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Design of a Two Degree-of-freedom Tuned Mass Damper for a Suspension Bridge Model

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Abstract. Tuned Mass Dampers (TMDs) are very useful when one aims to mitigate vibration-related problems. This device, once attached to a structure, considerably attenuates the effects of dynamic loads (e.g, wind and seismic activities). Although TMDs exhibit a favorable performance over one specific frequency range, structures have multiple natural frequencies, and sometimes this raises the necessity of attaching a different TMD for each one of them. However, this kind of solution often overloads the primary structure and limits TMDs' damping performance (weight penalty). The purpose of this work is to design a TMD capable of damping two vibration modes of a suspension bridge's deck. For this, the primary structure is modeled by the Finite Element Method, using ANSYS® combined with a MATLAB® routine, then it is conceived its modal and harmonic analysis, identified the targeting modes, and applied the "Equal Peak Design" technique to estimate the TMD's optimum damping and frequency ratio. Based on those factors, it is determined the final TMD dimensions. The results shows the developed TMD has the potential to significantly reduce deck's vibration levels, for its parameters can be effectively tuned to realize the target modes control.

Keywords: Two Degree-of-freedom Tuned Mass Dampers, Finite Element Method, Passive Vibration Control, Natural Frequencies, Numerical analysis.

1 Introduction

Over the last century, due to the advance of the construction technology, ever increasing computational power, and a refined control of the materials quality, the number of complex and long span structures have increased. Many are the forms engineers found to overcome those long spans, and one of the most popular were the suspension bridges. However, this type of structure, when responding to dynamic loading, tend to present a very low natural damping, i.e. below 1%, which makes the dissipation of the generated energy the main issue when designing [1].

As the span increases, the structure's stiffness decreases, so it does its natural frequency, overlapping with low frequency excitation sources, e.g., traffic induced vibration, flutter instabilities, and cable-structure interactions [2–4]. When intending to solve this problem, it is needed to analyze feasible damping systems capable of attenuating aerodynamic effects that may occur. Some of them are Tuned Mass Dampers (TMDs), viscous damper for the cables [5], and active control using active tendons [6]. In this study a TMD (or Dynamic Vibration Absorber) is employed to mitigate vibration effects on a numerical suspension bridge model.

Since Frahm [7], TMDs have been widely used when aiming to decrease - "eliminate" - vibration related problems in machinery. However, their application in civil structures began to intensify only after the important work of Den Hartog and Ormondroyd [8], e.g. London Millennium Bridge, Rio-Niterói Bridge, and Taipei 101. A Tuned Mass Damper is a secondary vibrating mass attached to a structure and adequately tuned with the its resonant frequency [9]. When optimally tuned, the only parameter responsible for its performance is the mass ratio in relation to the primary structure's [10].

Nonetheless, a structure has as many vibration modes, as degrees-of-freedom (DOF). This situation may raise the necessity of linking a TMD for each critical mode. This kind of solution, in practical applications, would probably overload the main structure. The annexation of excessive masses, could easily compromise the performance of the devices appended thereon. This phenomenon is called "weight penalty" and it tends to demand the limit for the TMD mass to be between 1-3% of the primary structure total mass [11].

Many works have shown countless TMD applications. Yoon et al. [12] developed and optimized a resonance-based mechanical dynamic absorber able to damp three natural frequencies of two cantilever beams. Zhu et al. [13] studied and optimized distributed dynamic vibration absorbers for suppressing vibrations in plates, achieving optimal suppression effect of higher order modal vibration. Cieplok and Sikora [14], and Pan and Zhang [15], studied the effect of having more than one mass within the same dynamic absorber showing how this type of solution is able to widen the device's bandwidth of operation. Yang and Dai [16], and Ma et al. [17], designed a two and three degree-of-freedom TMD, respectively, aiming only the main structure's fundamental mode. In both cases the result was a significant reduction of the vibration levels.

