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Stochastic live load model for buildings and its application in reliability based code calibration

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Abstract. While the exact loading to which a structure will be subjected cannot be precisely assessed during the design phase due to its stochastic nature, probabilistic models are useful for the rational determination of nominal values, partial safety factors and load combination factors employed in limit state design that accurately reflects the variability of these loads. In this paper, a simple probabilistic model describing the spatial and temporal variabilities of live loads in buildings is presented. This model consists of a sum of a sustained load and an intermittent load stochastic processes. Due to the lack of national data to back up the model, parameters are taken from the Joint Committee on Structural Safety (JCSS), based on international surveys. Using this stochastic model, sample values for live loads are generated for buildings with different occupancy types, and statistics for the fifty-year extreme and arbitrary point-in-time distributions of live loads are derived using Monte Carlo simulations. These values are then compared with those of Brazilian design codes ABNT NBR 6120:2019 (Design Load for Structures), and other major international standards. The resulting statistics are also employed in a reliability-based calibration of the partial safety factors presented in Brazilian design codes for steel (ABNT NBR 8800:2008) and concrete (ABNT NBR 6118:2014) structures. It is shown that, with the resulting set of optimized partial safety factors, reliability is made more uniform over different load ratios, with a smaller variation around a chosen target reliability value, while attaining no significant economic impact when compared with the currently employed factors.

Keywords: Live load model, Probabilistic model, Code calibration, Partial safety factors, Structural reliability.

1 Introduction

When designing a structure using a semi-probabilistic approach such as the limit states format employed by Brazilian codes and most international design codes as well, representative values of the loads are considered. These can be the characteristic (or nominal) values, design values, reduced combination values, or reduced serviceability values (frequent and quasi-permanent, for example).

Particularly for the case of buildings, one of the most fundamental loads to be considered is the live load (sometimes also referred to as imposed load), which consists, according to the definition found on ASCE/SEI 7-16 [1], of all loads produced by the use and occupancy of the building that does not include construction or environmental loads, such as wind load, snow load, rain load, earthquake load, or dead load.

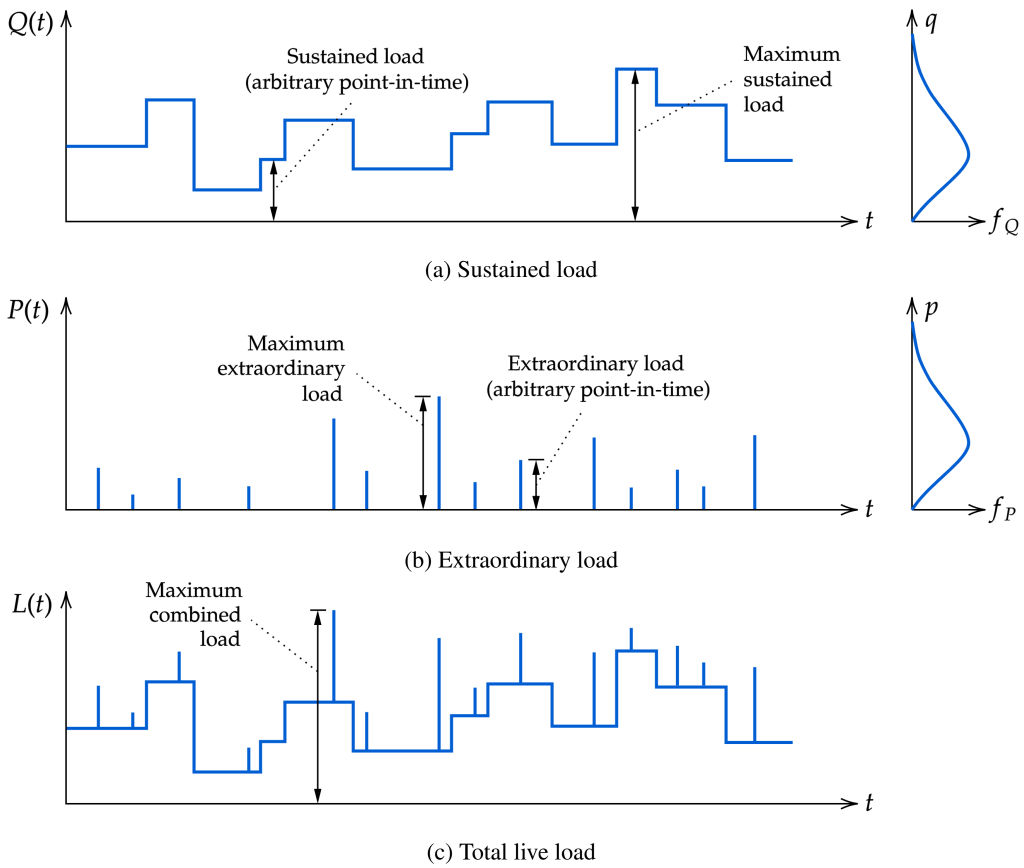
Live load values are usually specified in design codes as a uniformly distributed load, sometimes also accompanied by a concentrated load, both depending on floor occupancy type. In Brazil, design live loads are prescribed by design code ABNT NBR 6120:2019 [2]. This code states, in agreement with the definition given by ABNT NBR 8681:2003 [3], that the presented characteristic values "have between 25 % to 35 % probability of being exceeded, in the unfavorable sense, in a period of 50 years", but also that these values are "established by consensus". In fact, to the authors best knowledge, there is no study or probabilistic model to back up the claim that the recommended characteristic values in fact correspond to the aforementioned exceedance probabilities. Instead, those values were arrived at upon comparison with international design codes or building standards such as ASCE/SEI 7-16 [1], EN 1991-1-1:2002 [4], and ISO 2103:1986 [5].

