Approach to connecting non-matching meshes applied to concrete beams of different sizes in multiscale modeling

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Abstract. This work contributes by presenting an alternative way for connecting non-matching meshes. It uses coupling elements that do not add degrees of freedom to the problem. These coupling elements are used for connecting the mesoscale with the macroscale in concrete beams subjected to three-point bending tests in multiscale modeling. The beams are of different sizes, so it is possible to analyze the quality of the coupling between meshes with different ratios between the dimensions of their elements. The problem is solved using a geometrically exact version of the Finite Element Method (FEM). At the mesoscale, a damage model allows representing the degradation of concrete, including the formation of discrete cracks. The studies carried out showed that the coupling elements used allow connecting meshes with different ratios between the sizes of the elements.

Keywords: Coupling element, Non-matching meshes, Mesoscale concrete, Multiscale

1 Introduction

Composite materials show gains in physical or economic properties compared to their individual components. However, they are heterogeneous materials, so structural analysis becomes limited when considering homogenized mechanical properties, mainly when the analysis deals with non-linear mechanical behavior. The representation of the heterogeneity of the composite material improves the quality of the structural analyses, as highlighted by Unger and Eckardt [1], who simulated the degradation of concrete by representing it as a composite of particles (mesoscale representation). The numerical modeling at mesoscale allows the use of simple constitutive models for each phase of the composite and still leads to complex responses for the global behavior of the structures [1].

The simulations of mesoscale structures can be performed using discrete [2] and continuous models. In continuous models, the FEM stands out. In concrete modeling using FEM, in the context of this work, the strategies that use interface elements to represent the Interfacial Transitional Zone (ITZ) and the possible paths for the cracks [3, 4] are efficient. The computational effort and memory consumption in numerical simulations are high regardless of the form of representation of the mesoscale. Therefore, unless supercomputers are used, analyzes are limited to small samples.

The concurrent multiscale analysis technique allows to represent structures larger than those that can be simulated in mesoscale. In this technique occurs the reduction in computational cost and maintenance of the quality of responses when compared with mesoscale analysis. The representation of concrete through concurrent multiscale analysis has been successfully used [1, 5]. The two major challenges of this technique are coupling the scales correctly and determining the domain location that must be discretized on the mesoscale [1].

In concurrent multiscale analysis, the connection between the meshes of each scale can be through transition elements. In this case, the dimension of the finite elements near the interface between scales decreases in size until they reach the dimension of the elements of the smallest scale. So it is possible to connect the scales properly. In this way, matching meshes are obtained. However, the quality of the problem response in this region can be impacted by the presence of distorted elements [6]. Coupling techniques that allow direct connection between non-matching meshes solve this problem and reduce the number of elements, since this transition region no longer exists. Some strategies use penalty methods [7] to couple the subdomains. The use of penalty does not add degrees of freedom to the problem, but it can leave the stiffness matrix ill-conditioned. Other strategies use Lagrange multipliers [8] to enforce the boundary condition between subdomains. A disadvantage of Lagrange multipliers is that they add degrees of freedom to the problem. The connection can be made even using non-conventional elements created from elements of each subdomain in the region to be coupled [9]. The element formulation is modified in order to convert the non-matching interface into a matching one. In this case, there is an important computational cost to build these elements. These studies presented evidence that the non-matching meshes connection techniques still need improvement.

In this work, the elements developed by Paccola and Coda [31], previously used only to represent particles, are used as coupling elements of non-matching meshes with different size ratios between scale elements. Here, the use of these elements to couple macroscale and mesoscale meshes in concurrent multiscale analysis of three-point bending concrete beams is explored. In this way, it is possible to verify the behavior of the coupling strategy when the constitutive model of one of the scales is inelastic. The main advantages of the strategy are that these coupling elements do not add degrees of freedom and have the same mechanical properties of the material used. Therefore, this option does not have the inconvenience of adding degrees of freedom to the problem of methods that use Lagrange multipliers, nor the risk of obtaining an ill-conditioned matrix observed in penalty methods.

