

# Numerical-experimental analysis of frost formation on copper flat plates

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**Abstract.** In the present work, a numerical-experimental analysis of frost formation on copper flat plates was performed. The process of formation of frost is a physical phenomenon that occurs through the change of phase by resublimation, in which a flow of moist air solidifies when it comes into contact with surfaces with temperatures below 0°C, which occurs in various equipment present in refrigeration processes. Such accumulation gives rise to a porous structure, acting as a thermal insulator, decreasing the heat transfer rate from the surface and the efficiency of the cooling systems. A low-cost experimental apparatus was developed, enabling an experimental analysis of the phenomenon under study. Through numerical simulations, based on a mathematical model of this phenomenon, it was possible to determine the frost thickness. The numerical-experimental analysis demonstrates measurements and simulations of the frost thickness as a function of the elapsed time. The numerical results were in good agreement with the experimental results.

**Keywords:** frost formation, numerical-experimental analysis, flat plate, copper.

## 1 Introduction

The physical phenomenon of frost formation occurs when the flow of the mixture air and water vapor meets surfaces that have temperatures below 0°C, being present in several commercial and industrial applications of low temperature, such as in aircraft wings, compressor rotors, evaporators of refrigeration systems, gas purification systems, and many others [1].

The frost layer acts, at first, as a fin, intensifying the heat exchange with the external environment, however, with the increase of the thickness of the frost layer there is also an increase in thermal resistance, reducing the heat transfer rate of the surface [2]. The deposition on cold surfaces changes the performance of refrigeration equipment, reducing efficiency and demanding a higher energy expenditure for its correct functioning [3].

The increase in thickness or the accumulation of frost formation generates two problems, the first is the reduction of heat transfer rate and the second is the increase in load loss due to the accumulation of ice in evaporators, reducing the flow area in the tubes [4]. Therefore, avoiding the formation and accumulation of ice on the surface of the equipment used in refrigeration systems is essential to obtain a good performance, thus enabling the proper functioning of the set [5].

In this context, the present work performed a numerical-experimental analysis of frost formation on a flat surface, whose base material is copper, under controlled environmental parameters.

## 2 Experimental

The experimental apparatus according to Fig. 1, consists of a test section, which contains a Peltier *TEC1-12706* thermoelectric chip, a finned heat sink with *Cooler Master™* Hyper T4 heat pipes, a *Keysight™* 34970A data acquisition system with a *Keysight™* 34901A multiplexer with 18 channels, a *Keysight™* U8002A power supply, a *Sony™* Cyber-Shot DSC-W530 digital camera with 14.1 MP and 90 DPI, a *Polaroid™* tripod, a *Dell™* notebook and, an *NHS™* UPS. The test section consists of an acrylic box (casing), an aluminum support base and a *Multilaser™* axial fan. A schematic of the experimental apparatus is shown in Fig. 2.



Figure 1. Experimental apparatus

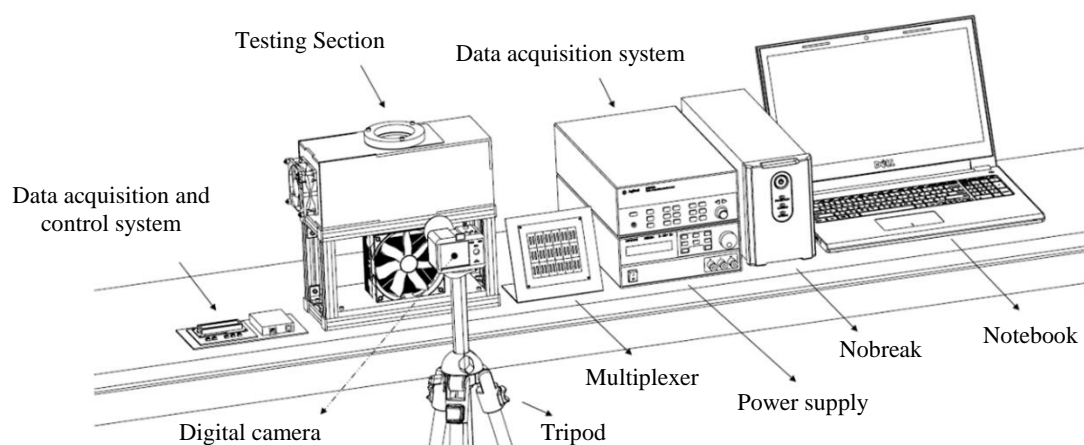


Figure 2. Experimental apparatus scheme

The flat surfaces used in the experiment consist of square plate of copper, with a 40mm edge and 2mm thick, as shown in and Fig. 3.