When addressing the control of bridge's many vibration modes, some studies promoted the use of multiple TMDs to control each mode separately, Tubino and Piccardo [18] applied it to pedestrian induced vibration, and Wang et al. [19] to high speed trains induced excitation. Nonetheless, as the number of targeted modes increase, so does the weight penalty associated with the extra masses attachment to the primary structure. Seeking to avoid so, in this study, it is designed a Two Degree-of-freedom (2DOF) TMD targeting two vibration modes at the same time (1^{st} bending, and 1^{st} torsional) for a Finite Element Suspension Bridge Model, applying the theoretical background proposed by Den Hartog [20]. Other works pursued the same goal, while using different approaches. Lin et al. [21], e.g., considered the 2DOF TMD with its two modes coupled. Here, the followed procedure was similar to the executed by Mokrani et al. [11] and Meng et al. [22], in which the 2DOF TMDs had their physical coordinates decoupled.

2 Methodology

2.1 Suspension bridge scale model

This study's primary structure is exposed in Fig. 1a. The deck has two main girders of 120 cm. Their cross-section is 1 cm by 1.5 cm. Above them, there are 11 transverse beams (15 cm by 3 cm; thickness: 0.5 cm). The transverse beams are equally spaced by 10 cm (axis to axis). There are 11 suspending cables (Ø0.2 mm) connecting the bridge's deck to a main cable (Ø0.5 mm). All suspenders are spaced by 10 cm. The total height of the structure is 31.5 cm, whereas the main cable's sag (ratio between the structure's height and span) is 15 cm. Total weight is 5.28 kg.

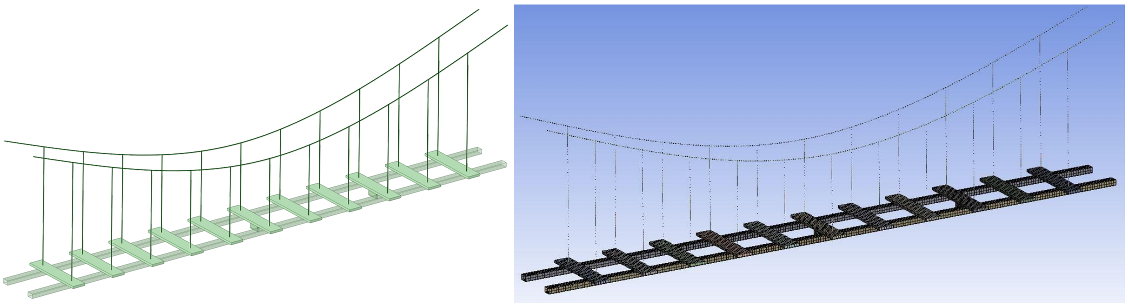
The model's boundary conditions were defined as recommended by Serap et al. [4] and Li et al. [23]. The girders are articulated, free to rotate about any axis, while not translating. Whereas the main cable pair are fixed at both ends. All contacts between the parts are defined as bonded. The structure is modeled in ANSYS® SpaceClaim, and a precise mesh, allowing the natural frequency values' convergence, is set. The girders are modeled using a linear two-dimensional element. The transverse beams with a quadratic three-dimensional element. Finally, all cables were meshed by an also quadratic, but one-dimensional element [24], see Fig. 1b.

Once everything is properly configured, a general static analysis is carried to evaluate the structure's behavior as predicted by Wang et al. [25] and Meriam et al. [26] - inspecting tension on the suspension and main cables, maximum displacement of the deck, and reaction forces values. Aiming to simplify the 2DOF TMD's design calculations, the dynamics of the cables were ignored [27], as their interaction with the deck is restricted to tension only behavior, and linear-elastic assumptions are made.

