago, USA, 2008.
[9] X.-S. Yang. *Nature-Inspired Metaheuristic Algorithms*, 2010.
[10] R. Storn and K. Price. Differential evolution: A simple and efficient adaptive scheme for global optimization over continuous spaces. *Journal of Global Optimization*, vol. 23, 1995.
[11] A. Kaveh. *Advances in Metaheuristic Algorithms for Optimal Design of Structures*, 2014.
[12] K. Wansasueb, N. Pholdee, N. Panagant, and S. Bureerat. Multiobjective meta-heuristic with iterative parameter distribution estimation for aeroelastic design of an aircraft wing. *Engineering with Computers*, vol. 38, 2022.
[13] S. Kukkonen and J. Lampinen. Gde3: The third evolution step of generalized differential evolution. In *IEEE Congress on Evolutionary Computation (CEC 2005)*, pp. 443–450. IEEE, 2005.
[14] ABNT. Associação brasileira de normas técnicas - cargas para o cálculo de estruturas de edificações. Technical report, ABNT, Rio de Janeiro - Brasil, 1980.
[15] ABNT. Associação brasileira de normas técnicas - forças devidas ao vento em edificações. Technical report, ABNT, Rio de Janeiro - Brasil, 1988.
[16] C. H. B. d. Resende. *Otimização estrutural de pórticos espaciais de aço via algoritmos de evolução diferencial.*, 2020.
[17] J. Carvalho, A. Lemonge, E. Carvalho, P. Hallak, and H. Bernardino. Truss optimization with multiple frequency constraints and automatic member grouping. *Structural and Multidisciplinary Optimization*, vol. 57, pp. 1–31, 2018.