Hence, a simple hierarchical model for the spatial modeling of time-dependent live load in buildings is presented in section 2, which consists of a sum of two stochastic processes representing the sustained and extraordinary parts of the loading. This model is then employed in order to derive the statistics for the arbitrary point-in-time and the 50 and 140-year extreme distributions of live loads, and assess if the loads given in NBR 6120:2019 are consistent with the prescribed exceedance probabilities. These statistics can be used in a wide variety of reliability analyses. One particular application where the live load statistics play a very important role is in the reliability-based calibration of partial safety factors employed in structural design codes. In section 3, the calibration of Brazilian codes for steel (ABNT NBR 8800:2008 [6]) and concrete (ABNT NBR 6118:2014 [7]) structures, originally performed by Santiago et al. [8], is re-processed using the live load statistics found herein.

2 Live load model

Live loads are intrinsically stochastic in nature, with variability both in space and time. However, buildings within the same occupancy type – such as office or residential buildings, for instance – tend to exhibit similar behavior when it comes to this variabilities, which is why live loads are usually specified in design codes according to the intended use of the building. Floor live loads in buildings can be decomposed in two parts with very distinct characteristics: a sustained load and an extraordinary load. The sustained part consists of the weight of all furniture, equipment, and working/living personnel present on a regular basis. The extraordinary load, on the other hand, is associated with localized crowding of people and/or furniture that may lead to high-intensity loading. While the sustained load is "on" for pretty much the entirety of the building lifetime, the extraordinary load is much more infrequent, only happening on average once every few years, and staying "on" for relatively short periods, in the order of a few minutes or hours.

The model employed in this study is the hierarchical model presented in Part 2 of the Probability Model Code by the Joint Committee on Structural Safety (JCSS) [9], which, in turn, is based on a well-known model originally proposed by Peir [10]. It represents the sustained load $Q(t)$ as a Poisson square wave process (Fig. 1a), and the extraordinary load $P(t)$ as a Poisson spike process (Fig. 1b). The temporal variability of the combined process $L(t) = Q(t) + P(t)$ is significantly more convoluted than each of its parts (Fig. 1c), which makes finding its maximum value within a reference period a non-trivial task.



(a) Sustained load

(b) Extraordinary load

(c) Total live load

Figure 1. Schematic time histories of typical live loads in buildings

2.1 Sustained load

The load intensity at a location (x, y) of a given floor in a given building is represented as a stochastic field $W(x, y)$ that can be described as follows:

$$W(x, y) = m + V + U(x, y), \tag{1}$$

where m is a general mean value for the whole population of buildings within the same use category; V is a zero-mean random variable which represents the deviation of the loading on that floor from the general mean m; and $U(x, y)$ is a zero-mean random field which describes the spatial variability of the load within the floor.

