XLIII CILAMCE
21-25, November, 2022
FOZ DO IGUAÇU - PR - BRAZIL

Optimization of a 2D reinforced concrete frame considering a seismic load via cross-entropy method

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Abstract. Finding an optimal design for reinforced concrete structures to attend a specific goal is a desirable step for engineers. This paper aims to optimize the dimensions of columns and beams of a 2D reinforced concrete frame subjected to a dynamic load. The structure considered has the specific goal to attend for a seismic action, and this is an important search, since structures in Brazil, for many years, were built with no consideration regarding seismic events. Even after ABNT NBR 15421 (2006) [1] was released, seismic actions were still not considered by most engineers during the design phase, according to Miranda et al. [2], which can lead to catastrophic structural damages if events like these happens in urban regions. The problem is formulated in terms of the dynamical seismic response of 2D reinforced concrete frame, and the optimization algorithm used considers a stochastic solution strategy combining penalization and the cross-entropy method, proposed by Cunha [3]. The consideration of the probability of failure and cost of failure on the objective function makes it a Risk Optimization problem.

Keywords: Risk optimization, reinforced concrete frame, seismic actions

1 Introduction

Earthquakes represent a concern for several countries, since they have the potential to cause a great number of casualties and damages in structures. Brazil is a mid-plate country located in the South America tectonic plate, which is considered a stable region when compared to places near the boundaries of tectonic plates. For the sake of comparison, a seismic event of magnitude 5 occurs in Brazil once in five years on average, while in the Andean region an earthquake of this magnitude happens on average twice a week [4]. Even though Brazil is located inside a tectonic plate, it presents a considerable history of small to moderate earthquakes, also including two events with moment magnitude (M) higher than 6. Studies also report damages occurred in the João Camara earthquake, in Rio Grande do Norte state and Itacarambi earthquake, in the state of Minas Gerais [5], [6]. Such damages are explained in places with small to moderate hazards with the definition of risk, which considers hazard, exposure, vulnerability, and consequences. One should note, therefore, that a low hazard does not imply low seismic risk in a region [7], especially if buildings are not properly designed to withstand seismic loads.

In order to evaluate the risk of building damage and collapse, the concept of Performance-based earthquake engineering (PBEE) has been developed over the years [8]. According to Krawinkler [9], it corresponds to the design, evaluation and construction of structures whose performance under extreme loads responds to the needs and objectives of owners-users and society. As a preliminary step, design professionals, owners and other stakeholders identify the desired building performance, and, as the design decisions are made, it is necessary to evaluate if the final building can achieve the indicated performance.

Improving the structural behavior can also be done with optimization techniques. In this respect, the optimization process under uncertainties has many advantages over deterministic ones, since deterministic optimization considers uncertainties in an indirect way. When the cost of failure is incorporated in the objective function, the optimization problem becomes a Risk Optimization, or also Life Cycle Cost Optimization [10]. So, this paper aims to apply the cross-entropy optimization technique to minimize the cost of a 2D reinforced concrete frame subjected to a seismic action.

2 Optimization problem and Methodology

2.1 Problem definition

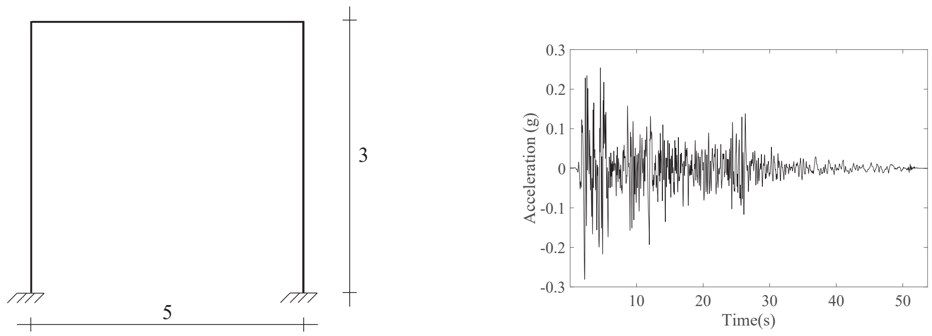
The objective of this work is to find a set of parameters for column and beam cross section that minimizes the total expected repair costs and the probability of collapse of a 2D reinforced concrete frame subjected to *El Centro* earthquake. The design random variables for the problem are column cross section width and height (columns are considered to be squared) (c_h) and beam height (b_h).