Figure 3. Flat plates used

The methodology adopted during the execution of the experimental tests, described in detail in Biglia [6], can be summarized as: 1) isolate the test environment; 2) connect the systems responsible for cooling, control and data acquisition; 3) wait for the stabilization of environmental parameters; 4) fix the plate to be tested; 5) connect the electrical components of the experimental device; 6) prepare and check measurement systems; 7) perform the initial measurement at zero time; 8) activate the cold surface by means of the power supply; 9) to perform the measurements at each ten-minute time interval during the total time of ninety minutes, collecting all the values in a digital spreadsheet; 10) Save the data obtained for evaluation.

The uncertainty analysis consists of quantifying the validity of the data and its accuracy, enabling the estimation of the random error present in the experimental results, and the error is defined as the difference between the actual and the indicated value [7].

The experimental uncertainties analysis was associated with the uncertainties of the frost thickness, temperatures, humidity, air velocity and time. The data collected in the experimental tests have the uncertainties shown in Table 1.

Table 1. Measurement uncertainties

Parameter	Measurement tool	Unit	Uncertainty
Air velocity	Digital Anemometer	[m/s]	±0.215
Cold surface temperature	Infrared thermometer	[°C]	±2.05
Frost thickness	Treatment of the images	[mm]	±0.265
Humidity	DHT22 Sensor	[%]	±5.0
Room temperature	Type T Thermocouple	[°C]	±0.5
Time	Digital Chronometer	[s]	±0.01

The experimental tests were performed at the Laboratory of Porous Media and Energy Efficiency (LabMPEE) linked to the Graduate Program (Master) in Mechanical Engineering (PPGEM) of the Department of Mechanical Engineering (DAMEC) of the Federal University of Technology - Parana (UTFPR), Ponta Grossa Campus.

### 3 Mathematical model

The mathematical formulation of the phenomenon of frost formation in flat plates is based on the models developed by Sedano [8] and Tao et al. [9], as described in detail by Biglia [10].

In the first stage of frost formation occurs the formation of nuclei, with one-dimensional growth in the direction perpendicular to the surface. The heat and mass balances are made in the unit model of the growth of the ice crystal, illustrated by Fig. 4(a), to obtain the differential equations that govern the phenomenon.

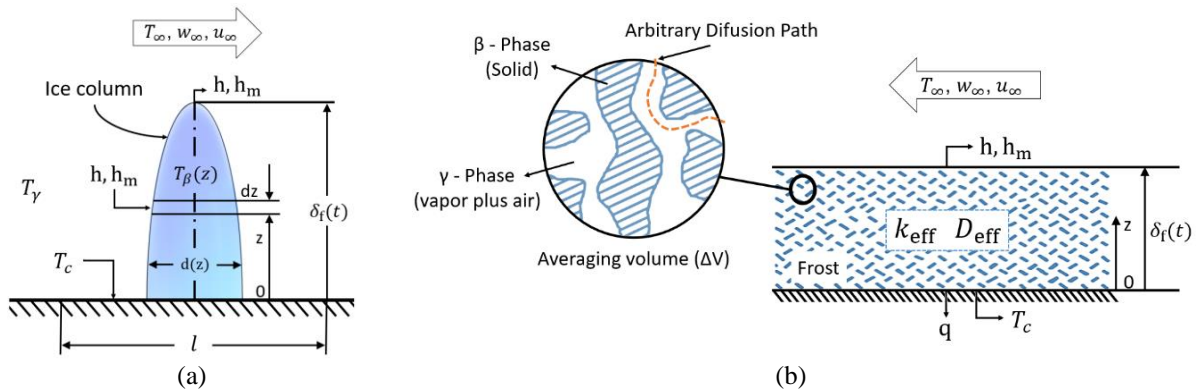


Figure 4. (a) Unitary model of ice crystal growth; (b) Volume element used in the model