When designing a structure, the point intensities $W(x, y)$ are not of practical interest. Instead, what we want to know is the load effect S caused by the stochastic field $W(x, y)$. Assuming linear elastic behavior, where the superposition principle is valid, the load effect S can be written as:

$$S = \iint_A W(x, y) I(x, y)\, dy\, dx \tag{2}$$

where $W(x, y)$ is the load intensity, and $I(x, y)$ is the influence surface for the desired load effect over an influence area A. It should be noted that the influence area A is defined as "that floor area over which the influence surface for structural effects is significantly different from zero" [1], which is different from the tributary area. The influence area is usually equal to twice the tributary area for beams and four times the tributary area for columns.

For design purposes, it is of practical interest to define a uniform load intensity that would produce the same load effect S corresponding to the original load field $W(x, y)$ when applied to the appropriate floor area. This load is denoted by q_{EUDL}, where EUDL stands for "equivalent uniformly distributed load", and can be written as:

$$q_{\text{EUDL}} = \frac{\iint_A W(x, y) I(x, y)\, dy\, dx}{\iint_A I(x, y)\, dy\, dx} \tag{3}$$

Assuming that the statistical properties of the random field $W(x, y)$ do not depend on the location (x, y) – that is, $W(x, y)$ is a homogeneous field –, the mean and standard deviation of q_{EUDL} can be calculated as:

$$\text{E}[q_{\text{EUDL}}] = \frac{\iint_A \text{E}\big[W(x, y)\big] I(x, y)\, dy\, dx}{\iint_A I(x, y)\, dy\, dx} = m \tag{4}$$

$$\text{Var}[q_{\text{EUDL}}] = \frac{\iint_A \iint_A I(x_1, y_1) I(x_2, y_2) \text{Cov}\big[W(x_1, y_1), W(x_2, y_2)\big]\, dy_1\, dy_2\, dx_1\, dx_2}{\big[\iint_A I(x, y)\, dy\, dx\big]^2} \tag{5}$$

Equation (5) depends on the autocovariance of the random field $W(x, y)$. It is reasonable to assume that, if the load intensity at a particular location (x_1, y_1) is greater than the floor average, it is likely that the load intensity at a nearby point (x_2, y_2) would also be high – or, in other words, that there is a generally positive correlation to the field $W(x, y)$ that tends to vanish as the distance separating the points increases. Three different autocorrelation functions for the random field $W(x, y)$ were proposed by Hauser [11].

Data from live load surveys show that the correlation length over which the correlation is significantly different from zero is usually around $1\,\text{m}$ to $2\,\text{m}$. This indicates that, as long as the influence area A is not too small, a reasonable approximation can be obtained by treating $W(x, y)$ as a "white-noise" process, meaning that the load intensities in two points are statistically uncorrelated if any separation between them exists. Under this assumption, $\text{Var}[q_{\text{EUDL}}]$ can be conservatively bounded by:

$$\text{Var}[q_{\text{EUDL}}] = \sigma_V^2 + \sigma_U^2 \min\left[\frac{A_0}{A}, 1\right] \kappa \tag{6}$$

where σ_V^2 is the variance of the random variable V; σ_U is the variance of the random field $U(x, y)$; A_0 is a reference area; and κ is a peak factor that depends on the shape of the influence surface, given by:

$$\kappa = A \frac{\iint_A I^2(x, y)\, dy\, dx}{\big[\iint_A I(x, y)\, dy\, dx\big]^2} \tag{7}$$

Values for the peak factor κ for different load effects are given in McGuire and Cornell [12] and in the CIB Report 116 [13] (Fig. 2). In this study, $\kappa = 2.0$ was adopted.

The arbitrary point-in-time equivalent load q_{EUDL} is assumed to be gamma distributed, following the observations of Peir and Cornell [14] and Corotis and Doshi [15], with mean and variance given by eq. (4) and eq. (6).