$$\begin{gathered}\text{Determine: } \mathbf{d}^* = \{c_h^*, b_h^*\}; \\ \text{That minimizes: } f(\mathbf{d}) = \left[c_c\left(2 \times c_h^2 + b_h \times b_w\right) + \sum_{k=1}^{m,fail} c_{f,k} \times p_{f,k}\right] \\ \text{Subjected to: } 0.19 \leq c_h \leq 0.40;\ 0.30 \leq b_h \leq 0.50 \text{ (in meters)} \\ \text{With: } \mathbf{M} = \{C_h, B_h\}; C_h \sim \mathcal{TN}(c_h, 0.04, -4.27, 0.97);\ B_h \sim \mathcal{TN}(b_h, 0.04, -4.53, 0.47).\end{gathered} \tag{1}$$

Where: b_w corresponds to the beam width; c_c represents the initial cost of construction; c_f corresponds to the expected cost associated with the failure mode considered; $m, fail$ corresponds to the modes of failure considered herein by the Damage Limit States. It is important to make clear that c_h and b_h are independent parameters.

The initial cost of construction will be considered herein as the Brazilian "*Basic Unity Construction Cost*" (CUB) for the state of Sao Paulo, converted to dollar. For the month of June, the cost is estimated as $R\$1894.49/m^2$, and the dollar exchange rate is $R\$5.34$, which means that the CUB value in dollar is $\$354.77/m^2$.

A representation of the 2D frame studied herein is presented in Figure 1a, with units in meters. Figure 1b shows the time-series for *El Centro* earthquake. The evaluation of the dynamic behavior of the 2D frame was done using *OpenSees* software [11], and details of the Finite Element Model used is presented in section 2.4.



(a) (b)

Figure 1. Representation of the 2D RC frame studied herein and the time series of *El Centro* earthquake

The design of the 2D frame considers the two columns containing 6 longitudinal reinforcement bars with diameter of 12.5 mm and a transverse reinforcement composed of one stirrup with two shear legs of 6.3 mm with 15 cm spacing. Beams have a fixed width (b_w) of 19 cm, and contain 2 longitudinal reinforcement bars with 12.5 mm diameter for tensile strength, and 2 longitudinal bars with 20 mm diameter for compression. The transverse reinforcement if done considers one stirrup with two shear legs of 5 mm diameter with 12 cm spacing. The compressive strength of concrete is 23 MPa, steel used is CA-50 and the concrete cover is 2.50 cm. Expected gravity loads are applied in the structure as an uniformly distributed load on the beam and are used to define seismic masses on the model. These loads include 1.05 times the dead load and $0.5\,kN/m^2$ for live load, which consists of 25% of the $2.0\,kN/m^2$ maximum live load for residential buildings established in ABNT NBR 6120 [12]. P-Delta effects are considered in the columns.

In this paper, collapse is defined with the consideration of structural capacity (C) presented in Wen et al. [13], corresponding to the maximum response a structure can withstand without reaching a limit state. This work considers the qualitative and quantitative definitions of the Damage Limit States proposed in Hazus manual [14] for the building type reinforced concrete moment resisting frames (C1), being: Slight (SSD), Moderate (MSD), Extensive (ESD) and Complete (CSD) Structural Damage. Quantitative values for each Limit State considering a low-rise structure (1 to 3 pavements) and a pre-code classification, which means that structures do not take seismic actions in the design phase, are adopted considering values of median interstory drift (which means interstory displacement divided by story height) capacity (Sc), represented in Table 1.

