XLIII CILAMCE
21-25, November, 2022
FOZ DO IGUAÇU - PR - BRAZIL

Analysis of Isolated Battered Pile and Soil Interaction via BEM/FEM Coupling

Ray C. S. Silva[1], João B. Paiva[1], Endi S. Luamba[1]

[1]*Dept. of Structural Engineering, São Carlos School of Engineering, University of São Paulo*
Av. Trabalhador São-Carlense, 400- Parque Arnold Schimidt, 13566-590, São Carlos- SP, Brazil
raycalazans@usp.br, paiva@sc.usp.br, luamba@usp.br

Abstract. The use of battered piles is more and more recurrent in view of their efficiency in certain problems, these are deep-foundation elements widely used in civil engineering due to their ability to reach more resistant soil layers and to support large loads. Bearing in mind that the numerical analyzes appear as an alternative in relation to the empirical analyzes, the present work aims to perform the numerical analysis of isolated battered piles and in groups through the BEM/FEM coupling. The pile is modeled by several finite elements of three-dimensional frame with two nodes, five nodal parameters and any inclination. The soil is modeled by the boundary element method, being considered a semi-infinite, elastic-linear, homogeneous and isotropic medium. Having been the fundamental solution of Mindlin used, the soil discretization is done only on the contact surface with the pile, it is not necessary to discretize the soil surface. The coupling of the BEM / FEM formulation is done considering the transformation of the matrix of soil coefficients (BEM) into a matrix equivalent to the FEM, which added to the stiffness matrix of the three-dimensional frame provides the final system. The interaction forces at the stake-ground interface have a linear distribution. The results obtained were validated by comparison with those available in the literature, showing effectiveness and robustness of the formulation. Finally, the results of which showed little influence of the angle of inclination on the displacements of isolated battered piles.

Keywords: Battered Piles, Soil-Structure Interaction, BEM/FEM Coupling.

1 Introduction

Piles are structural elements that are part of the pile-soil foundation system. They are generally used when the surface layers of soil do not have sufficient strength, and their resistant capacity is composed of two parts: shaft strength and tip strength. The analysis of the pile-soil system is usually carried out with the piles arranged vertically. However, battered (or inclined) piles are usually used in some specific situations when this element starts to actively participate in the balance of the structure, caused by horizontal efforts of great preponderance, as in the case of bridges, retaining walls over piles, etc. In na engineering Project it is necessary to determine the maximum efforts that the pile can withstand and also the maximum deflection that this foundation element will suffer. Given the complexity to determine these quantities precisely analytically, numerical methods appear as an alternative, such as the Finite Element Method (FEM), Boundary Element Method (BEM), Finite Difference Method (FDM), among others.

The pile-soil interaction has long been one of the topics that has received special attention from researchers around the world, given its wide practical applicability [1]. One of the first numerical studies on pile-soil behavior was the work of Poulos & Davis [2], in which FDM was used to solve the elasticity equations of the problem of an incompressible cylindrical pile immersed in an infinite medium. In the context of the BEM, the use of Mindlin's fundamental solutions [3] in the work of Nakaguma [4] made the method more competitive, since it would no longer be necessary to discretize the free surface of the soil. When FEM is used in semi-infinite domain analysis, as is the case with soil, it is necessary to discretize the entire domain or a large part of it, generating a large computational cost for storage and resolution of the final system. Even with the disadvantages presented by the FEM for this type of problem, some works in this direction have already been carried out, as is the case of the works Ottaviani [5] and Ghasemzadeh et al. [6].

The BEM/FEM combination has been shown to be suitable for solving problems comprising infinite domains interacting with finite domains, allowing a better use of both methods [7]. The piles, as they are media with finite dimensions, are modeled by the finite element method (FEM). This method has proven to be the best option for the so-called “domain problems” due to its practicality and efficiency, becoming the most widespread numerical method in the technical environment [8]. However, when it is necessary to analyze infinite or semi-infinite environments such as the soil, this method becomes limited, in view of the large number of elements to be used. An alternative that has been shown to be viable for modeling these media is the boundary element method (BEM), where in elastic analysis the medium is discretized only in its contour, reducing the number of equations and variables involved and reducing the computational cost. [9]. One can cite the works of Mendonça and Paiva [10] and Luamba and Paiva [11].