Equations (1) and (2) represent the mathematical modeling of the first stage of the frost formation process, for energy and diffusion, respectively.

$$\rho_{\beta} c p_{\beta} \frac{\partial T}{\partial t} \frac{\pi}{4} d^2 dz - \rho_{\beta} h_{sg} \frac{\pi}{4 dt} \left[ \left( d + \frac{\partial T}{\partial t} \right)^2 - d^2 \right] dz = q_z + q_{conv} - q_{z+dz} , \quad (1)$$

where:  $\rho_{\beta}$  is the specific mass (solid phase);  $cp_{\beta}$  is the heat specific to constant pressure (solid phase);  $T$  is the temperature;  $t$  is the time;  $d$  is the diameter of the surface of the frost layer;  $h_{sg}$  is the sublimation enthalpy;  $q_z$  is the heat transfer flow by conduction in  $z$ ;  $q_{conv}$  is the heat transfer flow by convection; and  $q_{z+dz}$  is the heat transfer flow by conduction in  $z+dz$ .

$$\rho_{\beta} \frac{\partial d}{\partial t} = 2h_m (w_{\gamma} - w_{\beta}) , \quad (2)$$

where:  $h_m$  is the convective coefficient of mass transfer;  $w_{\gamma}$  indicates the concentration evaluated at the temperature of the gas phase ( $T_{\gamma}$ ), and  $w_{\beta}$  is the concentration evaluated at the temperature of the solid phase ( $T_{\beta}$ ).

In the modeling of the second stage, the model developed by Whitaker [11] for porous media is used, based on the local volumetric mean. The governing equations for the  $\beta$  and  $\gamma$  phases were formulated by Sedano [8], using a volume element ( $\Delta V$ ), within the porous medium, as illustrated in Fig. 4(b).

The energy balance is given by Eq. (3).

$$\rho_{\beta} c p_f \frac{\partial T}{\partial t} + \dot{m} h_{vg} = \frac{\partial}{\partial z} \left( k_{eff} \frac{\partial T}{\partial z} \right), \quad (3)$$

where:  $\dot{m}$  represents the phase change rate of water vapor and  $k_{eff}$  the effective thermal conductivity of porous frost layer.

Through the hypothesis of the presence of saturated air in the porous medium, we obtained the equation of the continuity of the ice phase ( $\beta$ ), given by Eq. (4).

$$\frac{\partial \varepsilon_{\beta}}{\partial t} = \dot{m} = w - w_{sat} . \quad (4)$$

Again, through the hypothesis of the presence of saturated air in the porous medium, the diffusion equation of the gas phase ( $\gamma$ ) is obtained, given by Eq. (5).

$$\frac{\partial \varepsilon_{\gamma}}{\partial t} = \frac{\partial}{\partial z} \left( D_{eff} \frac{\partial w}{\partial z} \right) \quad (5)$$

where:  $D_{eff}$  is the effective diffusion coefficient.

## 4 Numerical model

The program was implemented in Python using the Jupyter Notebook Environment, which consists of an open-source application (BSD license) that allows the creation and resolution of routines that contain iterative codes and equations, as well as obtaining graphics (Matplotlib). The equations that model the first stage of porous frost layer growth are solved using finite differences. To develop the discretization, the Finite Difference Method is used centered on the derivative in space and intermediate points, and implicit formulation for the transition time.

Applying a fixed mesh, the values of the properties are interpolated to each new position of the boundary, through the Spline Method, which is an approximation technique, by means of polynomial interpolation, where the interval of interest is divided into several subintervals, which are interpolated with polynomials of smaller degrees.

The differential equations that model the second stage are again solved using the Finite Difference Method. Centered Finite Differences are used for the derivatives in space, and for the intermediate points, the Upwind Technique is used for the time derivative. A fixed mesh is used, as before, and the values of the variables and properties under study are interpolated to each of the new boundary positions, the differential equations are solved by iterations, until a difference equal to or less than  $10^{-5}$  is obtained between the values of the variables in two successive iterations.

