Beams non Linear Analysis from Envelope Concept

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Abstract. The heterogeneous nature of the Portland cement concrete promotes its irregular mechanical behavior. Due to the shrinkage deformations, the mass of concrete presents cracks, even before loading. Such material exhibit non linear performance since the stress level around 30% of its compressive strength, and, in this way, some researchers developed mathematical models designed to describe that kind of mechanical phenomenon. A model proposed by Branson, is especially useful to analyze reinforced concrete beams from beam finite elements resulting, consequently, computational effort economy. It is known that, in some cases, it may be suitable to carry out continuous beam analysis considering several loading arrangement, culminating over a bend moments envelope. Preliminary comparative studies made over continuous beam, considering the bend moments due to full load and bend moments envelope drafted from different load arrangement alternatives show remarkable differences between these two kinds of analyses versions. The purpose of this work is to report the nonlinear mechanical performance limit analysis of reinforced concrete continuous beams considering, include, the bend moments envelope. To accomplish such a subject, a computational code was developed, based on the finite element approach, applied upon the Branson's formulation.

Keywords: numerical simulation, finite elements, reinforced concrete, continuous beams, envelope.

1 Introduction

Concrete is a solid mass resulting from the hardening of a homogenized mixture involving aggregates, Portland cement and water, that experiences cracking already in the first days of its synthesis, with no loading, due to the volumetric contraction during its natural drying process, Wight and McGregor [1].

Due to the concrete heterogeneous nature, in the face of the stresses imposition, the above-mentioned cracking is intensified resulting in a nonlinear mechanical behavior for such a material, which can culminate in instability, Wight and McGregor [1].

The low tensile strength of the highlighted material determines the needing to use steel bars, as a resource to supply that deficiency, resulting in the reinforced concrete, Carvalho and Figueiredo [2].

Thus, the mechanical performance analysis of reinforced concrete structural members requires the adoption of nonlinear modeling, which can be effective from the Finite Element Method using upon a nonlinear orthotropic model, at least, in Plane State of Stresses, Madureira [3].

The attention to the deformation limits is as fundamental for the cross-sections dimensioning of reinforced concrete structural members as the observance of the strength requirements of the involved materials.

The use of nonlinear analysis in its most rigorous meaning, in the tasks of calculating displacements and stresses for ordinary structural design, represents excessive work expenditure and high computational effort, so that, it is suitable to consider the use of simplified alternative procedures, such as the formulation proposed by Branson [4], thus enabling the approximation from beam finite elements and, consequently, resulting in a computational effort economy.

Preliminary studies over the calculation of displacements of reinforced concrete beams using Branson's model [4] indicated that, its magnitudes obtained in this way, are underestimated in comparison with experimental test results published by Burns and Siess [5], justifying, in this way, the effort expenditure in accurate analyses aimed to the appropriate adjustments of such model.

This paper refers itself to the report of the mechanical performance numerical simulation of reinforced concrete continuous beams by use of a computational program based on beam finite elements applied over the Branson's nonlinear model approaching, include, the envelope concept.

In this way, the obtained results will be compared to those ones obtained from the material linear behavior consideration and, therefore, adopting the Hooke's constitutive relationship, in its one dimensional version.

2 Modeling

2.1 Beam Finite Element Formulation

The beam element nodal displacements and forces are presented in Fig. 1.a and Fig. 1.b, and, the corresponding stiffness terms in Fig. 1.c, Fig. 1.d, Fig. 1.e and Fig. 1.f. According to Cook at al [6], the element stiffness matrix are:

$$[K]^{(EI)} = \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \tag{1}$$

The EI product in Eq. (1) must be replaced by the bend equivalent stiffness formulated according to the Branson's model.