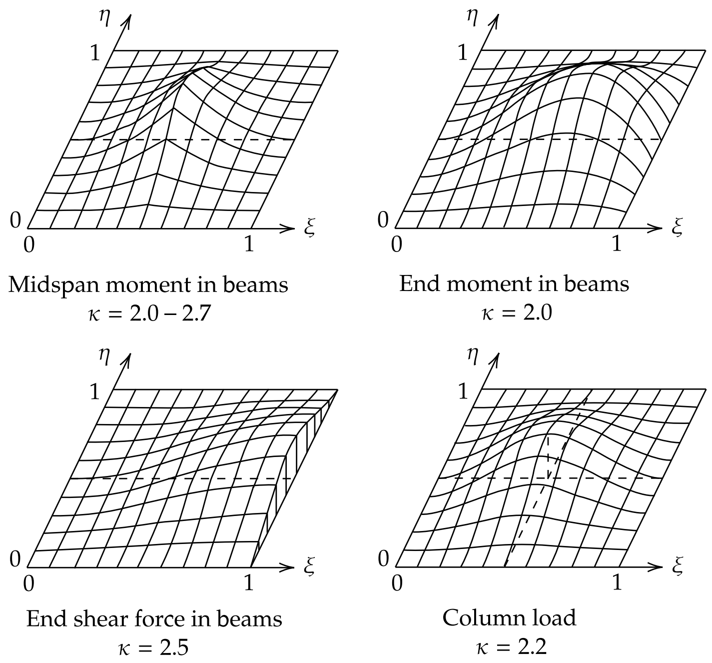


Figure 2. Influence surfaces and corresponding peak factor values for different load effects. Reproduced from [13]

As for the time variability, the sustained load value is assumed to remain constant between Poisson-arriving occupancy changes, when it jumps to a new (constant) load level. The mean occurrence rate of sustained load changes is denoted by λ, so that the expected number of occupancy changes during a reference perior T is λT, and the duration of an occupancy is exponentially distributed with mean $1/\lambda$.

2.2 Extraordinary load

A similar model is employed for the extraordinary load, where the moments for the extraordinary EUDL p_{EUDL} are given by:

$$\mathrm{E}[p_{\mathrm{EUDL}}] = m_p \tag{8}$$

$$\mathrm{Var}[p_{\mathrm{EUDL}}] = \sigma_{U,p}^2 \min\left[\frac{A_0}{A}, 1\right]\kappa \tag{9}$$

where the subscript p is used to differentiate extraordinary load parameters from sustained load parameters, which will from this point forward be denoted by a subscript q.

The extraordinary load occurrence is also assumed to be Poisson-arriving, with an occurrence rate ν and a deterministic duration d_p. According to the JCSS, the arbitrary point-in-time load intensities should be assumed to be exponentially distributed, but herein it is assumed to be gamma (similar to the sustained load), since that is what most other studies seem to consider.

2.3 Model parameters

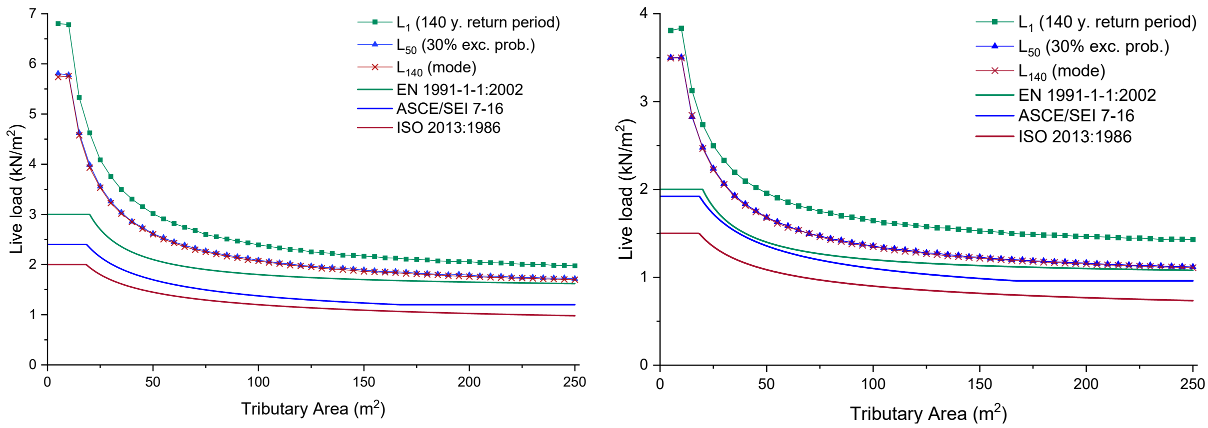
Ideally, model parameters $\sigma_{V,q}$, $\sigma_{U,q}$, λ for the sustained load and $\sigma_{U,p}$, ν for the extraordinary load should be calibrated using data from local live load surveys. Since there are no such surveys for Brazilian live loads, these parameters were instead taken from JCSS [9], except for classroom use, where JCSS parameters were found to be excessively conservative, and the parameters suggested by Honfi [16] were used instead. The load parameters presented by JCSS are mostly based on multiple surveys dating from 1893 to 1976 and presented in the CIB Report 116 [13], which refers to an extensive review by Sentler [17] and the work of Chalk and Corotis [18], among others.