All Damage Limit States are evaluated in this work, with an associated probability of failure for each one of them and an associated cost of failure. The reference of the cost is taken from Del Vecchio et al. [15], that evaluated the actual repair cost of a database of 120 RC residential buildings damaged by the 2009 earthquake in L'Aquila, Italy. The costs were normalized and them calculated in dollars, making it possible to use the values for different regions from the study. The Damage States definition used by the authors are associated with the definitions from Hazus manual [14], and the associated costs of failure (c_f) are summarized in Table 1.

Table 1. Damage Limit States considered, capacity (S_C) and cost of failure (c_f) associated (in dollars)

Damage Limit State	S_c (%)	c_f (\$)
Slight Damage	0.40	66.56
Moderate Damage	0.64	89.45
Extensive Damage	1.60	108.91
Complete Damage	4.00	140.34

In each dynamic evaluation of the building, the algorithm check if any of the Limit States is reached, and save the information on the Index Function. Latter, using RWAS algorithm (described in section 2.3), the probability of failure (p_f) for each LS is calculated and, multiplied by the cost of failure, are summed in the objective function. An overview of the proposed methodology is presented in Figure 2, with all the implementation done using Matlab software [17]. All the steps of the methodology are described in details on the following sections.

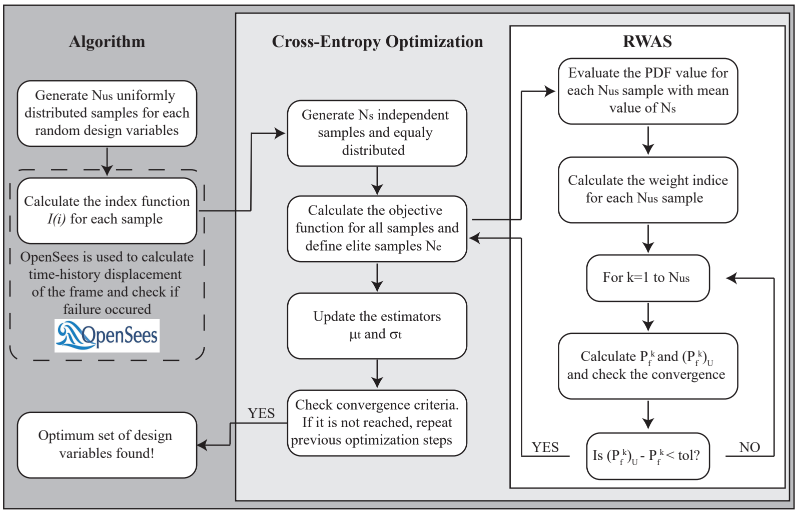


Figure 2. Flowchart of the methodology

2.2 Cross-entropy optimization method

This paper uses the cross-entropy method to find the optimum parameters of the 2D frame considering seismic actions. The full presentation and discussion of the method is done by Cunha [3]. The main idea of the method used herein is to transform the non-convex optimization problem into an equivalent problem to estimate a rare event, which can be efficiently treated as a Monte Carlo like algorithm. It is only necessary that the problem has a single solution. The feasible region is sampled considering a chosen probability distribution, and mean and standard deviation of the samples are used to update the optimum point estimation.

The two steps of the process are defined as sampling and learning. In the sampling step, the feasible region is sampled considering the chosen probability distribution (truncated Gaussian in this case), and the objective function is evaluated in each one of the samples. Next, on the learning step, a subset of these samples, named as *elite sample set*, is defined considering the samples that produces the highest values for the objective function. After that, the distribution is updated using statistics from this elite sample set, modifying the given distribution in a way to make it as close as possible to a Dirac delta centered on the global optimum. The distribution mean value gives an approximation to the global optimum, and its update is done to move the center of the distribution toward the optimization problem optimum, while decreasing the standard deviation and "shrinking" the distribution around its central value [3].