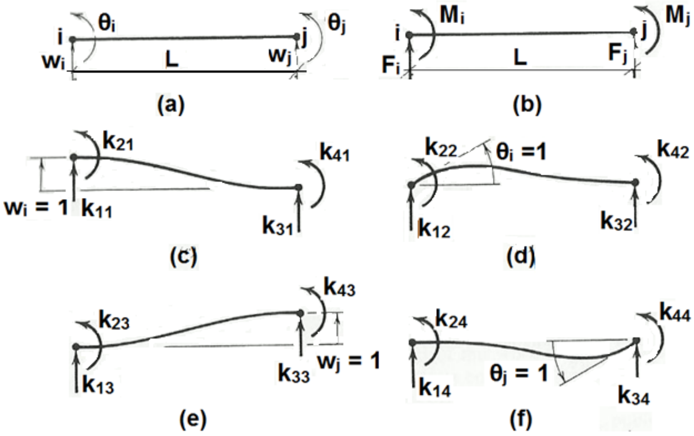

Figure 1. Element nodal displacements, forces and stiffnesses

2.2 Branson's model

Branson's model is intended for the calculation of displacements in reinforced concrete beams idealized as reticular bars, distinguishing the behavior of the structural member whose critical cross section is on the stage I from that one referring to the stage II. stage I corresponds to that condition in which the concrete in the critical cross-section still absorbs tensile stresses, and, the condition in which the stretched region of such a section is cracked, characterizes the stage II, Carvalho and Figueiredo Filho [2].

According to the model highlighted, the boundary between the stage I and the stage II is defined by the bending moment magnitude that would lead to the first crack arising at the beam critical cross section, Fig. 2. Such a moment is given by:

$$M_r = \frac{\alpha f_{ct} I_c}{y_t} \tag{2}$$

since the α parameter correlates tensile strength in bending and direct tensile strength, y_t is the distance from the gravity center of the cross section to its stretched edge, I_c is the cross section inertia moment and f_{ct} is the concrete tensile strength defined by:

$$f_{ct} = 0{,}3 f_{ck}^{(2/3)} \tag{3}$$

if te f_{ck} parameter represents the concrete compressive characteristic strength.

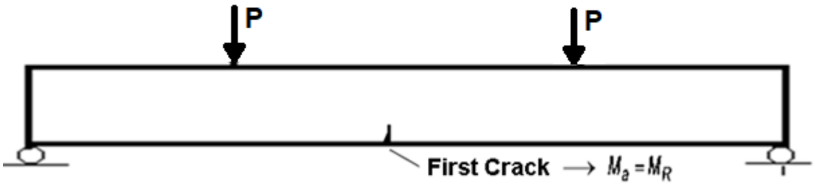

Figure 2. Flexure cracking

The bending stiffness of the cross-section in stage I must be calculated by:

$$EI = E_{cs} I_c \tag{4}$$

if E_{cs} is the concrete elasticity secant modulus defined according to the ABNT NBR 6118 [7].

On the other hand, for bending moments in the critical section whose magnitude is greater than M_r, and so, the critical cross section, therefore, is in the stage II, the bending stiffness of the cross-section should be calculated from:

$$\mathrm{EI} = E_{cs}\left\{\left(\frac{M_r}{M_a}\right)^3 I_c + \left[1 - \left(\frac{M_r}{M_a}\right)^3 I_{II}\right]\right\} \tag{5}$$

since the M_a parameter represents the bending moment magnitude in the critical section and I_{II} the inertia moment of the cracked critical section in stage II.

3 Computational support

The results of support to the analysis which is approached in this paper were obtained from the computational code called VIGEFNL drafted according to the FORTRAN automatic language and approximation by beam finite elements of two nodal points and two degrees of freedom by nodal point.

The algorithmic pattern of that computational code includes a calculation structure adjusted to the Branson's model [4] applied to the analysis of continuous beams consisting of reinforced concrete considering its nonlinear mechanical behavior and idealized from reticular bars. Such an implementation strategy culminates in computational effort savings.

In the initial part of the referring software, the finite element mesh is generated, and, after that, the geometrical characteristic and the material physical property parameters of the analyzed model are read. In the next step, in turn, the load is read and so, it is divided into some incremental parcels, according to a quantity suitable to get a good precision.

The algorithmic scheme is idealized so that the beam loading evolve in a progressive incremental way, and, in each step, the forces are calculated and the bend stiffness are fitted to the force level according to Branson's formulation [4].

The results obtained from the automatic program described above were compared to those ones, obtained by a computational code developed in FORTRAN language and finite elements approximation on a nonlinear orthotropic calculation structure, in plane state of stresses carried out in [8], culminating in a good agreement.

4 Studied specimens

Continuous beams with rectangular cross-section 0.20 m width and 0.60 m height were analyzed, Fig. 3, cast in C 30 concrete, reinforced with CA 50 steel bars. Such a structural member is submitted to uniformly distributed loads. One of them is a permanent load that is applied throughout its length, whose magnitude is about g = 7,0 kN/m. The second one is a variable load whose magnitude is q = 18,0 kN, placed according to ten different arrangements, Tab. 1, characterizing ten cases of loading, and in this way, becoming possible the bend moments envelope determination.

