Reliability of built-up cold-formed steel columns designed by the direct strength method

Celmar P. de Andrade[1], Marcílio S. R. Freitas[1], André L. R. Brandão[2]

[1]*Dept. of Civil Engineering, Federal University of Ouro Preto*
Ouro Preto, 35.400-000, Minas Gerais, Brazil
celmar.andrade@aluno.ufop.edu.br, marcilio@ufop.edu.br
[2]*Institute of Integrated Engineering, Federal University of Itajubá*
Itabira, 37.500-903, Minas Gerais, Brazil
andreriqueira@unifei.edu.br

Abstract. Built-up cold-formed steel (CFS) columns are composed of two or more sections that are joined together with welds, bolts or screws. The use of built-up sections may be an option at certain locations in a CFS-framed building when higher axial capacity or local frame rigidity is required. This paper presents a study of the reliability of built-up cold-formed steel columns. Reliability indexes are evaluated by First Order Reliability Method (FORM method) for usual nominal live-to-dead load ratios. The reliability analysis used to assess the safety level of design specifications included the model error and other random variables such as strength parameters, geometric parameters, dead and live loads. For the statistical study of the model error variable, column test results obtained from literature were compared to the resistant capacities obtained by the direct strength method (DSM). A total of 266 column tests were selected in order to ensure representativeness of the buckling modes and section types. A lack of uniformity of reliability indices was observed in the analyzes organized by instability mode or by section type. It was found that the all-data case leads to unsatisfactory results, with the reliability index lower than the target value.

Keywords: Cold formed profiles; Composite bars; FORM; Reliability; DSM.

1 Introduction

The Cold-formed Steel (CFS) profiles have advantages over hot-rolled *profiles*, such as less weight and greater plate width/thickness ratio, and may suffer instabilities along their length. These profiles can be manufactured in a several sections, providing many advantages in their use. Therefore, an elegant and low-cost solution for the possibility of failure due to global buckling of individual profiles can be obtained by joining two or more profiles (built-up sections). The built-up cold-formed steel columns can be use in CFS-framed building, when greater higher axial capacity or local frame rigidity is required. There are currently several methods for designing composite members, among them there is the Direct Strength Method (DSM) originally proposed by Schafer e Peköz [1]. This method, although not calibrated for the design of built-up sections, presents several studies of application in built-up sections. Through the DSM, the ultimate forces of the section are obtained by means of an elastic stability analysis of the cross section. The composite member sections analyzed in this paper are shown in Fig. 1.

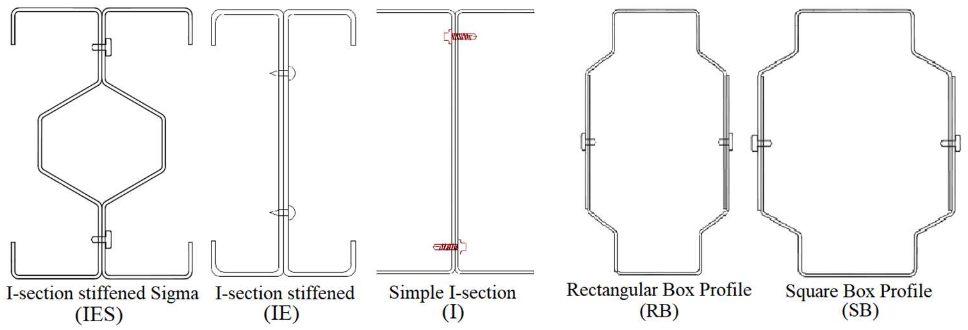

Figure 1. built-up sections used in this work.

Design equations of the Direct Strength Method are present in the current Brazilian Code ABNT NBR 14762:2010 [2]. The Brazilian standard presents a resistance factor, γ, which is related to ϕ by $\phi = 1 / \gamma$. The North American standard AISI S100:2016 [3] has design provisions for LRFD (Load and Resistance Factor Design), used by the United States and Mexico, and LSD (Limit States Design), used in Canada. A limit state corresponds to the circumstance in which a structural system fails to fulfill the purpose for which it was designed Hsiao [4]. The Reliability is associated with the assessment of the probability of violation of a limit state of a structural element, which can include either safety against failure or collapse of part of the structure. Regarding the limit state of strength, the current format of the LRFD method is represented by eq. (1). Both LRFD and LSD formats are based on the same philosophy, with the design load less than or equal to the resistance.