In this work, the formulation of Luamba and Paiva [11] is extended to the case of inclined piles subject to forces and moments at their top. For this, a rotation matrix is used for the finite pile elements and the degree of freedom related to the rotation in relation to the longitudinal axis is restricted. Finally, some validation examples are presented in order to demonstrate the robustness of the formulation.

2 Pile

2.1 Finite Element

The pile in this work is modeled as a finite element of a three-dimensional beam. Its discretization is made by a one-dimensional element and is represented by its barycentric axis. The element is formed by two nodes, with five degrees of freedom per node (three translations and two turns), totaling ten degrees of freedom. It is worth mentioning that the torsional stiffness was neglected and, therefore, the degree of freedom related to the rotation in the longitudinal axis was disregarded. As a result, we have the following finite element, nodal parameters, linear force distribution and nodal forces:

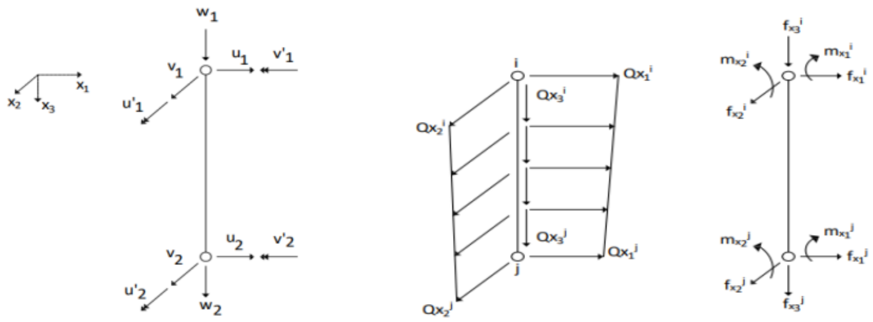


Figure 1. Finite Element

The FEM system of equations is given by:

$$[K]\{u\} = \{f_{nos}\} + [T]\{Q\} \tag{1}$$

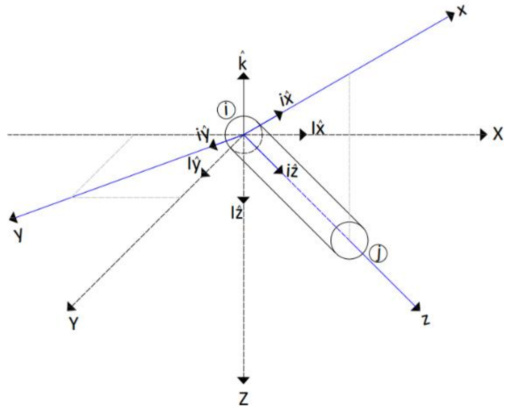
Where: [K] is the stiffness matrix of the pile, {u} is the vector of nodal displacements of the pile, $\{f_{nos}\}$ is the vector of the external forces applied to the top of the pile, [T] is the matrix that transforms pile-soil interface tractions into equivalent nodal forces, {Q} is the vector of pile-soil interface tractions.

The matrix of transformation for each element $[T^e]$ is given by:

$$\left[T^e\right]=\begin{bmatrix} \frac{7}{20}l & 0 & 0 & 0 & 0 & \frac{3}{20}l & 0 & 0 & 0 & 0 \\ 0 & \frac{7}{20}l & 0 & 0 & 0 & 0 & \frac{3}{20}l & 0 & 0 & 0 \\ 0 & 0 & \frac{l}{3} & 0 & 0 & 0 & 0 & \frac{l}{6} & 0 & 0 \\ \frac{1}{20}l^2 & 0 & 0 & 0 & 0 & \frac{1}{30}l^2 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{20}l^2 & 0 & 0 & 0 & 0 & \frac{1}{30}l^2 & 0 & 0 & 0 \\ \frac{3}{20}l & 0 & 0 & 0 & 0 & \frac{7}{20}l & 0 & 0 & 0 & 0 \\ 0 & \frac{3}{20}l & 0 & 0 & 0 & 0 & \frac{7}{20}l & 0 & 0 & 0 \\ 0 & 0 & \frac{l}{6} & 0 & 0 & 0 & 0 & \frac{l}{3} & 0 & 0 \\ -\frac{1}{30}l^2 & 0 & 0 & 0 & 0 & -\frac{1}{20}l^2 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{30}l^2 & 0 & 0 & 0 & 0 & -\frac{1}{20}l^2 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