2.4 Simulation

In this study, statistics for the 1-year, 50-year and 140-year extreme value distributions (L_1, L_{50} and L_{140}) of the total live load EUDL were obtained via Monte Carlo simulation, according to the following steps:

1. Definition of load parameters for a given building occupancy type;
2. Numerical generation of the time intervals t_q between sustained load changes, from an exponential distribution with parameter λ;
3. Numerical generation of sustained EUDL intensities corresponding to each occupancy, from a gamma distribution with moments given by eq. (4) and eq. (6);
4. Numerical generation of the arrival times t_p for extraordinary loads with fixed duration d_p;
5. Numerical generation of extraordinary EUDL intensities corresponding to each occurrence, from a gamma distribution with moments given by eq. (8) and eq. (9);
6. Addition of the sustained and extraordinary EUDL day-by-day over the reference time T;
7. Determination of the maximum total EUDL value in T;

Steps 1 to 7 are repeated for n samples, after which the mean and variance of all the n samples are evaluated. Herein, $n = 10\,000$ was adopted, and the resulting L_{50} and L_{140} distributions were found to be well adjusted by a Type I Extreme Value distribution (Gumbel). From the fitted L_{50} distribution, characteristic (nominal) values can be calculated as the L_{50} value with $30\,\%$ exceedance probability, corresponding to the median of the interval specified in the definition given by NBR 6120:2019 [2]. This characteristic value can also be calculated as the mode of the 140-year extreme value distribution, since the aforementioned exceedance probability corresponds to a mean return period of 140 years. Due to space constraints, only the results for office and residential buildings are shown in Fig. 3, in a comparison with some major international design codes. Values from NBR 6120:2019 are not shown in Fig. 3 because the Brazilian code does not allow for area-based live load reduction as these other codes. Results for other occupancies can be found in the M.Sc. thesis of the first author [19].



(a) Office (b) Residential

Figure 3. Simulation results for office and residential buildings

In can be seen in Fig. 3 that the results obtained using the JCSS [9] parameters seem to be slightly higher than those indicated in the considered design codes. However, a direct comparison is inappropriate, since each of these defines their characteristic values differently from the Brazilian codes. It can also be seen that the curves for the 0.7 fractile of $L_5$0 and the mode of $L_1$40 are practically coincident, but the 140-year return period values calculated from L_1 are somewhat higher. This is due to the fact that annual maxima are not fully independent (as is the case for wind loads), since the average time between sustained load changes is usually longer than 1 year.

Assuming that the total live load $L(t)$ is an ergodic process, the statistics for its average point-in-time distribution (L_{apt}) were also obtained via Monte Carlo simulation using a really long reference period T. The results are mainly influenced by the sustained load parameters, since the extraordinary load occurs relatively rarely, and are is well described by a gamma distribution. The resulting distributions for different occupancy types are shown in 1, expressed as a function of the nominal values L_n given in NBR 6120:2019 [2]. The tributary areas A_T were chosen so that the 0.7 fractile of L_{50} roughly corresponds to L_n.