$$\sum \gamma_i Q_i \leq \phi R_n \tag{1}$$

where Q_i is the load effect; γ_i is the load factor; R_n is nominal resistance; ϕ is the resistance factor, $\phi < 1$. The resistance factor, ϕ, encompasses the uncertainties and variability inherent in the nominal resistance. The nominal resistance R_n is obtained from a suitable analytical model, applying the nominal section proprieties and the cross-section properties and the material properties. The load factor, γ_i, considers the uncertainties and variability of the loads and the load effects, Ellingwood *et al.* [5]. Resistance factors are obtained so that structural reliability reaches an intended level. The Reliability of a structure is established in terms of a reliability index, β, defined by probabilistic methods. In the standard calibration process, in the design equations, the share of actions and resistance need to be factored in order to reach a certain level of safety. Therefore, the resistance factor presents in the design standards need to be calibrated by the technical committees of the standard, in order to reach the target reliability index β_0.

Calibration procedures were obtained by the work of several researchers, such as Ravindra e Galambos [6]; Ellingwood et al. [5]; Hsiao [4]. These procedures are still used in the verification of new propositions of design formulations or updates of existing equations in the standards. The resistance factor contained in the Brazilian standard NBR 14762:2010 [2] is describe as γ = 1.20 for bars subjected to compression. The resistance factor of the North American standard AISI S100:2016 [3] for cold-formed steel columns under compression are ϕ = 0.85 (LRFD) and ϕ = 0.80 (LSD), which were calibrated using the FOSM method. Calibrations were performed such that in the LRDF format the ratio between the nominal live load (Ln) and the nominal dead load (Dn) Ln/Dn = 5, with the combination of actions 1.2Dn + 1.6Ln, and target reliability index β_0 = 2.5. In the case of LDS, the ratio is Ln/Dn = 3, combinations of actions 1.25Dn + 1.5Ln and β_0 = 3.0. For NBR 14762 [2] there was no calibration procedure, however, the safety level of the design equations for columns subject to compression is close to that defined by the North American standard, due to the combination of the ultimate actions of the Brazilian standard also be 1.25Dn + 1.5Ln. In addition, the resistance factor (γ = 1.20) is equivalent to ϕ of 0.83 (in the North American format), that is, an intermediate value between the resistance factor adopted for LSD and the LRFD.

This article aims to evaluate the reliability of built-up cold-formed steel columns subjected to axial compression force. The resistance calculation was performed using the Direct Strength Method, present in the Brazilian standard, based on a database with 266 built-up columns. This study gathered profiles of I-sections stiffened with web and flange stiffeners, I-section stiffened with flange stiffeners, simple I-sections and Box-sections. The values of the professional factor, P, were calculated by the ratio between the results obtained experimentally and the theoretical results. The Reliability indices were obtained using the Reliability methods: FOSM – First Order Second Moment, FORM – First Order Reliability Method and MCM – Monte Carlo Method. Results are evaluated for resistance factor of ϕ = 0.80 (LSD), ϕ = 0.83 (NBR) and ϕ = 0.85 (LRFD). The same calibration data from the North American standard was used.

2 Methodology

2.1 Direct Strength Method

The Direct Strength Method considers the original geometric properties of the cross section and the general analysis of elastic stability. To obtain the elastic buckling loads and respective buckling modes, the software CUFSM was used, Li and Schafer [7]. According to NBR 14762:2010 [2] the axial forces of elastic buckling, N_l, N_{dist}, N_e must be obtained for the bars under centered compression. In this way, the characteristic value of the axial compressive strength, $N_{c,Rk}$, is considered the lowest value for global, local and distortional buckling, as expressed by eq. (2).

$$N_{c,Rk} = mínimum\left(N_{c,Re}, N_{c,Rl}, N_{c,Rdist}\right) \quad (2)$$

The formulations for calculating the characteristic value of the axial compressive strength, associated with global buckling ($N_{c,Re}$), local buckling ($N_{cR,l}$) and distortional buckling ($N_{c,Rdist}$) are presented below.
Global Buckling:

$$N_{c,Re} = \left(0.658^{\lambda_0^2}\right)Af_y \;\; for \;\; \lambda_0 \leq 1.5 \quad (3)$$

$$N_{c,Re} = \left(\frac{0.877}{\lambda_0^2}\right)Af_y \;\; for \;\; \lambda_0 > 1.5 \quad (4)$$