2.2 Rotation Matrix

To perform the BEM/FEM coupling, it is necessary to write the soil and pile equations in the same coordinate system, since it is known that the fundamental soil solutions are written in relation to the global system, while the mechanical magnitudes of the pile are in its system. place. Consequently, it is necessary to define a rotation matrix that relates the global coordinate system and the adopted local coordinate system.



.

Figure 2. Cordinate system

The rotation matrix for the BEM/FEM coupling system of equations is given by:

$$[R]=\begin{bmatrix} [r] & [\bar{0}] & \cdots & [\bar{0}] \\ [\bar{0}] & [r] & \cdots & [\bar{0}] \\ \vdots & \vdots & \ddots & [\bar{0}] \\ [\bar{0}] & [\bar{0}] & [\bar{0}] & [r] \end{bmatrix} \tag{3}$$

Where:

$$[r]=\begin{bmatrix} \frac{\cos\theta_{zX}\cos\theta_{zZ}}{|\hat{y}|} & \frac{\cos\theta_{zY}\cos\theta_{zZ}}{|\hat{y}|} & -|\hat{y}| \\ \frac{-\cos\theta_{zY}}{|\hat{y}|} & \frac{\cos\theta_{zX}}{|\hat{y}|} & 0 \\ \cos\theta_{zX} & \cos\theta_{zY} & \cos\theta_{zZ} \end{bmatrix} \tag{4}$$

$$\left[\bar{0}\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

3 Soil

In this work the soil is considered to be a semi-infinite medium and the variables of interest are obtained through the Mindlin solution [3]. For this, the Venturini approach [7] is used, which considers load lines applied in the semi-infinite medium as a way of representing the interaction forces along the pile. Thus, the representation of the soil is restricted to discretizing this load line where there is the pile-soil interaction. The discretization adopted is the same used in the FEM for piles, as shown in Figure 3.

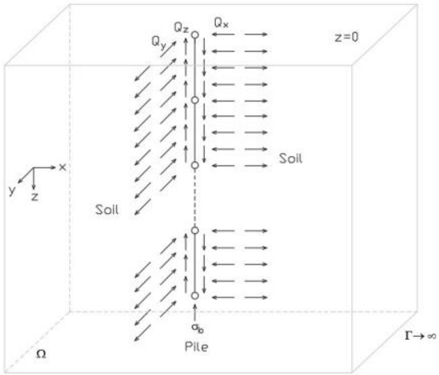


Figure 3. Interface tractions acting along the pile and in the soil domain.

Considering the medium as linear elastic, homogeneous, isotropic, neglecting the volumetric forces and the forces on the free surface of the soil, we have the following integral displacement equation for a domain with N load lines (piles):

$$u_i(s) = \sum_{e=1}^{N_e} \int_{\Gamma_e} u_{ji}^{(*)} q_i^e \, d\Gamma \tag{6}$$

For the numerical resolution of the problem, it is necessary to discretize the load line into a finite number of linear isoparametric elements. Thus, the numerical integration is done by the Gauss-Legendre quadrature. In order to avoid singularity problems, the strategy of Ferro [12] is used, where an integration is performed with the field points along the pile circumference. Finally, Equation (6) can be rewritten matrix as follows:

$$\left\{u_{MEC}\right\} = \left[G\right]\left\{Q_{MEC}\right\} \tag{7}$$

Where: $\{u_{MEC}\}$ is the vector of nodal displacements of the load line (pile) obtained by the BEM, [G] is the nodalized soil influence coefficient matrix, $\{Q_{MEC}\}$ is the vector of interface forces.