2.2 Tuned mass damper

Consider a single degree-of-freedom (DOF) oscillatory mass-spring system, to which an auxiliary mass is attached (spring-mass-damper). The equation that represents the system's dimensionless amplitude is [10]

$$\frac{X_1 k}{F_0} = \left\{ \frac{(2\xi r)^2 + (r^2 - \beta^2)^2}{(2\xi r)^2 (r^2 - 1 + \mu r^2)^2 + [\mu \beta^2 r^2 - (r^2 - 1)(r^2 - \beta^2)]^2} \right\}^{1/2} \tag{1}$$



a) b)

Figure 1. a) CAD presentation for the analyzed suspension bridge model, b) Meshing model for the structure

Where

$\mu = m_2/m_1$ – Ratio between the TMD and the primary structure's masses
$r = \omega/\omega_n$ – Ratio between the excitation and the system's natural frequency
$\xi = c_2/c_c$ – Ratio between TMD damping constant and its critical damping
$\beta = \omega_a/\omega_n$ – Ratio between TMD natural frequency alone and the system's

Equation 1 exposes how the amplitude of the main system may be solely controlled by setting the suitable parameters μ, ξ, and β. Therefore, it needs to be estimated those parameters optimum values.

In this study, the employment of the Equal Peak Design Method (by Den Hartog and Ormondroyd [8]) is enforced. Den Hartog [20] defined, using the "minimax" optimization problem-solving technique, minimizing the maximum value of the H_∞ norm, the optimum frequency ratio β for the Structure-TMD system, and optimal damping ratio are estimated by

$$\beta_{optimal} = \frac{1}{1+\mu} \tag{2}$$

$$\xi_{optimal} = \sqrt{\frac{3\mu}{8(1+\mu)^3}}. \tag{3}$$

Therefore, the only parameter left to be estimated so that one may design an optimal TMD is the mass ratio μ. In this work, it is designed two sets of 2DOF TMDs, one with $\mu = 1/20$ and another with a $\mu = 1/40$. The behavior of the primary structure, once having the 2DOF TMDs attached, is presented in the Results section.

2.3 Two Degree-of-freedom TMD

When aiming to damp multiple DOF, the first logical solution would be using multiple TMDs, however in this study, another approach is considered: using only one TMD for controlling two different frequencies of the suspension bridge model presented in Fig. 1, the first bending and torsional modes.

Consider the single DOF system exposed in Fig. 2a, subjected to an exciting force f applied at the distance a from its rotational center. Since the system is fully decoupled in global coordinates, one could control it by using two individual TMDs, one for each movement, as its shown in Fig. 2b. Each TMD should then be adequately tuned to decrease the main body vibration response.

Notwithstanding, a different solution is connecting the main structure to a 2DOF TMD, in which both movements could be approximated as to be happening independently from one another, addressing the weight penalty problem. Such case is exposed in Fig. 2c. There, the primary structure is connected to the 2DOF TMD consisted of two point masses at both ends of a rigid bar. The bar is "pinned" at its center by a rotational spring-damper set (k_t and c_t), as well as by "longitudinal" spring-damper set (k_2 and c_2).

Next, what needs to be done in order to tune independently both vibration modes of the 2DOF TMD with the bridge's, is solving both, the translational dynamic equation given by