where:

$$\lambda_0 = \left(\frac{Af_y}{N_e}\right)^{0.5}$$

λ_0 is the reduced slenderness factor associated with global buckling; A is the gross cross-sectional area of the column; f_y is the yield stress of the steel; N_e is the global elastic buckling axial force.
Local Buckling:

$$N_{c,R\ell} = N_{c,Re} \;\; for \;\; \lambda_\ell \leq 0.776 \quad (5)$$

$$N_{c,R\ell} = \left(1 - \frac{0.15}{\lambda_\ell^{0.8}}\right)\frac{N_{c,Re}}{\lambda_\ell^{0.8}} \;\; for \;\; \lambda_\ell > 0.776 \quad (6)$$

where:

$$\lambda_\ell = \left(\frac{N_{c,Re}}{N_\ell}\right)^{0.5}$$

λ_ℓ is the reduced slenderness factor associated with local buckling; N_ℓ is the axial elastic local buckling force.
Distortional Buckling:

$$N_{c,Rdist} = Af_y \;\; for \;\; \lambda_{dist} \leq 0.561 \quad (7)$$

$$N_{c,Rdist} = \left(1 - \frac{0.25}{\lambda_{dist}^{1.2}}\right)\frac{Af_y}{\lambda_{dist}^{1.2}} \;\; for \;\; \lambda_{dist} > 0.561 \quad (8)$$

where:

$$\lambda_{dist} = \left(\frac{Af_y}{N_{dist}}\right)^{0.5}$$

N_{dist} is the axial elastic distortional buckling force; λ_{dist} is the reduced slenderness factor associated with distortional buckling.

2.2 FOSM, FORM and SMC methods

The evaluation of structural reliability can be performed using approximate analytical methods and simulation methods, FOSM, FORM and SMC respectively. The FOSM method is based on the first-order of the Taylor series, and on the statistical parameters, mean and standard deviations, Beck and Souza [8]. The FORM method uses all statistical information from the random variables of the problem, so, in addition to the mean and standard deviations, the probability distribution and correlation coefficients are also used, Hasofer and Lind [9]. In the SMC random variables are generated based on their respective probability distributions and the verification of the structural response is performed. The failure probability is calculated by the ratio between the numbers of simulations, n, in which the limit state function is smaller than zero and the total number of simulations, Melchers and Beck [10].

2.3 Professional factor

The professional factor, P, is a random variable obtained by the ratio between the experimental resistance (tested column) and the theoretical resistance (DSM). This factor allows to verify how safe or unsafe the model studied is. In this way, the mean, P_m, approaches the unit value if the model adequately represents the physical phenomenon studied. The coefficient of variation, V_P, is obtained by the ratio between the standard deviation, σ_P, and the mean, P_m, and denotes the level of dispersion of the data around the mean. The database contains a total of 266 tests performed by different authors. In this study, profiles of I-sections stiffened with web and flange stiffeners (IES), I-sections stiffened with flange stiffeners (IE), simple I-sections (I), Rectangular Box Profile (RB) and Square Box Profile (SB), (Fig. 1) were collected. A summary of the database is shown in Tab. 1.

Table 1. Database summary

Section type	L (mm)	*h/t*	*References*
IES	299.60 - 3200.00	81.93 - 208.33	Abu-Hamd *et al.* [11], Zhang & Young [12] and [13], Aghoury *et al.* [14]; Li & Young [15]
IE	261.00 - 3038.00	61.07 - 410.71	Lu *et al.* [16], Fratamico *et al.* [17], Roy *et al.* [18]
I	999.69 - 1800.70	33.33 - 80.01	Selvaraj & Madhavan [19]
RB	270.00 - 3200.00	80.00 - 135.08	Zhang & Young [20], Nie *et al.* [21], Li & Young [15]
SB	299.80 - 3199.50	82.05 - 183.33	

The data for the variable P were organized into groups by failure mode and by section type. Table 2 presents the results of the statistical study and the quantity of data per group (Q). With the help of Minitab20® [22] software, Anderson-Darling tests were performed, obtaining the normal probability distribution function (PDF) as the one that best adjusted to each of the groups.