Table 1. Live load statistics for average point-in-time, 1-year, 50-year and 140-year extreme distributions

Occupancy type	A_T	L_{apt} (Gamma)		L_1 (Gumbel)		L_{50} (Gumbel)		L_{140} (Gumbel)	
	(m^2)	μ	c.o.v.	μ	c.o.v.	μ	c.o.v.	μ	c.o.v.
Office	55	0.20	0.94	0.37	0.63	0.93	0.26	1.11	0.21
Residential	70	0.20	0.75	0.36	0.67	0.93	0.22	1.09	0.18
Hotel room	110	0.21	0.27	0.54	0.25	0.95	0.14	1.05	0.13
Patient room	55	0.20	1.16	0.29	0.97	0.89	0.35	1.13	0.28
Classroom*	150	0.20	0.61	0.35	0.55	0.92	0.24	1.09	0.20
Average		0.20	0.75	0.38	0.61	0.92	0.24	1.09	0.20

* Model parameters for classrooms were taken from Honfi [16] instead of JCSS [9].

3 Reliability-based calibration of design codes

Results for the reliability-based calibration of partial safety factors of Brazilian design codes for for steel (ABNT NBR 8800:2008) and concrete (ABNT NBR 6118:2014) structures are reported in Santiago et al. [8]. The 50-year extreme live load statistic employed by the authors, however, had a bias factor of 1.0 ($\mu = L_n$) and a coefficient of variation equal to 0.40. This incurred in two major problems: (1) the nominal value L_n using this statistic corresponds to a $43\,\%$ exceedance probability in 50 years, in contradiction with the definition given by NBR 8681:2003 and NBR 6120:2019; and (2) the employed c.o.v. of $40\,\%$ is significantly higher than the one used by past studies, leading to smaller reliability indexes for some of the considered structural configurations when compared to previous studies.

The L_{50} statistic obtained in this study has a smaller c.o.v., more in line with the values reported by Ellingwood et al. [20] ($\mu = L_n$ e c.o.v. $= 0.25$) and Szerszen and Nowak [21] ($\mu = 0.93L_n$ e c.o.v. $= 0.18$), utilized in the calibration of American design codes. Using the L_{apt} and L_{50} statistics presented in Table 1, the reliability-based calibration was re-processed. It is basically an optimization problem which consists of finding the set of partial safety factors that minimize the weighted sum of the squared differences between the calculated reliability index β from a target reliability index β_T over a wide variety of structural configurations and load ratios. Reliability indexes are evaluated using the First Order Reliability Method (FORM), and the optimization problem is solved using the Particle Swarm Optimization (PSO) algorithm. For more information on the calibration procedure, the reader is referred Santiago et al. [8], where is is described in greater detail.

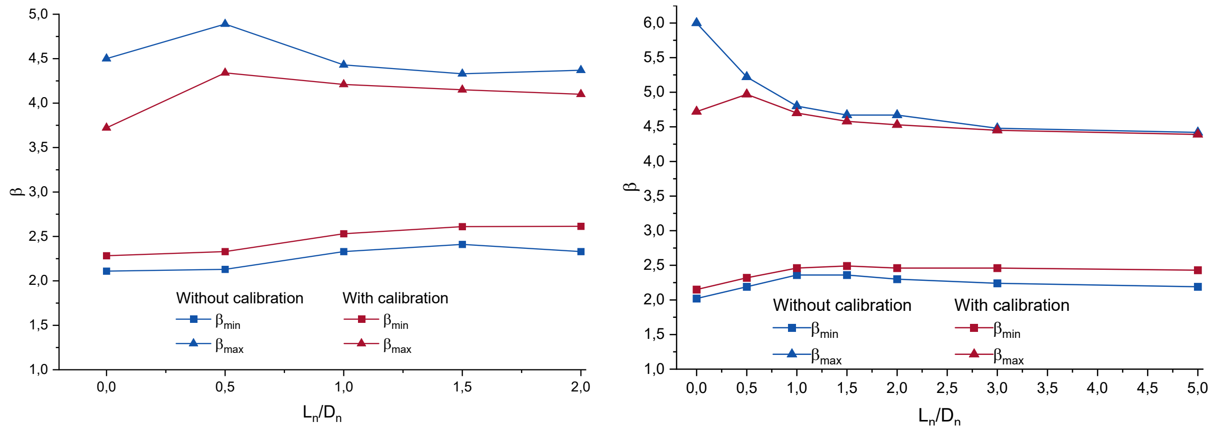
In the original calibration, Santiago et al. [8] considered the target reliability index to be $\beta_T = 3.0$. Herein, it is adopted as $\beta_T = 3.17$, which corresponds to the mean reliability index for concrete structures obtained using the live load statistics in Table 1 (the mean reliability index found for steel structures was 3.28). The results are briefly summarized in Table 2. Subscripts D, L and W refer, respectively, to dead, live and wind load, and γ_c, γ_s, γ_{a1} and γ_{a2} are partial safety factors for material strengths.