2 Numerical Modeling

2.1 Concurrent Multiscale Modeling

In the concurrent multiscale strategy adopted, it is necessary to create meshes for the macroscale and for the mesoscale. As in the macroscale there is only one type of material, only a regular mesh with an adequate linear elastic model is created. For the mesoscale, before generating the mesh, geometries are created to represent the coarse aggregates. The aggregates are randomly generated using the strategy presented in Wriggers and Moftah [10]. The developed code can generate aggregates in the form of regular polygons with different numbers of sides. Then a regular mesh is created for the matrix and the particles. Finally, between matrix elements and between matrix and aggregates, interface elements are inserted. The function of these interface elements is to represent the ITZ and the possible paths for the cracks between the elements of the mortar matrix. Mesoscale degradation is fully explained by the inelastic constitutive model of these interface elements. The regular finite elements that represent mortar and coarse aggregate are maintained with linear elastic behavior. The high aspect ratio interface elements developed by Manzoli et al. [11] are used in this work. This kind of elements are suitable for representing discontinuities. This happens because, in the limit, the behavior observed is the same as the continuum strong discontinuity approach, presented in [12], as demonstrated by Manzoli et al. [13]. Therefore, regular finite elements can be applied in the analysis of concrete cracking as interface elements since they have small height and an inelastic behavior, which in this work is represented by a damage model.

2.2 The positional approach of FEM

The processing code implemented uses the positional approach of FEM [14]. In this approach, the unknowns are the element node coordinates rather than the displacements used in the classical method. It emerged in the work of Bonet et al. [15], who used positions as unknowns. Coda and Greco [14] proposed the method and applied it to 2D frame elements. The positional approach is a naturally non-linear geometric version of FEM, therefore it can be advantageous in the development of more precise analyses, see [16] for instance. In the method, as it is geometrically exact, the internal force vector is not obtained from a linear relationship between the stiffness matrix and the nodal positions. Then the Newton-Raphson method is used for the iterative solution of the problem. To use the technique, it is necessary to calculate the internal force vector and the tangent stiffness matrix for each trial position. These terms are obtained as a function of the specific deformation energy and depend on the constitutive model and the type of element used. The constitutive model used is the Saint-Venant-Kirchhoff model, suitable to describe the behavior of materials that present large displacements and moderate strains. In this work, triangular finite elements with linear approximation are used. The way to calculate the internal force vector and the tangent stiffness matrix of these elements is detailed in Coda [17].

2.3 Damage model

The damage model presented by Manzoli et al. [11] is used on high aspect ratio interface elements. In this model only the tension component of the stress tensor is used, thus it is limited to cases where the mode I fracture predominates, as is the case of beams subjected to bending explored in this work. The damage variable d is given by

$$d = 1 - \frac{q(r)}{r}. \tag{1}$$

The function $q(r)$ defines the softening law and r is a strain-like internal variable.

Considering the loading-unloading conditions and the consistency condition, for a pseudo-time t associated with the loading process, the variable r is always given by the highest value between the effective Cauchy stress normal to the major base of the interface element $\overline{\sigma}_{nn}$ up to that moment and the initial value of the process given by the tensile strength f_t of the material. So it can be written as

$$r = max(\overline{\sigma}_{nn}(s), f_t) \ | \ s \in [0, t]. \tag{2}$$

To represent the softening, the same exponential law already used successfully by Rodrigues et al. [4] for mesoscale concrete was chosen. It is given by

$$q(r) = f_t \exp\left(\frac{f_t^2}{G_f \mathbb{E}} h(1 - r/f_t)\right), \tag{3}$$

where h is the smallest height of the interface element, G_f is the fracture energy for mode I, and $\mathbb{E}$ is the Young's modulus of the material.