Table 1. Variable load arrangement

Cases of loading	Spans loaded by the variable load
1	All spans
2	Only the first
3	Only the fifth
4	Only the second
5	Only the fourth
6	Only the third
7	Only the first and the second
8	Only the fourth and the fifth
9	Only the second and the third
10	Only the third and the fourth

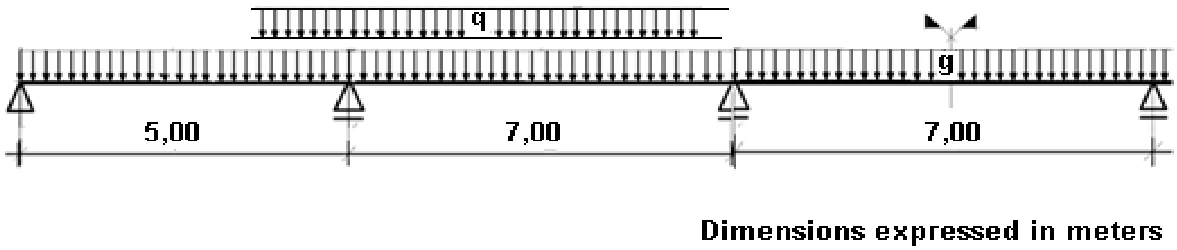

Figure 3. Studied specimens longitudinal structural scheme

5 Results

The positives and negatives maximum bend moments values, referring to the linear analysis, for every cross section, were considered to elaborate a spreadsheet, Tab. 2, that was used to draw up the corresponding bend moments envelopes, Fig. 4.

Table 2. Linear analysis maximum bend moments by cross section

Cross section coordinate (m)	Case 1 bend moment (kNm)	Maximum bend moments (kNm)	
		Positive – Envelope 1	Negative – Envelope 2
0.00	0.0	0.0	0.0
1.00	31.7	39.8	-1.3
2.00	38.4	54.6	-9.7
3.00	20.1	44.4	-25.0
4.00	-23.2	9.2	-47.3
5.00	-91.5	-12.0	-102.1
6.00	-18.3	3.6	-25.9
7.00	29.9	48.9	-7.8
8.00	53.1	74.2	2.6
9.00	51.2	74.5	-6.6
10.00	24.4	49.8	-22.7
11.00	-27.4	2.3	-45.9
12.00	-104.2	-16.9	-121.4
13.00	-29.2	3.0	-45.4
14.00	20.8	49.0	-23.1
15.00	45.8	74.0	-7.9

The cross sections that present maximum positive and maximum negative bend moments must be considered as the critical sections, Fig. 4 and Fig. 5. From the graphs of Fig. 4, it may be verified that, the bend moments associated to the envelopes, obtained from linear analyses, are higher than those ones referring to the beam loaded fully in all spans condition. At a general view, the positives bend moments presented the highest differences. This same tendency was verified to that bend moments corresponding to the non linear analysis.