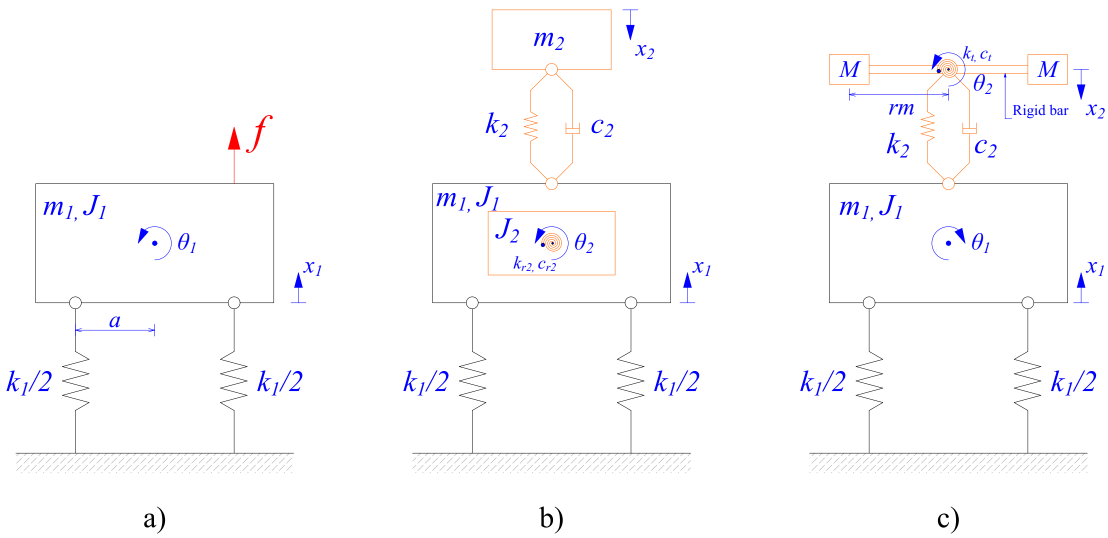


Figure 2. a) Single DOF oscillating system representing the suspension bridge, b) Attachment of two individual TMD, one for each movement (bending and torsion), c) Fully decoupled Two DOF TMD controlling bending with k_2 and c_2 and torsion with k_t and c_t.

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & k_2 \\ k_2 & k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{4}$$

and the rotational dynamics equation for the system (2DOF TMD-Bridge)

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} ak_1 + k_t & k_t \\ k_t & k_t \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} af \\ 0 \end{Bmatrix}. \tag{5}$$

After solving eqs. (4) and (5) for optimal values of β and ξ (eqs. (2) and (3)), the next procedure is finding the physical parameters that allow both vibrating modes of the 2DOF TMD to be optimally tuned with the primary system's. In this study, the chosen model is presented in Fig. 3. The physical parameters programmed to control the bending mode were the main beam length "l" and the point masses "M". Whereas the rotational movement is regulated by the arm "rm", distance between the main beam axis and each point mass.

The spring stiffness for the 2DOF TMD is given below (translation and torsional, respectively), according to Rao [10], Warren and Richard [28]:

$$k_2 = \frac{bt^3 E}{2l}, \quad k_t = \frac{CG}{l} \quad \text{where } C = bt^3 \left[\frac{16}{3} - 3.36 \frac{t}{b} \left(1 - \frac{t^4}{12b^4} \right) \right] \tag{6}$$

From the remaining physical variables, J_1 and m_1 are extracted directly from ANSYS®. Whereas J_2 and m_2 are functions of the first two (following the desired μ), considering the optimum design strategy (eq. (2)). Finally, it is recommended the 2DOF TMD to be placed where the biggest amplitude occurs, according to the targeting modes presented on Fig. 4, results from the Modal Analysis developed in ANSYS® Mechanical.

3 Results and Discussion

Once the theoretical background is properly set, a MATLAB® code is written in order to provide the optimal physical dimensions for the 2DOF TMDs. The chosen material for the design of the main beam, and the rigid bar, is aluminum alloy.