Table 2. Updated partial safety factors, following the procedure of [8] with new live load statistics

Safety factors	Before calibration		Original calibration [8]	New calibration*
	NBR 8800:2008 [6]	NBR 6118:2014 [7]	$\beta_T = 3.0$	$\beta_T = 3.17$
γ_c	–	1.40	1.40	1.40
γ_s	–	1.15	1.15	1.15
γ_{a1}	1.10	–	1.10	1.10
γ_{a2}	1.35	–	1.30	1.40
γ_D	1.25	1.40	1.25	1.20**
γ_L	1.50	1.40	1.70	1.50
γ_W	1.40	1.40	1.65	1.50
ψ_L	0.50 / 0.70 / 0.80	0.50 / 0.70 / 0.80	0.35	0.45
ψ_W	0.60	0.60	0.30	0.35
$\gamma_L \cdot \psi_L$***	0.70 / 1.05 / 1.20	0.70 / 0.98 / 1.12	0.60	0.68
$\gamma_W \cdot \psi_W$***	0.84	0.84	0.50	0.53

* Rounded values, to make NBR 6118 compatible with NBR 8800.
** This coefficient is suggested to remain 1.40 when live and wind loads are zero.
*** Effective combination value for secondary action.

Figures 4a and 4b illustrates shows the minimum and maximum reliability indexes obtained for concrete and steel structures before and after the calibration, for live-to-dead load ratios (L_n/D_n) up to 2.0 and 5.0, respectively. It is noticeable that the calibration procedure has the effect of reducing the dispersion of results, achieving more uniform reliability for different design configurations.



(a) Concrete structures

(b) Steel structures

Figure 4. Reliability index envelope for all structure configurations in terms of L_n/D_n

The calibration results reported in Table 1 points to an increase in the partial safety factor corresponding to the primary variable load (whether it is wind or live load) and a reduction in the secondary accompanying load, as evidenced by the reduction of the combination factors ψ_L and ψ_W. It is also seen that the partial safety factor for dead load can be reduced from $\gamma_D = 1.40$ currently adopted for concrete structures (except for the load combination where it is the only load acting on the structure, in which case we recommend that γ_D is kept as 1.40), since the variability of this kind of load is usually much smaller than for wind or live loads. Comparison with the results of the previous calibration, which utilized a L_{50} statistic with a c.o.v. of $40\,\%$, shows that the calibration results using the new live load statistics presented herein lead to a set of partial safety factors more in line with major international design codes such as EN 1990:2002 [22] or ACI 318:19 [23].

4 Conclusions

This paper addressed the temporal and spatial variability of live loads in buildings using a hierarchical model presented in JCSS [9]. Model parameters for both sustained and extraordinary loads were mostly taken from [9], since there are no available survey data for Brazilian buildings. Using this model, new statistics for the average point-in-time distribution (L_{apt}), as well as the 1-year (L_1), 50-year (L_{50}) and 140-year (L_{140}) extreme value distributions for live loads are derived. These statistics comply with the $25\,\%$ to $35\,\%$ exceedance probabilities in 50 years prescribed in NBR 8681:2003 and NBR 6120:2019. Particularly, the L_{50} statistic herein presented have a significantly lower coefficient of variation than the corresponding statistic reported by Santiago et al. [8]. The reliability-based calibration of Brazilian structural codes for concrete and steel structures is re-processed using these new statistics, in order to assess the impact they have on the optimal set of partial safety factors. The results show that more uniform reliability indexes can be achieved by increasing the partial safety factors corresponding to the main variable load and reducing the secondary load in the combinations, and that the partial safety factor for live loads obtained in the original calibration [8] was overestimated due to the larger coefficient of variation considered by the authors.

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