Based on the steps described, the algorithm for the computational implementation of the method, based on Cunha [3], is described below:

Step 1: Define the number of samples N_s and the number of elite samples N_e, where $N_e < N_s$; define the convergence tolerance tol, the maximum of iteration levels t_{max}, a family of probability distributions (this work uses Gaussian distribution, and the equations presented are valid for this distribution) and an initial vector of the parameters of the model;

Step 2: Generate the N_s independent and identically distributed samples to be evaluated;

Step 3: Evaluate the objective function in all N_s samples, sort the results and define the elite samples N_e with the points that better performed;

Step 4: Update the estimators of the mean value (μ_t) and standard deviation (σ_t) with aid of the elite samples set, as shown in Equation 2.

$$\begin{aligned} \boldsymbol{\mu_t} &= \alpha\mu_t + (1-\alpha)\,\mu_{t-1} \\ \boldsymbol{\sigma_t} &= \beta_t\sigma_t + (1-\beta_t)\,\sigma_{t-1} \end{aligned} \tag{2}$$

Where μ_t and σ_t are the estimators with the aid of the elite sample on the actual iteration, α, β and β_t are smooth parameters, and β_t is given by $\beta_t = \beta - \beta\left(1-\frac{1}{q}\right)^q$. The parameters are such that $0 < \alpha \leq 1$, $0.8 \leq \beta \leq 0.99$ and $5 \leq q \leq 10$.

Step 5: Repeat steps 2 to 4 until the stop criterion is met. Herein, $max\,(\sigma) < tol$.

2.3 Improved weighted average simulation (RWAS)

The improved weighted average simulation is a technique developed by Okasha [16] to solve structural reliability problems, based on the weighted average simulation method (WASM) proposed by Rashki et al. [18]. Its main goal is to determine the probability of failure by generating uniformly distributed samples and applying the probability density value as the weight index at each sample. After that, the probability of failure is computed by dividing the sum of the weight indices of all samples [18] [16]. The modification proposed allows the evaluation of the probability of failure with a small number of performance functions evaluation, since the p_f converges faster to the final result with only a fraction of the generated samples [16].

At the beginning of the method, it is necessary the evaluation of the Index function $I(i)$ for all the samples generated using uniform distribution (N_{us}). This step is necessary to distinguish if the samples are located in the failed region ($g_i < 0$) or in the safe region, ($g_i \geq 0$), based on the definition of failure of the problem. The Index function is represented in Equation 3. Since the problem evaluated herein consists on design values considered also as random values, the evaluation of the Index function can be performed only once, since a change in the mean value consists in only evaluate again the weight index.

$$I(i) = \begin{cases} 1, & \text{if } g_i < 0 \\ 0, & \text{if } g_i \geq 0 \end{cases} \tag{3}$$

The next step is to evaluate the PDF value ($f_X(x_n)$) of each random variable, based on the mean value for each variable considered in the step of the cross-entropy optimization method. With the PDF value, the weight indices of the samples are calculated as $W(n) = f_X(x_n) \times f_Y(y_n) \times f_Z(z_n)$, with X, Y and Z being the random variables considered as statistically independent.

The modifications proposed in RWAS consist on sorting the generated samples in a descending order according to the values of their weight indices, with the id number of each sample assigned to the rank r_i according to the place of this sample in the weight index sorting process [16]. The probability of failure is than calculated incrementally in the order of the ranks, considering Equation 4.

$$P_f^k = \frac{\sum_{i=1}^{k} I\left(r_i\right) \cdot W\left(r_i\right)}{\sum_{i=1}^{k} W\left(i\right)} \tag{4}$$

The incremental process can be terminated once a convergence criterion is reached. To evaluate this convergence criterion, it is necessary to evaluate the probability of failure already accumulated at increment k, as defined in Equation 5, which represents the upper limit value for the contributions of the remaining samples at increment k, assuming all of them are located in the failed region [16].