Table 2. Statistical data of the Professional factor

Data Group	Nomenclature	Q	P_m	σ_P	V_P
Global Mode	G	61	1.16	0.28	0.24
Local Mode	L	178	0.98	0.19	0.19
Distortional Mode	D	27	1.09	0.13	0.12
Ie Sigma Profile	IES	73	1.17	0.24	0.20
Ie Profile	IE	93	1.01	0.22	0.22
I Profile	I	45	0.93	0.09	0.10
Rectangular Box Profile	RB	29	0.80	0.06	0.08
Square Box Profile	SB	26	1.14	0.07	0.06
All Data	T	266	1.03	0.22	0.22

2.4 Performance function

The limit state function is based on the usual safety conditions related to limit states in relation to resistance variables, depending on the material resistance, the geometry of the section and its dimensions and the loads, expressed in terms of dead load and live load. Mathematically, the performance function can be expressed by eq. (9).

$$g(.) = R_n(MFP) - c(D + L) \quad (9)$$

where R_n is the nominal strength of the structural element, M is the material factor, which reflects material uncertainties, F is the fabrication factor, which reflects geometry uncertainties, P, is the professional factor (model error), and D and L are the dead load and live load. The $'c'$ is the deterministic coefficient of transformation of actions into effects. The statistical parameters of random variables M, F, D and L were obtained from the work of Ellingwood *et al.* [5] are presented in Tab. 3.

Table 3. Statistical parameters and probability distribution

Random Variable	Mean value/ Nominal value	Coefficient of Variation	Probability Distribution Function
M	1.10	0.10	Lognormal
F	1.00	0.05	Lognormal
D	1.05	0.10	Normal
L	1.00	0.25	Gumbel

3 Reliability Analysis

For the reliability analysis, the calibration procedure of the North American specification AISI S100:2016 [3] was taken into account, with resistance factor ϕ = 0.80 (LSD), ϕ = 0.83 (NBR) and ϕ = 0.85 (LRFD). Thus, the following situations were defined:

- For LRFD: (I) 1.2Dn + 1.6Ln, Ln/Dn = 5, ϕ = 0.85, β_0 = 2.5;
- For LSD: (II) 1.25Dn + 1.5Ln, Ln/Dn = 3, ϕ = 0.80, β_0 = 3.0;
- For NBR: 1.25Dn + 1.5Ln, Ln/Dn = 3, ϕ = 0.83, β_0 = 2.5.

Data grouping was defined according to Tab. 2. Reliability index were calculated using the FOSM, FORM and SMC methods and the results obtained are presented in Tab. 4.

Table 4. Reliability index for the SMC, FORM and FOSM methods.

Data Group	B_{SMC}			β_{FORM}			β_{FOSM}		
	LRFD	LSD	NBR	LRFD	LSD	NBR	LRFD	LSD	NBR
G	2.11	2.14	2.06	2.20	2.21	2.13	2.51	2.57	2.45
L	2.01	2.04	1.93	2.09	2.12	2.01	2.17	2.24	2.10
D	2.73	2.83	2.71	2.77	2.87	2.74	2.89	3.04	2.87
IES	2.38	2.43	2.32	2.47	2.51	2.41	2.70	2.79	2.65
IE	1.91	1.93	1.83	1.99	2.01	1.91	2.13	2.19	2.06
I	2.30	2.40	2.25	2.33	2.42	2.28	2.36	2.49	2.32
RB	1.84	1.90	1.76	1.86	1.94	1.78	1.82	1.92	1.74
SB	3.12	3.27	3.13	3.12	3.27	3.12	3.32	3.54	3.36
T	1.94	1.98	1.89	2.04	2.06	1.96	2.19	2.26	2.12

Analyzing the values obtained by the FORM method, only the values of groups D and SB were superior to β_0 = 2.5 for LRFD and NBR and for LSD only the SB group presented a result superior β_0 = 3.0. For SMC, in LRFD format, β values range from 1.84 to 3.12, for LSD, β values range from 1.90 to 3.27 and for NBR, β values range from 1.76 to 3.13. The highest value of β is observed in the SB group and the lowest value in the RB group. For the FORM method, in LRFD the β values range from 1.86 to 3.12, for LSD the β values range from 1.94 to 3.27 and for NBR the β values range from 1.78 to 3.12. The highest value of β is observed in the SB group and the lowest value in the RB group. And for the FOSM method, in the LRFD format the β values range from 1.82 to 3.32, for LSD the β values range from 1.92 to 3.54 and for NBR the β values range from 1.74 to 3.36, with the highest and lowest values of β identical to those observed in the FORM and SMC methods. Figure 2 and Fig. 3 presented the reliability index for the LRFD and LSD philosophies and NBR format.