## 5 Results

For the analysis of the frost thickness, the software ImageJ© was used in the treatment of the images, setting the same measurement scale according to the specifications of the digital camera used, such as resolution and DPI (dots per inch), which provides the width and height of the image file, in order to enable the conversion of length, in this case the height, in pixels into millimeters, and subsequently allowing the superimposition of all images, having as reference the initial time, as exemplified by Fig. 5.

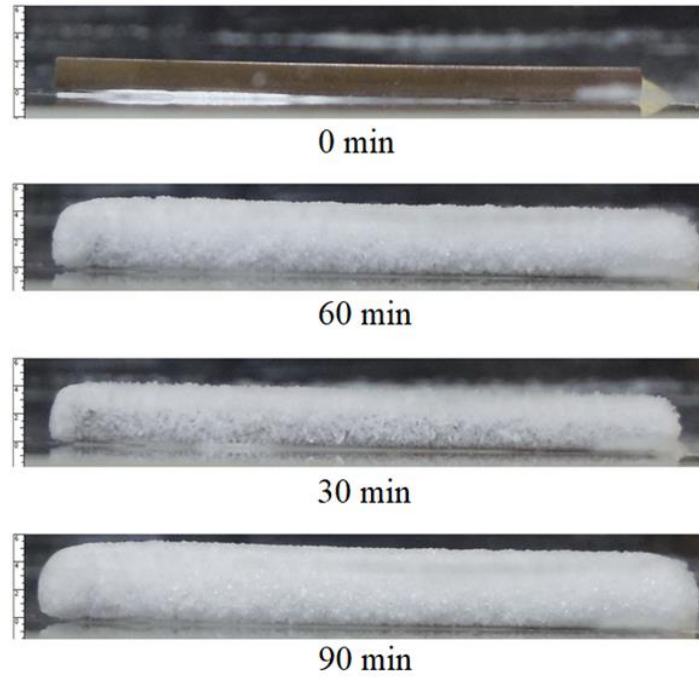


Figure 5. Formation of the porous ice layer in time

During the obtaining of the experimental results, the environmental parameters were kept constant according to Tab. 2.

Table 2. Environmental Parameters

Parameter	Symbol	Value	Unit
Environmental Temperature	$T_{\infty}$	18.0	[°C]
Cold Surface Temperature	$T_c$	-20.0	[°C]
Relative air humidity	$\theta$	50.0	[%]
Air Velocity	$v_{\infty}$	0.5	[m/s]

Table 3 and Figure 6 show the experimental results obtained in the tests, in relation to the formation of the porous frost layer, through the periodic recording and monitoring of its thicknesses over the course of ten minutes.

The results presented in Fig. 6 show good agreement in relation to the behavior of increasing the thickness of the porous frost due to time, with the results published in the literature, by Liu et al. [12], Piucco [13], and Sommers et al. [14].

Figure 7 presents the result of the numerical analysis of the second stage of frost formation, directed to the formation and growth of the porous frost layer, referring to copper, whose thermal conductivity, as indicated in Bergman et al. [15], was set at 401 W/m.K and the environmental parameters according to the Tab. 2.

Table 3. Evaluation of thickness versus time.

Time [min]	Frost thickness [mm]
0	0.000
10	1.499
20	2.158
30	2.353
40	2.589
50	2.955
60	3.150
70	3.520
80	3.765
90	4.009

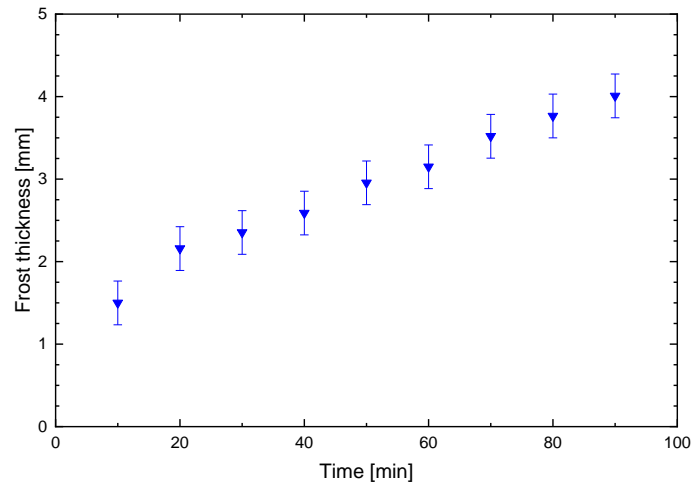


Figure 6. Experimental analysis of porous frost formation.