The damage implementation algorithm is adapted from the version used by Manzoli et al. [11] and it is the implicit-explicit integration scheme (IMPL-EX) developed in Oliver et al. [18] for convergence improvement.

3 Coupling non-matching meshes

Paccola and Coda [20] developed a strategy to coupling particles and matrix in particulate composite materials modeling. The technique consists of representing the nodal positions of the elements of the particles using the shape functions of the elements of the matrix. Once this is done, all the nodal information of the elements of the particles is written as a function of the coordinates of the nodes of the matrix. Paccola and Coda [20] showed that the internal force and the tangent stiffness matrix of the particles can be calculated as a function of their own nodes and then distributed over the nodes of the elements of the matrix. For this, they are weighted by the shape functions applied to the dimensionless coordinate point corresponding to each node of the elements of the particles within the elements of the matrix. The internal force vector and the tangent stiffness matrix global values are obtained by adding the values obtained for the elements of the matrix with those of the particles.

The formulation used by Vieira [19] to coupling non-matching meshes is the one presented in [20] to connect the matrix and particles with perfect adhesion. Thus, it is possible to perform a strong coupling between the scales. The idea of the strategy proposed by Vieira [19] is to consider that the coupling elements are elements of particles such as those of [20]. However, these elements are positioned in the border region between the scales with at least one node overlapping elements of each scale. Thus, when defining each node of the coupling element as a function of the nodes of the elements on which they are superimposed, the two subdomains are correlated, and consequently they are coupled. An advantage of the strategy is that the representation of coupling elements does not add degrees of freedom to the problem.

3.1 Creating coupling elements

To coupling non-matching meshes, consider the domain of Fig. 1. In the beginning, the Ω_1 and Ω_2 subdomains share the Γ_{12} border, but are decoupled. The subdomain Ω_1 has the elements e_1 and e_2 and the nodes 1, 2, 3 and 4. The other nodes and elements are in the subdomain Ω_2. The strategy chosen to generate the coupling elements was to transform elements of the smallest scale that have at least one node on the edge Γ_{12} in this type of element. The coupling elements l_1, l_2, l_3 and l_4 were created from the elements e_3, e_4, e_5 and e_6 of the subdomain Ω_2. The nodes a, c, and e of the coupling elements on the edge Γ_{12} are considered to belong to the subdomain of the largest scale. This ensures that every element will have at least one node at each scale. The coupling elements are identical to the elements from which they were created. So, in addition to connecting scales, they can also fulfill the structural function of the elements that gave rise to them and replace those elements. That is, the elements e_3, e_4, e_5 and e_6, and the nodes 5, 6 and 7, which are present only in these elements, can be excluded from the structural analysis. By making this substitution, the stiffness of the coupling element is kept equal to that of the element that gave rise to it. This point illustrates an advantage of the technique used, which is the fact that it does not need to use penalty to define the stiffness of the coupling element, which is needed in Bitencourt et al. [7] for instance.

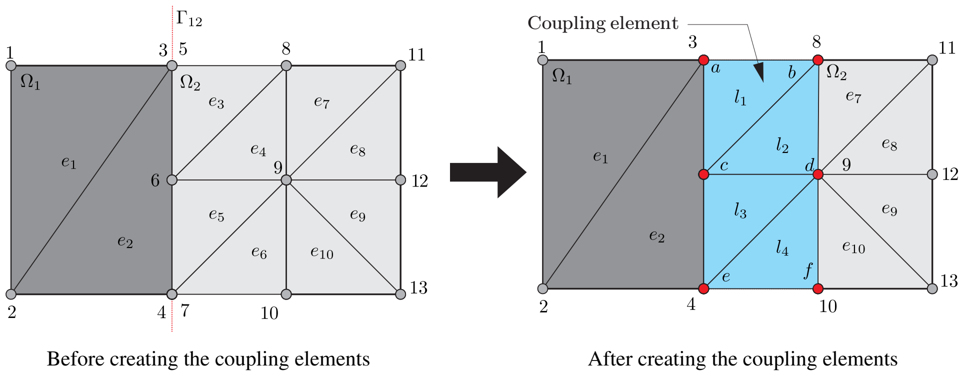

Figure 1. Procedure for creating coupling elements.