$$R_f^k = \frac{\sum_{j=k+1}^{N} W\left(r_j\right)}{\sum_{i=1}^{N} W\left(i\right)} \tag{5}$$

By doing this, the result is the highest probability of failure that can be predicted at increment k, even with the assumption that the rest of the samples all fail. The probability of failure cannot exceed the upper limit value of p_f, given by Equation 6 [16].

$$\left(P_f^k\right)_U = \frac{\sum_{i=1}^{k} I\left(r_i\right) \cdot W\left(r_i\right) + \sum_{j=k+1}^{N} W\left(r_j\right)}{\sum_{i=1}^{N} W\left(i\right)} \tag{6}$$

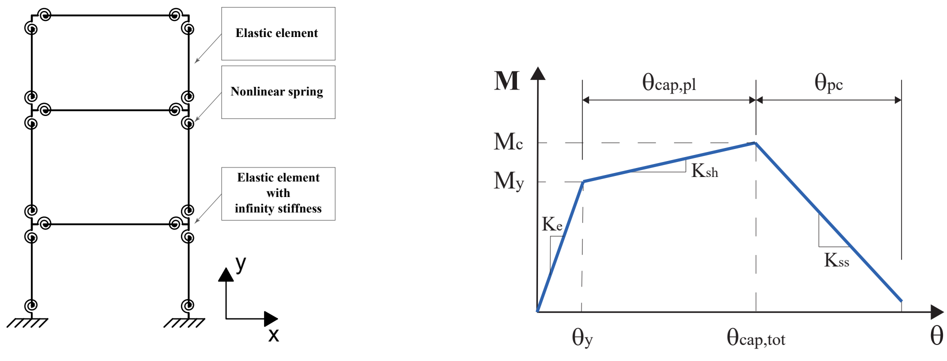
The convergence criteria can be considered as the step where the difference between the upper limit value of p_f and the value of the accumulated probability of failure of the kth increment is smaller than a specified tolerance (TOL), which means $\left(P_f^k\right)_U - P_f^k < TOL$.

2.4 Lumped Plasticity Model

The Finite Element Model used herein consists on a lumped plasticity model developed in OpenSees software [11] and its own library to create beams and columns elements. Figure 3a represents the model in a 2D frame. The model is used to simulate the nonlinear hysteretic response of reinforced concrete (RC) beams or columns under large deformation and is also developed to enable simulation of the nonlinear dynamic response of RC frame buildings under earthquake ground motions.

To properly model the inelastic behavior of beams and columns elements, a nonlinear spring model developed by Ibarra et al. [19] is used. The material, named *uniaxialMaterial IMKPeakOriented* on OpenSees library, is applied to a zero-length element represented by the springs on Figure 3a. Joints are represented by an elastic element with the length of the joint and infinity stiffness. The rest of the element is modeled also with *elasticBeamColumn* element with its area and Young's Modulus of resistance of the material. To account for the degradation of strength and stiffness associated with large deformations, suitable geometric transformations, and a leaning (P-Δ) column are used in the analysis. The effects of foundation flexibility have not been considered at this part of the model development.

The nonlinear spring model consists of a monotonic backbone curve and hysteretic degradation rules to capture post-peak in-cycle softening which are associated with concrete crushing and reinforcing bar buckling at large cyclic deformations [8]. Figure 3b represents the monotonic curve by an idealized trilinear end moment (M) versus chord rotation (θ) response of an equivalent cantilever column. The curve is defined considering five parameters: yield moment capacity M_y; initial elastic secant stiffness to yield point K_e; maximum moment capacity M_c; plastic chord rotation from yield to cap point $\theta_{cap,pl}$; post-capping plastic rotation capacity θ_{pc}. The flexural yield strength M_y generally is computed using strain compatibility approach. It is assumed that sections remain plane and uses an equivalent rectangular compressive stress distribution under ultimate loads with a concrete crushing strain of 0.003 [8]. The eqautions for the model parameters can be found on [8], [20] and [21].