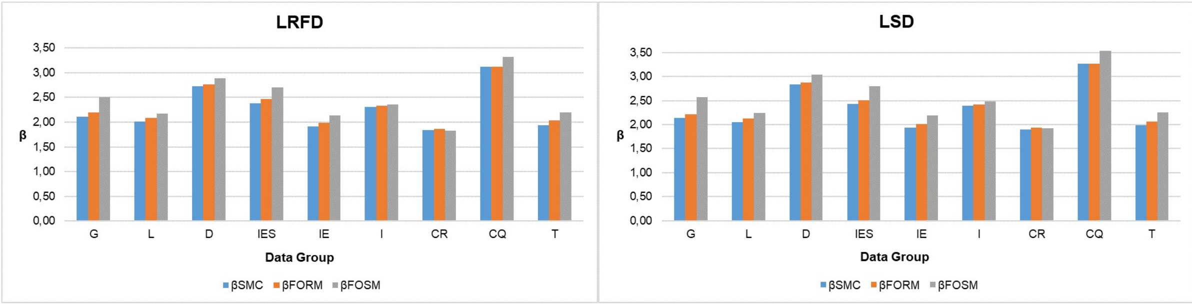

Figure 2. Reliability index for LRFD and LSD philosophies

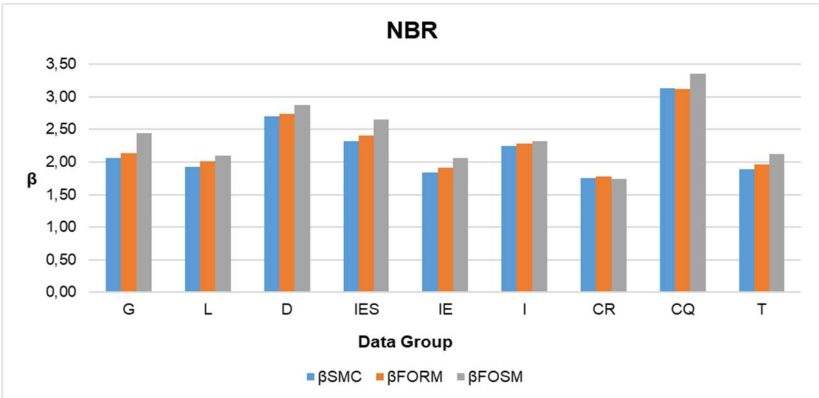

Figure 3. Reliability index for NBR *format*

4 Conclusions

This study aimed to evaluate the reliability of built-up columns of cold-formed profiles under compression, using the resistance factor, γ, equal to 1.20 adopted by NBR 14762:2010 [2]. The following conclusions were drawn from the results of this analysis:

- The FORM method showed good accuracy in the results compared to the Monte Carlo Method. Furthermore, the FORM proved to be efficient for the performance function and the number of variables in the problem, converging with 5 iterations.
- The results of the FOSM method were superior to the results of the other reliability methods for most data group. It is noteworthy that this is a simplified method, used in the calibration of the AISI S100:2016 standard, but with lower precision.
- For the LRFD philosophy, groups D and SB presented adequate results in relation to the required safety level, with values of reliability indexes higher than the target β_0 = 2.5.
- For the LSD philosophy, only the SB group presented satisfactory results, with a reliability index higher than the target β_0 = 3.0.
- For the NBR format, groups D and SB presented adequate results in relation to the required safety level, with values higher than the target value β_0 = 2.5.
- Therefore, it is necessary to deepen the studies to improve the DSM and its application in the design of built-up bars of cold-formed profiles, since the method is not calibrated for such profiles.