4 Numerical example

In this example, the behavior of the technique of connecting non-matching meshes in the coupling of meshes with different ratios between element sizes is verified. Three-point bending tests of 4 beams were simulated. The beam geometry and boundary conditions are shown in Fig. 2. The dimensions of each beam are written as a function of the parameter D that was adopted with the values of 50 mm, 100 mm, 200 mm and 400 mm. The lengths (l), heights (h) and spans of each beam are shown in Table 1. The beams were named $V1$, $V2$, $V3$, and $V4$ in ascending order of size. The beams have a notch at the bottom edge at the midpoint of the length that induces the cracking. Therefore, a region centered on the notch of length and height D was represented in mesoscale. The thickness (e) of all beams is 50 mm. Each beam size was simulated considering the scales coupled directly through the element size transition and then using non-matching meshes connected by the developed coupling elements. This was done to compare the answers with each other.

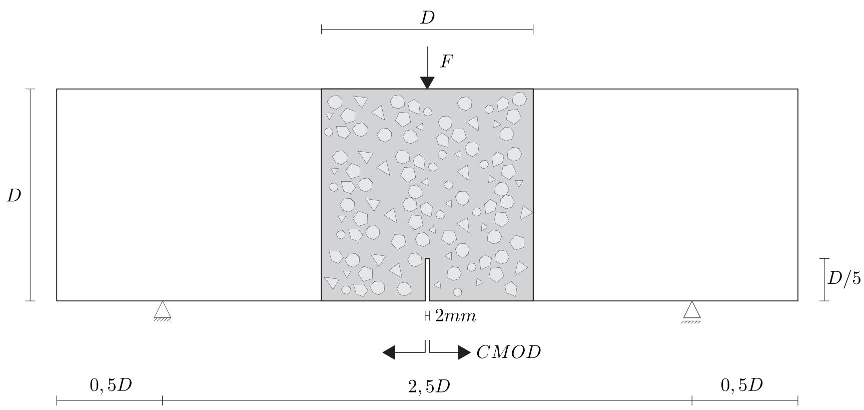

Figure 2. Geometry and boundary conditions.

The coarse aggregates were generated in the form of regular polygons with 5, 6, 7 and 8 sides and randomly positioned considering that they occupy 40% of the sample area. They are represented in the size range limited by $d_{min} = 4$ mm and $d_{max} = 10$ mm and follow the average grading curve obtained from those presented by Grégoire et al. [21].

In all analyses, the generated mesh is formed of triangular elements with linear approximation. In cases where the scale meshes are conforming, the number of nodes for beams $V1$, $V2$, $V3$ and $V4$ was, respectively, 2882, 10027, 41789 and 163957. In cases where the scale meshes are non-conforming, the number of nodes for beams $V1$, $V2$, $V3$ and $V4$ was, respectively, 2842, 9609, 41203 and 163350.

The Table 2 presents the mechanical properties adopted. The Young modulus of concrete is known from the experimental study by Grégoire et al. [21] and the other properties, except for those of the coupling elements, were adopted with the same values estimated by Rodrigues et al. [4]. Rodrigues et al. [4] simulated these beams in mesoscale and these property values made it possible to obtain responses close to the experimental ones [21].

Table 1. Dimensions of the simulated beams.