4 Coupling

In possession of the BEM equations for the load line Eq. (7) and the FEM equation for the pile Eq. (1), it is possible to perform the coupling through the equilibrium and compatibility conditions between its nodes. In addition, the rotation matrix of Eq. (3), (4) and (5) allows transforming the local coordinate system of the pile to a global coordinate system that coincides with the fundamental solution of the BEM. Thus, assuming that the distributed forces are the pile-soil interaction forces, we have:

$$\left[\bar{K}\right]_G \left\{u\right\}_G = \left\{f_{nos}\right\}_G \tag{8}$$

Where:

$$\left[\bar{K}\right]_G = \left(\left[K_{MEF}\right]_G + \left[K_{MEC}\right]_G\right) \tag{9}$$

$$\left[K_{MEF}\right]_G = \left[R\right]^T \left[K\right]_L \left[R\right] \tag{10}$$

$$[K_{MEC}]_G = [T]_G [G]_G^{-1} \tag{11}$$

$$[T]_G = [R]^T [T]_L [R] \tag{12}$$

Where the subscript G and L represent the global and local coordinate system, respectively.

5 Results

In this section, three examples of validation of the proposed formulation are presented. The examples seek to analyze the behavior of the battered pile for three types of stress (axial force, transverse force and concentrated moment) as a function of the inclination angle. 20 elements are used to discretize the pile.

5.1 Battered pile under axial loading

An isolated pile with inclination of relative to the vertical axis is subjected to an axial force of 1000kN. The pile geometry, soil and pile properties are shown in Figure 4 (a). The results obtained are compared with Poulos and Madhav [13] in Figure 4 (b).

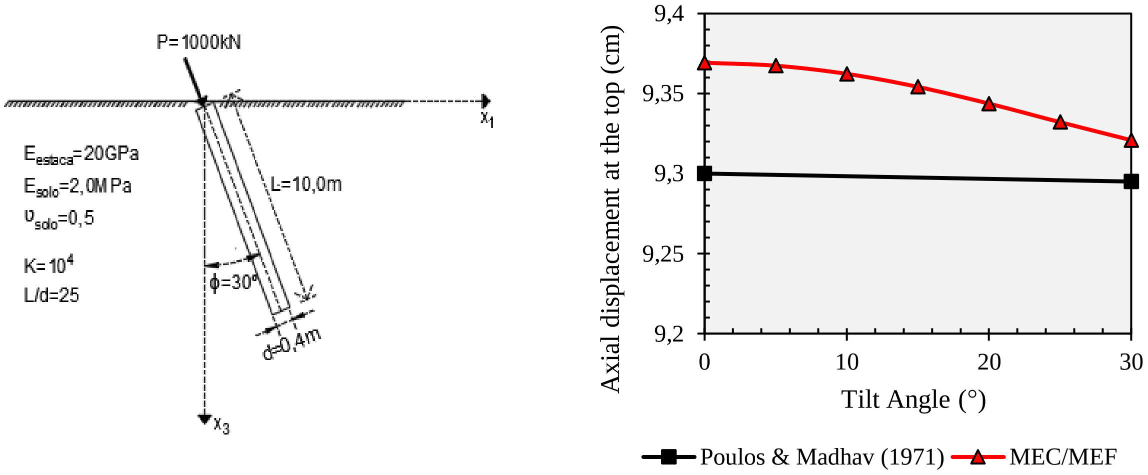


Figure 4. Battered pile under axial loading (a) Axial displacement at the top (b)

A good fit between the authors' results can be seen, with differences smaller than 1%. The results obtained by Poulos & Madhav [13] have very little variation, while those obtained by the BEM/FEM (MEC/MEF) coupling show that as the inclination angle increases, the displacements are smaller. However, for both formulations, the pile inclination angle has little influence on the axial displacement (less than 0.5%).

5.2 Battered pile under transverse loading

This example consists of an inclined pile with a transverse force applied to its top. The force is always applied in the direction perpendicular to the pile direction. As a result, even when the angle of inclination is varied, only bending will occur in the pile. Problem data is provided in Figure 5.