Acknowledgements. The authors would like to thank UFOP (Universidade Federal de Ouro Preto) and CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior) for supporting the development of this study.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] SCHAFER, B. W., & PEKÖZ, T. "Direct Strength Prediction of Cold-Formed Steel Members Using Numerical Elastic Buckling Solutions". In International Specialty Conference on Cold-Formed Steel Structures: Recent Research and Developments in Cold-Formed Steel Design and Construction, pp. 1-8, 1998.
[2] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. ABNT NBR 14762: Dimensionamento de estruturas de aço constituídas por perfis formados a frio - Procedimento. Rio de Janeiro: ABNT, 2010.
[3] AMERICAN IRON AND STEEL INSTITUTE. AISI-S100-16: North American specification for the design of cold-formed steel structural members. Washington, D.C.: ANSI/AISI, 2016.
[4] HSIAO, L. E. Reliability based criteria for cold-formed steel members. PhD thesis, University of Missouri, Rolla, 1989.
[5] ELLINGWOOD, B.; MACGREGOR, J.G.; GALAMBOS, T.V.; CORNELL, C.A. Development of a Probability-Based Load Criterion for American National Standard A58 – NBS Special Publication. National Bureau of Standards, United States Department of Commerce, Washington: D.C, 1980.
[6] RAVINDRA, M. K.; GALAMBOS, T. V. Load and resistance factor design for steel. *Journal of the Structural Division*, v. 104, n. 9, pp. 1337-1353, 1978.
[7] LI, Z.; SCHAFER, B. W. Buckling analysis of cold-formed steel members with general boundary conditions using CUFSM: Conventional and Constrained Finite Strip Methods. Versão 4.5. Cornell University. 2010.
[8] BECK, A.T.; SOUZA JR, A.C. A first attempt towards reliability-based calibration of Brazilian structural design codes. *Journal of the Brazilian Society of Mechanical Science*, 2010.
[9] HASOFER, A.M.; LIND, N.C. Exact and invariant second moment code format. *Journal of the engineering mechanics division: ASCE*, v. 100, n. 1, pp. 111-121, 1974.
[10] MELCHERS, R. E., & Beck, A. T. *Structural Reliability Analysis and Prediction*, Hoboken: John Wiley and Sons, 2018
[11] ABU-HAMD, M.; ABDEL-GHAFFAR, M. M.E.; EL-SAMMAN, B. M. Buckling strength of axially loaded cold formed built-up I-sections with and without stiffened web. *Ain Shams Engineering Journal*, Vol. 9, n. 4, pp. 3151-3167, 2018.
[12] ZHANG, J.H.; YOUNG, B. Compression tests of cold-formed steel I-shaped open sections with edge and web stiffeners. *Thin-Walled Structures*, Vol. 52, pp. 1-11, 2012.
[13] ZHANG, J.H.; YOUNG, B. Numerical investigation and design of cold-formed steel built-up open section columns with longitudinal stiffeners. *Thin-Walled Structures*, Vol. 89, pp. 178-191, 2015.
[14] AGHOURY, M. E.; TAWFIC, M.; AMOUSH, E. Compressive Strength of Axially Loaded Built-up Sigma Cold Formed Sections Columns. *Future Engineering Journal*, Vol. 1, n. 1, Article 6, 2020.
[15] LI, Q.Y.; YOUNG, B. Experimental and numerical investigation on cold-formed steel built-up section pin-ended columns. *Thin-Walled Structures*, Vol. 170, 2021.
[16] LU, Y.; ZHOU, T.; LI, W.; WU, H. Experimental investigation and a novel direct strength method for cold-formed built-up I-section columns. *Thin-Walled Structures*, Vol. 112, pp. 125-139, 2017.
[17] FRATAMICO, D. C.; TORABIAN, S.; ZHAO, X.; RASMUSSEN, K. J.R.; SCHAFER, B. W. Experiments on the global buckling and collapse of built-up cold-formed steel columns. *Journal of Constructional Steel Research*, Vol. 144, pp. 65-80, 2018.
[18] ROY, K.; TING, T. C. H.; LAU, H. H.; James LIM, J. B.P. Effect of thickness on the behaviour of axially loaded back-to-back cold-formed steel built-up channel sections - Experimental and numerical investigation. *Structures*, Vol. 16, pp. 327-346, 2018.
[19] SELVARAJ, S.; MADHAVAN, M. Design of cold-formed steel built-up columns subjected to local-global interactive buckling using direct strength method. *Thin-Walled Structures*, Vol. 159, 2021.
[20] ZHANG, J.H.; YOUNG, B. Experimental investigation of cold-formed steel built-up closed section columns with web stiffeners. *Journal of Constructional Steel Research*, Vol. 147, pp. 380-392, 2018.
[21] NIE, S.; ZHOU, T.; EATHERTON, M. R.; LI, J.; ZHANG, Y. Compressive behavior of built-up double-box columns consisting of four cold-formed steel channels. *Engineering Structures*, Vol. 222, 2020.
[22] MINITAB. MINITAB®: Version: 20.4. Available at: http://www.minitab.com/pt-br/