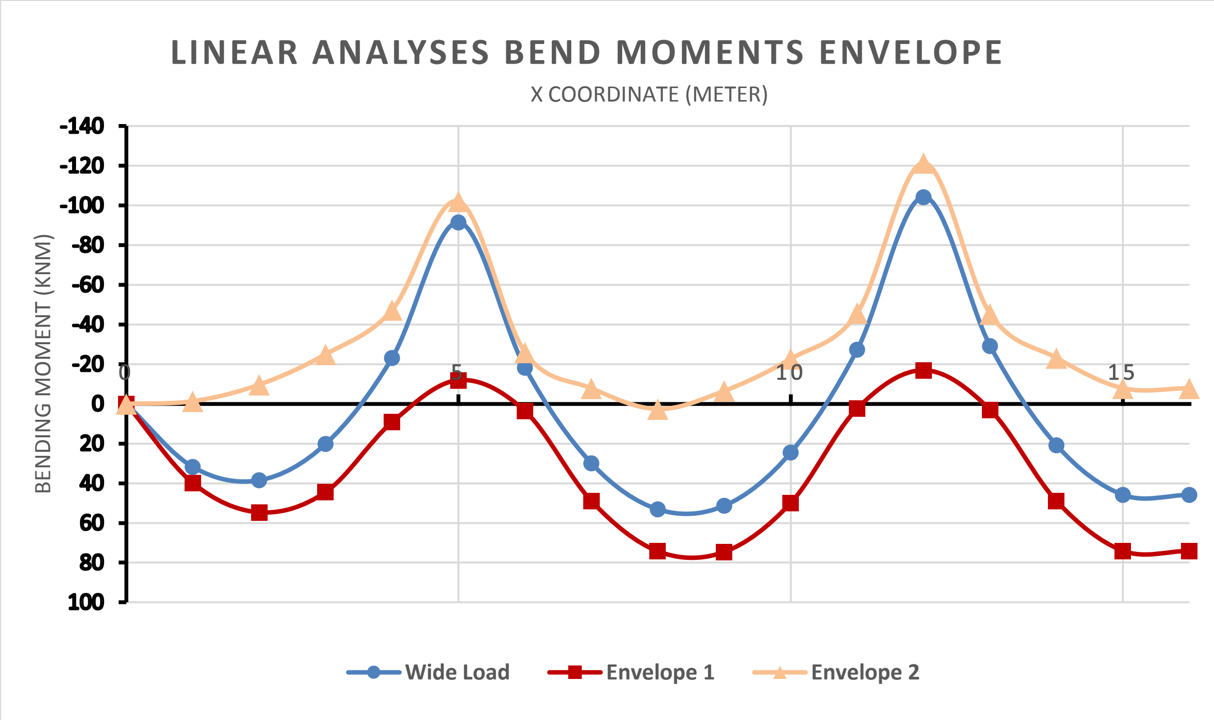

Figure 4. Linear analyses bend moments envelope

The positives and negatives maximum bend moments values, referring to the non linear analysis, for every cross section were considered to elaborate Tab. 3, that was used to draw up the bend moments envelopes, Fig. 5.

Table 3. non Linear analysis maximum bend moments by cross section

Cross section coordinate (m)	Case 1 bend moments (kNm)	Maximum bend moments (kNm)	
		Positive – Envelope 1	Negative – Envelope 2
0.00	0.0	0.0	0.0
1.00	32.7	39.7	-1.5
2.00	40.4	54.4	-10.0
3.00	23.1	44.1	-25.5
4.00	-19.1	8.8	-47.9
5.00	-86.4	-11.2	-95.3
6.00	-13.4	2.8	-26.3
7.00	34.6	48.2	-8.1
8.00	57.7	73.6	2.7
9.00	55.7	73.9	-6.6
10.00	28.7	49.2	-23.0
11.00	-23.3	3.1	-46.4
12.00	-100.2	-15.8	-112.3
13.00	-25.3	0.6	-45.6
14.00	24.6	48.5	-23.2
15.00	49.4	73.6	-7.7

