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The effect of geometric stiffness on vibration frequencies of bulkhead frames of pressurized aircraft fuselages

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Abstract. Aircraft fuselages are structured by, among other elements, planar portal frames called bulkheads. Among their main loads, is the internal pressurization, which causes considerable traction efforts. It is well known, from the Matrix Structural Analysis theory, that the stiffness of frame elements is composed of two parts, the elastic stiffness and the geometric stiffness. The latter depends on the level of axial forces acting on the members. If in traction, it increases stiffness, and, consequently, raises the frequencies of free vibration of the structure related to it. This effect is widely explored in so-called tensile structures, such as inflated blimps. If of compression, it decreases the total stiffness and lowers the frequencies, leading, in the limit, to the buckling of the element. In this work, a bulkhead portal frame of a fictitious aircraft is numerically analyzed, demonstrating that the presence of high levels of traction that such structures support, due to internal pressurization, considerably increase their frequencies.

Keywords: geometric stiffness, bulkhead, aircraft fuselage, frequency.

1 Introduction

Aircraft fuselages are aerodynamic structures, that provide as external protection of a structure, being formed by flat portal frames, called bulkheads. Such elements are subject, among other loads, to internal pressurization, which causes high traction forces in the element, that can affect some of its dynamic characteristics, which are dependent on the stiffness and mass of the structure.

In this sense, the stiffness and mass of a structure are properties that are used to determine its natural frequency, which in turn indicates the free oscillation rate of a structure, after the removal of incident forces [1]. In the portion of the total stiffness, we have the elastic stiffness and the geometric stiffness. Geometric stiffness is characterized by being a property dependent on the level of axial force acting on the element. Studies that evaluated the influence of axial forces on the variation of structural stiffness can be observed in [2-9].

Based on the works mentioned above, it can be said that compressive forces tend to reduce the stiffness of a structure, which can even lead to the element collapse, which is often caused by the buckling of the element. Tensile forces, on the other hand, have the opposite effect, thus increasing the stiffness of structures and, consequently, raising their natural frequencies of vibration. This effect is widely explored in so-called tensile structures, such as inflated blimps [10].

In this sense, this paper evaluated, numerically, a bulkhead portal frame of a fictitious aircraft, to assess the variation of the structure's natural frequency, considering different levels of axial (tensile) effort. As such elements are normally subject to traction levels under working conditions, due to internal pressurization, it is expected that their natural frequency will be increased, due to the portion referring to geometric stiffness, therefore, this is the hypothesis investigated.

2 Numerical formulation

The numerical procedure developed was based on the concepts of matrix structure analysis [11]. The mathematical development of matrix analysis for linear systems can be adapted to nonlinear ones, what happens is, that in this case, the solutions cannot be obtained explicitly, requiring the use of iterative solutions.

Consider the column with both ends pinned of Fig. 1, where *E* represents the longitudinal deformation modulus; *I* the moment of second order; *L* the total length; *A* is the cross-sectional area and *p* is the density of the material. Numbers 1-4 refer to the degrees of freedom considered.

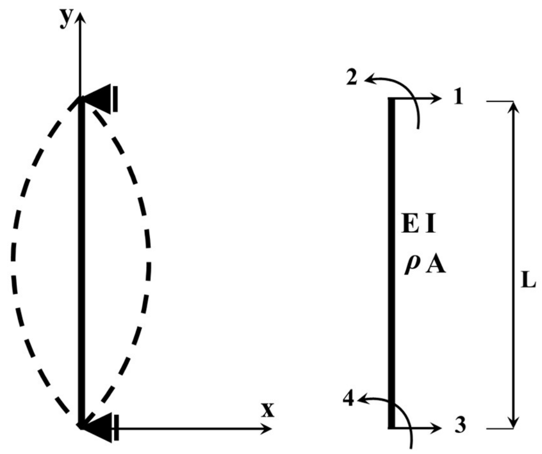

Figure 1. Schematic representation beam/column with both ends pinned

The total stiffness of the structure being given by:

$$K_T = K_E + K_G, \tag{1}$$

where K_E is the conventional stiffness portion, given by the eq. (2), and K_G is the geometric stiffness, that is determined by eq. (3):

$$K_E = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}, \tag{2}$$

with *E* being the Young's modulus; *I* the moment f second order; *L* the total length of the element.

$$K_G = \frac{P}{L}\begin{bmatrix} \frac{6}{5} & \frac{L}{10} & \frac{-6}{5} & \frac{L}{10} \\ \frac{L}{10} & \frac{2L^2}{15} & \frac{-L}{10} & \frac{-L^2}{30} \\ \frac{-6}{5} & \frac{-L}{10} & \frac{6}{5} & \frac{-L}{10} \\ \frac{L}{10} & \frac{-L^2}{30} & \frac{-L}{10} & \frac{2L^2}{15} \end{bmatrix}, \tag{3}$$

where *P* is the axial load applied to the element. The mass matrix of the element is given by:

$$M = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}, \tag{4}$$

considering ρ the density of the material. Therefore, taking into account the stiffness and mass matrix of the system, the equation of motion is given by:

$$\left(-\omega^2\left[M\right]+\left[K_T\right]\right)\left[q\right]=0\,, \tag{5}$$

where $[q]$ is the matrix of the generalized coordinates of the system and *w* the natural frequency. Substituting eq. (2), (3) and (4) in eq. (5), it comes to:

$$\left(-\omega^2\frac{\rho AL^2}{420}\begin{bmatrix} 4L^2 & -3L^2 \\ -3L^2 & 4L^2 \end{bmatrix}+\frac{EI}{L^3}\begin{bmatrix} 4L^2 & -2L^2 \\ -3L^2 & 4L^2 \end{bmatrix}+\frac{P}{L}\begin{bmatrix} \frac{2L^2}{15} & \frac{-L^2}{30} \\ \frac{-L^2}{30} & \frac{2L^2}{15} \end{bmatrix}\right)\begin{bmatrix} q_2 \\ q_4 \end{bmatrix}=0\,. \tag{6}$$