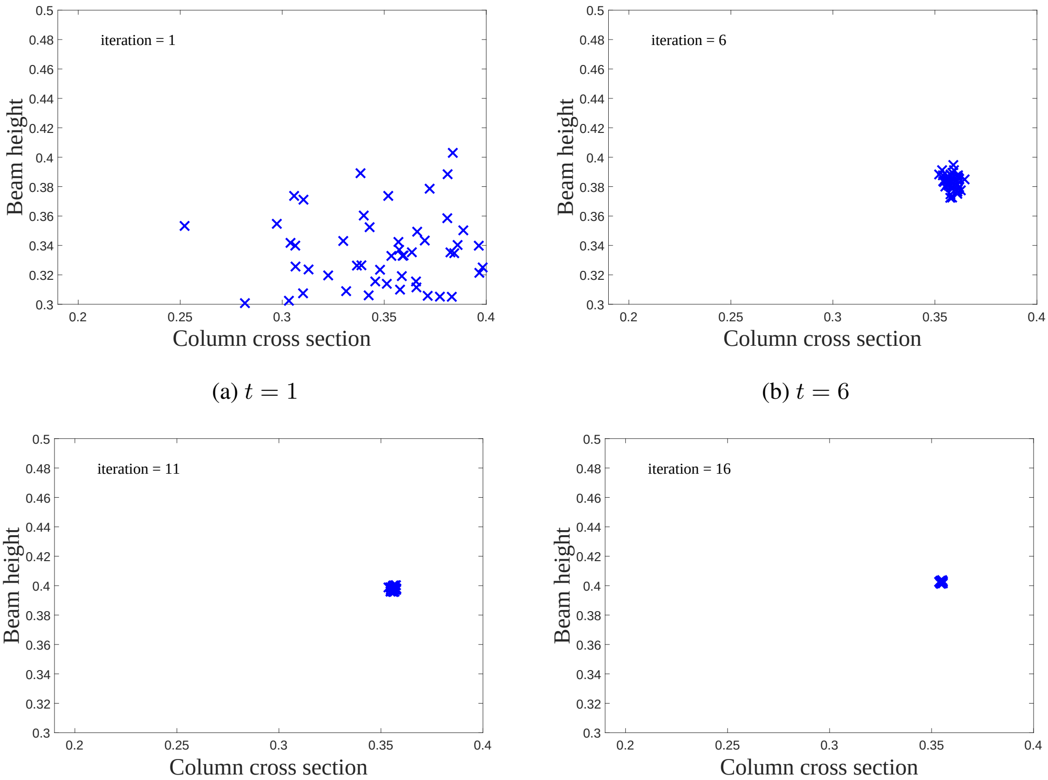
(a) (b)

Figure 3. Representation of the Finite Element Model and Idealized trilinear end moment versus chord rotation

3 Results and Discussion

The solution of the optimization problem is presented. The domain is randomly sampled considering $N_s = 50$ points, considering a truncated Gaussian distribution. The elite samples are selected considering $N_e = round\,(N_s/10)$, the maximum number of iteration is $t_{max} = 150$, and the convergence criteria is set as $tol = 5{\times}10^{-4}$. The smoothing parameters are $\alpha = 0,7$, $\beta = 0.8$ and $q = 5$. The number of uniformly distributed samples for the RWAS method is $N_{us} = 5000$ samples.

Figure 4 shows the domain sampling at different iterations of the algorithm, illustrating the CE method. The evolution of the algorithm is also represented in Table 2, where each line displays the iteration, the total cost obtained by the objective function $(f(d))$, mean and standard deviation of c_h and b_h. In this simulation, the optimum value obtained is \$128.99 with the optimum dimensions of $c_h = 35.49\,cm$ and $b_h = 40.32\,cm$.