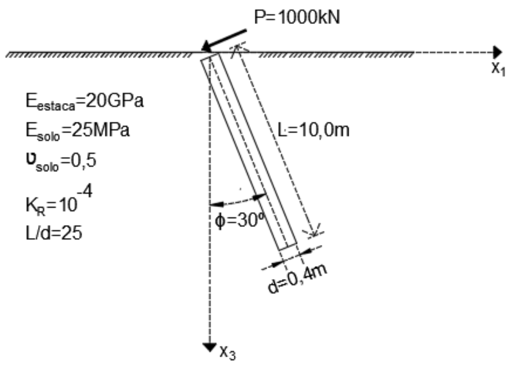


Figure 5. Battered pile under transverse loading

The rotations and transverse displacements at the top of the pile as a function of the inclination angle can be seen in Figure 6, where the results obtained by Poulos and Madhav [13] are also presented.

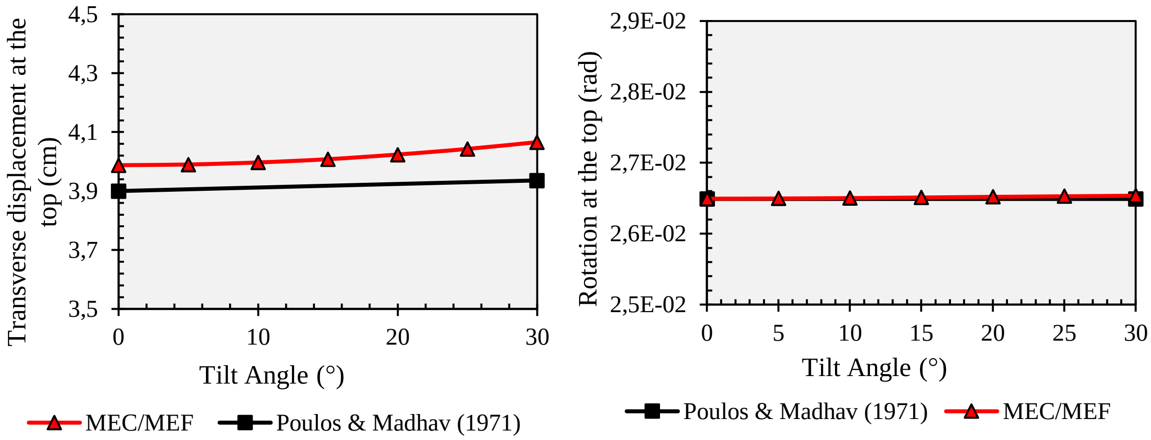


Figure 6. Transverse displacement at the top (a) Rotation at the top (b)

A good agreement is observed between the authors' results, especially for rotations, as the difference for transverse displacements is less than 3%. In the present example, when analyzing the transverse displacements of Figure 6 (a), it can be seen that the angle of inclination increases the displacements, in the BEM/FEM coupling more than in Poulos and Madhav [13]. However, this increase is still not significant, since it remains below 2% in the range of angles analyzed. As for the rotations, there is almost no change as a function of the angle of inclination in both formulations.

5.3 Battered pile under concentrated moment

The same pile as in the previous example is now subjected to a moment loading at its top.

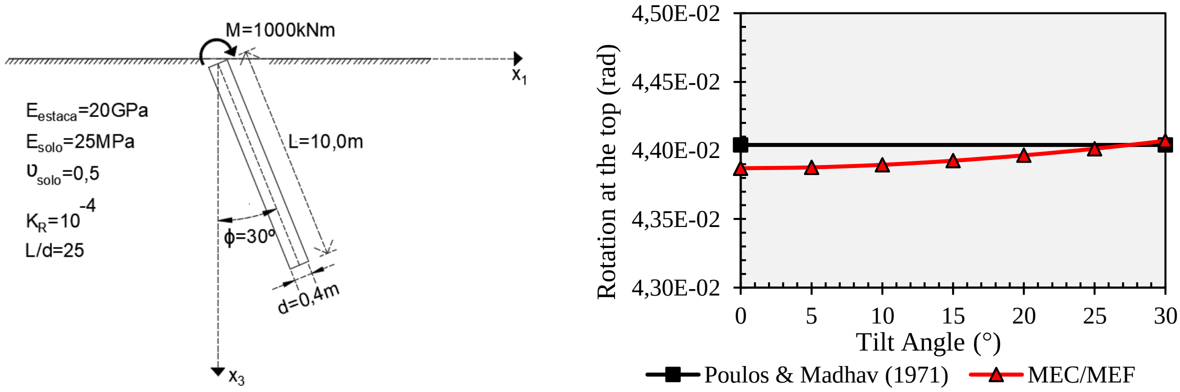


Figure 7. Battered pile under concentrated moment (a) Rotation at the top (b)