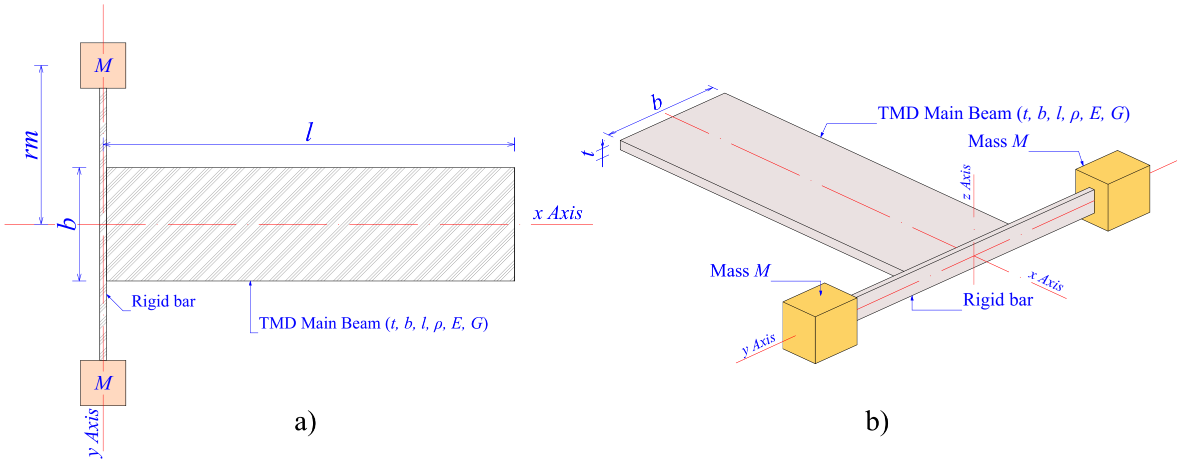


Figure 3. a) Plan view of the TMD, b) Isometric view of the TMD

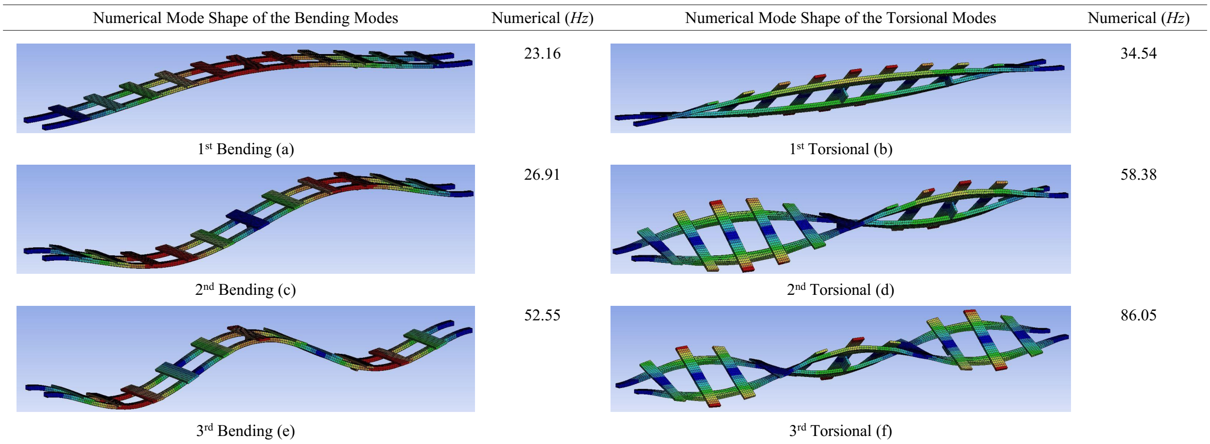


Figure 4. a-f) Main estimated vibration modes. The right side presents the torsional modes, while the left, the first three bending modes

Table 1 exposes the estimated values for both proposed 2DOF TMD cases. Using this data, the 2DOF TMDs are modeled in ANSYS® and connected by its end to the primary structure right at its center (following Fig. 3). A Harmonic Analysis, disregarding the calculated optimal damping, was employed and the Frequency Response Response (FRF plot) is presented in Fig. 5. It can be perceived that, in despite of the absence of damping, both devices were able to “split” the first bending and torsional frequencies, as expected in such cases [11].

Table 1. Geometric values for both 2DOF TMD cases considered

Physical Variables	l	t	b	M	rm
TMD Case 1	8.87 cm	0.2 cm	2.5 cm	130.6 g	4.0 cm
TMD Case 2	11.0 cm	0.2 cm	2.5 cm	64.2 g	5.1 cm

Figure 5 points out the vibration level decrease occurring for both targeted frequencies. The reduction at the first bending frequency considering Case 1, and Case 2 were, approximately, 55% and 52.7%, respectively. Whereas at the first torsional mode, 55.4% (Case 1) and 57.6% (Case 2).