Name	D (mm)	Dimensions $h \times l \times e$ (mm)	Span(mm)
V1	50	$50 \times 175 \times 50$	125
V2	100	$100 \times 350 \times 50$	250
V3	200	$200 \times 700 \times 50$	500
V4	400	$400 \times 1400 \times 50$	1000

The coupling elements used in the simulations with non-matching meshes present properties identical to those of the adopted mortar. In the simulations with matching meshes, the interface elements between the macroscale and the mesoscale were maintained with elastic behavior. These elements do not exist in cases where non-matching meshes are used. All analyses in this study were performed for plane stress state. The load is applied in the form of displacement, with application of vertical increments where the force F is observed in Fig. 2. The total applied displacement was $0.14mm, 0.20mm, 0.32mm$ and $0.42mm$ for beams V1, V2, V3 and V4, respectively.

Table 2. Material parameters for the three-point bending tests.

Material	Young modulus (MPa)	Poisson ratio	G_f (N/mm)	f_t (MPa)
Concrete	37000	$0,20$	$-$	$-$
Coarse aggregate	50000	$0,20$	$-$	$-$
Mortar	30200	$0,20$	$-$	$-$
Coupling Element	30200	$0,20$	$-$	$-$
ITZ	30200	$0,00$	$0,05$	$2,6$
Int. matrix-matrix	30200	$0,00$	$0,1$	$5,2$
Int. macro-mesoscale	30200	$0,00$	$-$	$-$

Figure 3 presents the experimental curves and those obtained in this work for the 4 beams with the two ways of connecting the domains. The curves relate the reaction force F with the measurement of Crack Mouth Opening Displacement(CMOD). The CMOD was calculated as the relative horizontal displacement between the two lower nodes of the notch's vertical faces. Analyzing these curves, it is possible to see that, in general, in all cases the answers were close to the experimental results of Grégoire et al. [21]. The numerical curves showed some differences in the softening stage, but acceptable, since this stage is marked by a strong influence of the heterogeneity of the structure. This analysis allows us to conclude that the concurrent multiscale concrete modeling technique used in this study works well regardless of sample size. Therefore, once the numerical model is correct, we start to analyze the coupling between the scales. Comparing the responses obtained with matching and non-matching meshes, it is possible to see that they were very close for the 4 beam sizes. Noticeable differences in the curves appeared only in beams V2 and V4, however they are small and only appeared in the final stage of the analysis. Unger and Eckardt [1] emphasize that the strong coupling of non-matching meshes can cause stress concentration in the proximity of the edge between the subdomains, which influences the quality of the structural responses. However, even with an inelastic model at the mesoscale, it was not observed the degradation of the concrete close to the coupling elements, demonstrating the robustness of the proposed strategy. The dimension of the mesoscale elements is the same for all beam sizes, since it is related to the dimension of the coarse aggregates, which is the same in each case. However, the macroscale mesh is refined just enough to obtain a numerical solution that converges. In the case studied, the macroscale elements have sides of the order of 2, 4, 8, 16 times greater than the sides of the coupling elements (or mesoscale elements) for beams V1, V2, V3 and V4, respectively. The presented results confirm the efficiency of the coupling elements developed for different size ratios between the elements of each scale.

The images of the cracks obtained in the final step of the analysis are shown in Fig. 4, which increases the displacement of each beam by 30 times. In all cases, the cracks are as expected for a structure with the boundary