The characteristic equation of the system being given by:

$$\left|-\omega^2\left[M\right]+\left[K_T\right]\right|=0\,. \tag{7}$$

Solving the system presented in eq. (7), i.e., an eigenvalue problem, can obtain an approximate numerical solution of the first natural frequency which, after some mathematical manipulations, is given by:

$$\omega_1 = 10.954\sqrt{\frac{EI}{\rho AL^4}}\sqrt{\left(1+\frac{PL^2}{12EI}\right)}\,. \tag{8}$$

It can be seen that the frequency increases as the tensile loads increase and decreases if these values are compression. The increase in frequency is directly related to the increase in stiffness of the structural element and the decrease in frequencies with the loss of stiffness, which has a limit value given when the total stiffness equals zero, being possible, in this case, to determine the critical system load. The frequency, given by eq. (8), is 11% higher than the exact solution of the system, without considering the load P ($P = 0$) [12].

3 Case study

To investigate the formulations presented in the previous item, which incorporates the effect of the geometric non-linearity of the element in determining the natural frequency of the system, a plane frame was analyzed, which is an existing element in the aircraft fuselages, called bulkheads. Such an element is characterized by being subject to internal pressurization, causing considerable tensile forces in the system, which in turn considerably affect the vibration frequency of the system. Fig. 2 presents the geometric characteristics of the bulkhead portal frame of a fictitious aircraft fuselage.

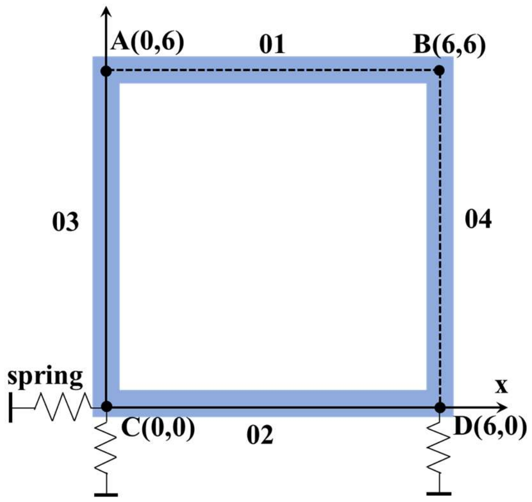

Figure 2. Characteristics of the bulkhead portal frame

The bars formed by elements 1-4 have longitudinal deformation modulus E = 360 GPa, density ρ = 3000 kg/m^3 and cross section 12x10 cm. To determine the axial force in each bar, an internal pressurization of 10kN/m was considered (Fig. 3). Note that springs were used at points C and D to support the structure. Such springs have low stiffness (r = 98 kN/m).

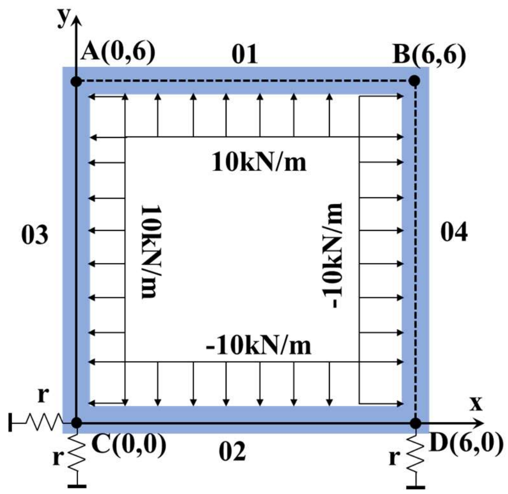

Figure 3. Loads considered of the bulkhead portal frame structure

4 Results and discussions

Considering the efforts presented in Figure 2, the axial forces in each bar were determined and such values were considered, applying Equation (8), to determine the variation of the natural frequency of the structure in two situations, one without axial loads and other with consideration of internal pressurization, that generate tensile load. Tab. 1 shows the structure frequency variation results.

Table 1. Structure frequency variation results

Condition	Frequency (rad/s)
Without interne pressure	0.0232
With 10kN/m interne pressure	0.1588
Variation (%)	**584.5**

Table 1 shows that there was an increase of almost 6 times in the value of the structure's natural frequency, comparing a state without the application of internal pressurization. This increase is mainly due to the gain in stiffness of the structure as a function of the portion referring to the geometric stiffness.

5 Conclusions

This paper evaluated, numerically, the variation of the natural frequency in bulkhead portal frame, used for the composition of the elements, of the fictitious aircraft fuselage, subject to considerable tensile forces due to the effect of the internal pressurization efforts in which such elements are submitted. For this, the portion referring to the geometric non-linearity of the structure was considered.

In the end, it was possible to observe that the high tensile forces (30 kN) of the bars frame caused a variation of 548.5% of the natural frequency of the structure. This shows the increase in the stiffness of the element due to the axial effort existing in the element.

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