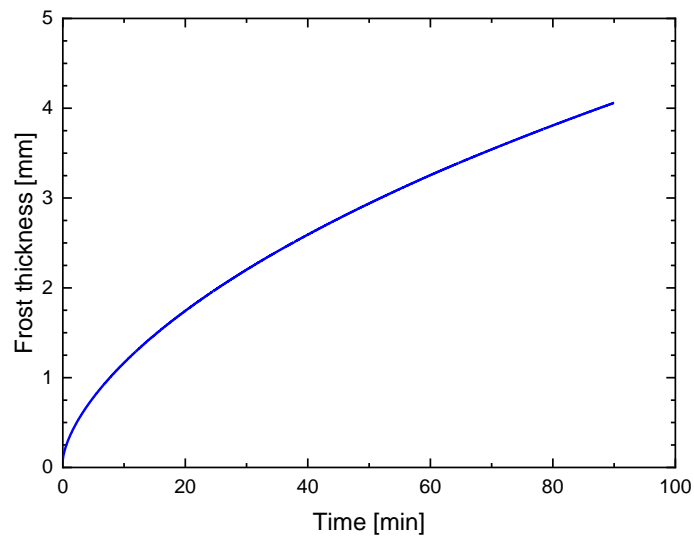


Figure 7. Numerical analysis of pore ice formation.

Figure 8 allows you to compare the experimental results with those obtained numerically.

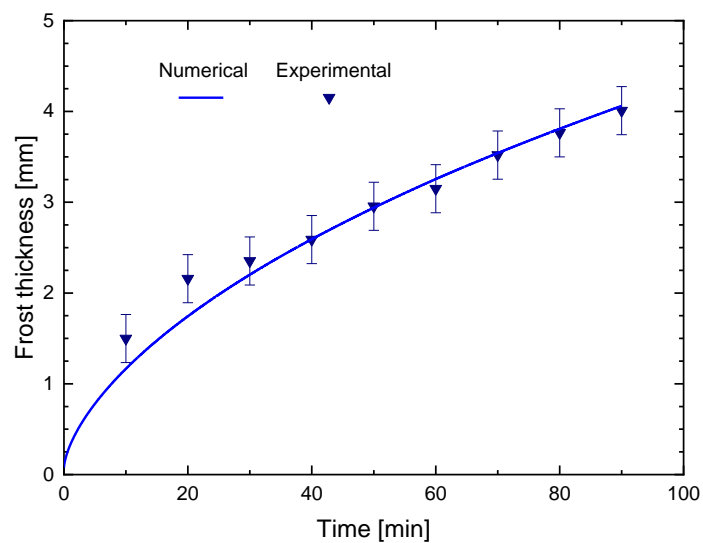


Figure 8. Numerical and experimental analysis of frost formation.

Through numerical-experimental analysis performed from the comparison with experimental data, it is observed that the mathematical model used presents good agreement with the experimental results, with the highest relative percentage errors of ~24% for the time of 20 minutes. It is also observed, the highest errors are present in the first-time intervals, presenting, in general, greater deviations in the first thirty minutes, probably due to the initial conditions of the phenomenon used.

## 6 Conclusions

In this work, a numerical-experimental study of pore ice formation on copper flat surfaces was performed. The experimental results obtained indicate a good agreement with the literature regarding the behavior of frost formation. From numerical analysis, it can be concluded that the mathematical model and numerical code used can predict the increase in the thickness of the porous frost formation on flat surfaces. Finally, this research reinforces results present in the literature, being able to assist the scientific community and serve as support for future work.

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