(a) $t = 1$ (b) $t = 6$ (c) $t = 11$ (d) $t = 16$

Figure 4. Representation of the optimization method at different iterations (t) of the algorithm

Table 2. Evolution of the CE algorithm

t	$f(d)$ ($)	μ_{c_h} (m)	μ_{b_h} (m)	σ_{c_h} (m)	σ_{b_h} (m)
001	130.88	0.3622	0.3278	0.0128	0.0285
002	130.71	0.3601	0.3505	0.0070	0.0140
003	130.18	0.3587	0.3669	0.0038	0.0101
004	129.63	0.3577	0.3785	0.0029	0.0050
005	129.57	0.3578	0.3836	0.0024	0.0044
006	129.40	0.3577	0.3888	0.0024	0.0035
007	129.28	0.3561	0.3920	0.0015	0.0024
008	129.21	0.3563	0.3944	0.0013	0.0016
009	129.17	0.3564	0.3961	0.0010	0.0014
010	129.12	0.3560	0.3978	0.0008	0.0011
011	129.09	0.3559	0.3990	0.0007	0.0009
012	129.07	0.3556	0.4000	0.0006	0.0008
013	129.04	0.3549	0.4010	0.0006	0.0007
014	129.02	0.3548	0.4019	0.0005	0.0006
015	129.00	0.3546	0.4025	0.0005	0.0005
016	128.99	0.3549	0.4032	0.0004	0.0005

To check the accuracy of the results, the process is repeated 5 times, all with the same algorithm set up, to check the values of the objective function and dimensions of the frame. The summary of the process is shown in Table 3. One should notice that the results presented in Figure 4 and Table 2 correspond to simulation number 5.

Table 3. Results of all the simulations performed

Simulation	$f(d)$ ($)	μ_{c_h} (m)	μ_{b_h} (m)	Number of iterations
01	128.91	0.3556	0.3866	20
02	128.36	0.3525	0.4060	20
03	128.19	0.3640	0.3002	10
04	129.41	0.3558	0.3950	14
05	128.99	0.3549	0.4032	16

The results obtained in the 5 simulations are very close to each other, with the minimum cost of, approximately, $\$129.00$. For simulation 5, the cost of construction corresponds to $\$116.14$ (90.04% of total cost) and the cost of failure is $\$12.86$ (9.96% of total cost). One can also notice that an increase on the mean value of the column cross section makes the beam height decrease to the minimum value, as observed in simulation 03. The performance of the algorithm can be evaluated by the time necessary to complete all 5 simulations, which are 17.84 hours in a Desktop Intel Core i5-10400 2.90 GHz, with RAM 16GB. The step that consumes most of the time on the simulations is the dynamic evaluation of the frame on OpenSees to calculate the Index Function for the RWAS, which also indicates the need to have a Finite Element Model that captures precisely the response of the building with a good processing time, justifying the choice for the Lumped Plasticity Model.

4 Conclusions

The results of this paper show the accuracy of the cross-entropy optimization method, associated with the improved weighted average simulation (RWAS) to calculate the probability of failure of a 2D reinforced concrete frame subjected to a seismic load. All 5 simulations performed converged to the same minimum of the objective function, making the algorithm suitable to find the optimum dimensions of column and beam cross section, considering a Risk Optimization process. This is an important step for the development of a optimization process of reinforced concrete structures with seismic actions, considering the PBEE.

Acknowledgements. This study was financed by the Sao Paulo Research Foundation (FAPESP) - Finance Codes 2020/14072-7 and 2019/13080-9; Brazilian Higher Education Council (CAPES); Brazilian National Council for Research (CNPq)- Finance Code 309107/2020-2; Fundação Carlos Chagas Filho de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ) - Finance Codes 210.167/2019, 211.037/2019 and 201.294/2021. The opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors only and do not necessarily reflect the views of the sponsors or affiliates

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