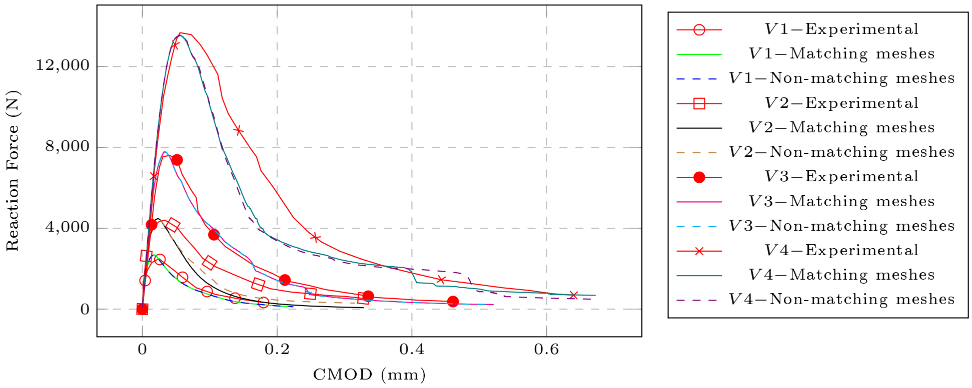

Figure 3. Comparison of the numerical results with those obtained experimentally by [21]: reaction force-CMOD curves for beams of different sizes and ways to couple scales.

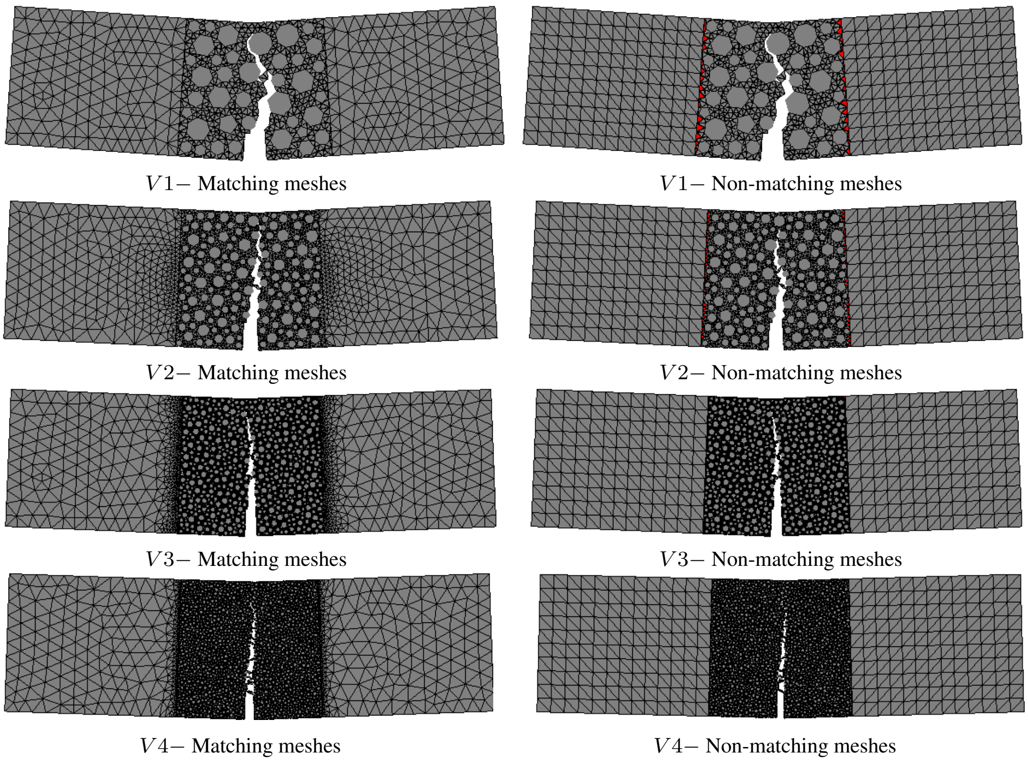

Figure 4. Numerical crack patterns obtained for beams of different sizes without and with use of coupling elements.

conditions used. It was observed that, regardless of the size of the beam, the use of coupling elements to connect non-matching meshes had little influence on the way the crack evolved. The cracks obtained for each beam size with matching and non-matching meshes were very similar.

5 Conclusions

The non-matching meshes coupling strategy applied to concurrent multiscale modeling of concrete beam degradation proved to be efficient. For the different ratios between the dimensions of the elements of each scale, the proposed coupling elements fulfilled their function as if the connection between the scales were with matching meshes. The coupling elements did not cause excessive stress concentration in the border region between scales to result in incorrect inelastic deformations.

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