It was found that the transverse displacements of this pile are equal to the rotations of the pile in the previous example (Fig. 6 (b)). This fact is justified because it is a linear-elastic analysis with forces/moments of the same intensity, and the Maxwell-Betti reciprocity theorem is valid.

The rotation at the top of the pile as a function of the angle of inclination is given in Figure 7 (b). Again, very little variation of the response in relation to the angle of inclination is observed (less than 1%). There is great agreement between the results and the formulation by Poulos and Madhav [13], in which the variation of rotation as a function of the angle of inclination is not considered.

6 Conclusions

The proposed BEM/FEM formulation for inclined (or battered) piles showed good robustness, as can be seen in the exposed examples. As for the variation of results as a function of the angle of inclination of the pile, a priori, there was little importance. However, this finding still needs further studies, such as a parametric analysis.

Acknowledgements. The authors are grateful to the Coordination for the Improvement of Higher Education Personnel – CAPES, for the scholarship provided to the first and third author.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] E. S. Luamba. Análise da interação casca plana - estaca - solo via acoplamento MEC/MEF tridimensional e suas aplicações. MSc. Dissertation. University of São Paulo, São Carlos, 2018.
[2] H.G. Poulos; E.H. Davis. The settlement behavior of single axially loaded incompressible piles and piers. Géotechnique, v. 18, p. 351-371, 1968.
[3] R. D. Mindlin. Force at a point in the interior of a semi-infinite solid. Office of naval research project nº. 064-388. Technical report nº. 8, May, 1953.
[4] R. K. Nakaguma. Three-dimensional elastostatics using the boundary element method. Ph.D. Thesis - University of Southampton, Southampton, 1979.
[5] M. Ottaviani. Three-dimensional finite element analyses of vertically loaded pile groups. Géotechnique, v. 25, p. 159-174, 1975.
[6] H. Ghasemzadeh, M. Tarzaban; M. M. Hajitaheriha. Numerical Analysis of Pile-Soil-Pile Interaction in Pile Groups with Batter Piles. Geotechnical and Geological Engineering, v. 36, p. 2189-2215, 2018.
[7] W. S. Venturini. Boundary Element Method in Geomechanics. 1º Ed., Springer Berlin, Heidelberg, 1983.
[8] O. C. Zienkiewicz; R. L. Taylor; J. Z. Zhu. The Finite Element Method: Its Basis and Fundamentals. 7º Ed. Butterworth-Heinemann, Oxford, 2013.
[9] C. A. Brebbia; J. C. F. Telles; L. C. Wrobel. Boundary Element Techniques: Theory and Applications in Engineering. 1º Ed., Springer-Verlag Berlin, Heidelberg, 1984
[10] A.V. Mendonça; J.B. Paiva. An elastostatic FEM/BEM analysis of vertically loaded raft and piled raft foundation. Engineering Analysis With Boundary Elements, v. 27, pp. 919-933, 2003.
[11] E. S. Luamba; J. B. Paiva. Static analysis of axially loaded piles in multilayered soils using a BEM/FEM formulation. Engineering Analysis With Boundary Elements, v. 135, p. 63-72, 2022.
[12] N. C. P. Ferro. Uma combinação MEC/MEF para análise da interação solo-estrutura. Ph.D. Thesis – University of São Paulo, São Carlos, 1999.
[13] H. G. Poulos; M. R. Madhav. Analysis of the movement of battered piles. Proc. 1º Aust. N. Z. Conf. On Geomechs., Melbourne, p. 268-275, 1971.