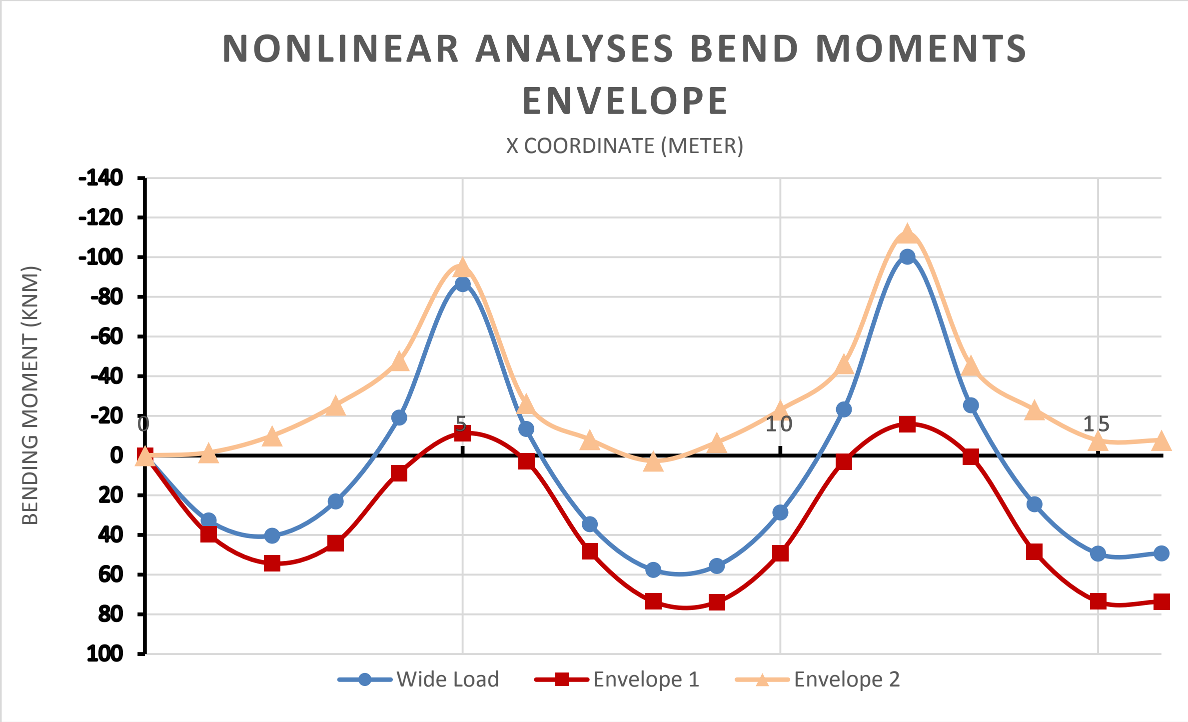

Figure 5. non Linear analyses bend moments envelope

From the Fig. 4 and the Fig. 5 it was identified the beam critical cross sections and then, from the Tab. 2 and Tab. 3 data, it was selected the bend moments magnitudes at these sections, Tab. 4.

From Tab. 4 it may be found that, for the case 1, referring to the beam fully loaded on all spans, the positive bend moments in the critical sections, obtained from the non linear analysis, presented higher magnitudes than those ones associated to the linear analysis, while for the negative bend moments such a trendy occurred according to a inverse fashion, as the same mode that was announced by Madureira at al [8].

For the linear analysis modeling, the greatest difference for the positives bend moment was about 61.6% and it was recorded for the cross section whose coordinate is x = 15.00 m. On the other hand, regarding the negative bend moment magnitudes, the greatest difference was by 16.5% and it was registered for the cross section whose coordinate is x = 12.00 m.

On the other hand, for the non linear analysis version, the differences in the bend moments magnitudes between the load case 1 and the those ones corresponding to the envelope presented smaller values, resulting to the positives and negatives moments, respectively, the 49.1% and 12.1%, at the same cross sections that are mentioned in the last paragraph.

Table 4. Critical sections bend moments

Cross Section Coord. (m)	Linear Analysis			Non Linear Analysis		
	Bend Moments (MNm)		Difference(%)	Bend Moments (MNm)		Difference(%)
	Case 1	Envelope		Case 1	Envelope	
2.00	38.41	54.58	42.1	40.4	54.4	34.7
5.00	-91.48	-102.11	11.6	-86.4	-95.3	10.3
8.00	53.07	74.21	39.8	57.7	73.6	27.6
12.00	-104.20	-121.43	16.5	-100.2	-112.3	12.1
15.00	45.80	73.99	61.6	49.4	73.7	49.1

6 Conclusions

This paper highlights the divergences between elastic linear constitutive modeling and plastic nonlinear modeling in the analysis of continuous beams cast in reinforced concrete, considering, including, the envelope concept.

For the purposes of achieving that subjective a computational application based on finite element approximation on Branson's nonlinear model was used to perform the analysis of a beam subjected to ten cases different alternatives of load arrangements.

The obtained results revealed that, for the case 1 referring to the beam fully loaded on all spans, the negative bending moments that was derived from the nonlinear modeling presented lower magnitudes than those ones arising from the linear analysis, while, for the positive ones, such a tendency has occurred according to a inverse way.

In addition of the needing to adopt higher steel reinforcement areas for the cross sections subjected to the maximum positive bend moments the non linear analysis reveals that adjustment reinforcement spreading along the beam longitudinal direction is suitable.

It must consider itself, including, that the envelopes showed themselves useful, once, the internal forces obtained from them presented values, significantly greater the those one associated the beam fully loaded condition.

It is worthwhile to highlight, however, that, about the envelopes adoption, the difference between the linear modeling and the non linear analysis results did not show themselves, expressive.

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