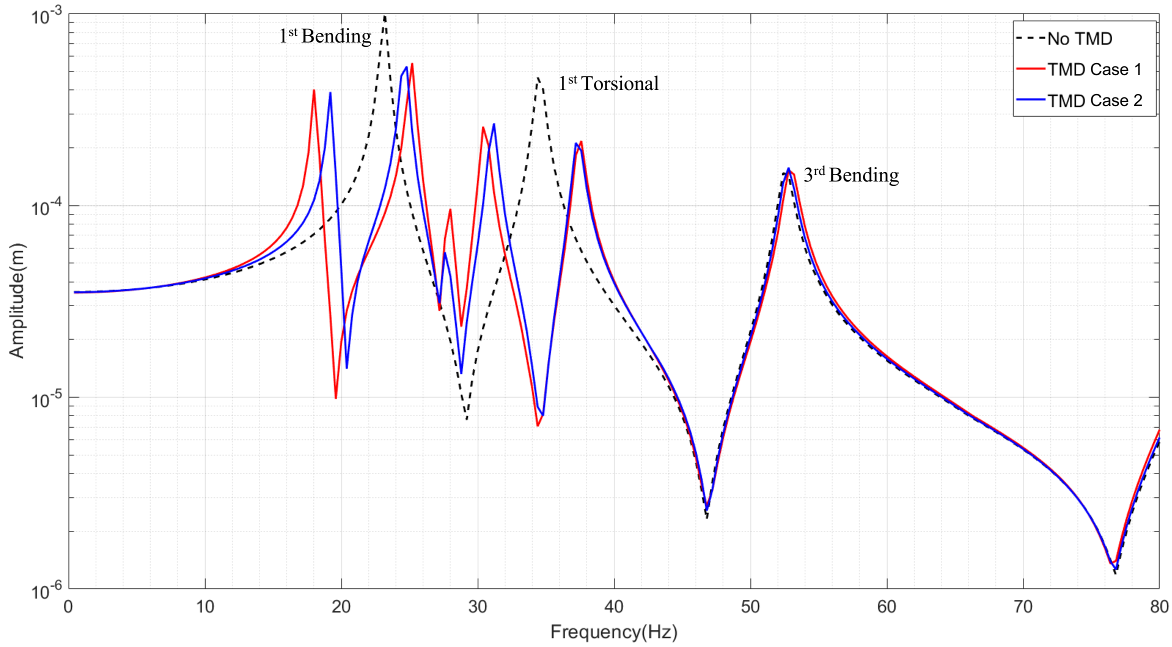


Figure 5. Frequency response plot comparing the behavior of the primary structure with no TMD attached in black, Structure-TMD Model 1 ($\mu = 1/20$) in red, and Structure-TMD Model 2 ($\mu = 1/40$) in blue.

4 Conclusions

The purpose of this work was to design a 2DOF TMD capable of targeting, at the same time, the first bending and torsional natural frequencies of a suspension bridge numerical model, by the Equal Peak Design method proposed by Den Hartog and Ormondroyd [8]. In order to do so, a MATLAB® routine was written to help estimate the optimal physical dimensions for the dampers, according to the proposed chosen model (Fig. 3). After estimating those measures, the devices were modeled, using a Finite Element Analysis software (ANSYS®), along with the primary structure (Bridge).

It was observed the 2DOF TMD cases were able to "split" both of the targeted frequencies at the same time. Its behavior, even while neglecting damping, can be considered suitable, for it demonstrate the methodology's effectiveness and the device's potential to damp both modes. In future works it is planned to introduce the optimal damping calculated to the 2DOF TMDs and observe how the suspended bridge numerical model behaves through a Transient Analysis.

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