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Analysis of reinforced concrete beams with opening using incompatible mode elements and strut-and-ties model

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Abstract. The need to reduce the execution time has led to the negligence in the process of compatibility project. One of the consequences is the need to drill holes in beams for the passage of ducts and piping. This practice occurs without criteria for analysis and structural design. Openings in beams performed without an accurate analysis result in reduced strength considered in the design, instability and compromised safety. In the regions of openings, usually called "Regions D", the Bernoulli hypothesis becomes invalid and, therefore, the strut-and-ties model associated with the finite element method has been used. This work proposes to analyze the distribution of stresses in beams with opening using classical finite elements and enhanced with incompatible modes. Precise results of internal efforts allow an adequate arrangement of reinforcements and fulfillment of safety in the ultimate and service limit states. In order to validate the performance of the elements implemented for the analysis of structures with discontinuities, the results obtained are compared with those found in the literature and calculated using a commercial package. The results obtained attest to the quality of the implemented formulation.

Keywords: reinforced concrete, incompatible mode elements, strut-and-ties model.

1 Introduction

Construction modern necessity many ducts to accommodate essential services like water supply, sewage, airconditioning, electricity, etc. Sometimes these pipes are placed in openings made in reinforced concrete beams. Ordinary beams with openings and deep beams (with and without openings) are considered disturbed regions where their strains within any section are significantly nonlinear. Therefore, it is not adequate to design those regions using either bending theory or conventional shear design equations. Hence, it is essential to rely on a rational method such as the strut-and-tie model. Strut-and-tie models (STMs) are derived from the truss analogy models from researches by W. Ritter [1] and E. Mörsch [2]. Afterwards, Schlaich et al. [3] expanded the concepts to evaluate the discontinuity conditions of several structural elements. According to Schlaich et al. [3], if the design engineer is not yet sufficiently experienced with modelling, he will first employ an elastic finite element program and plot the elastic stresses for orientation of the strut-and-tie-model. The present work focused on study of beams and deep beams with openings through the evaluation of the elastic stress field using finite elements enriched with incompatible modes.

2 Strut-and-ties model

Strut-and-tie models (STMs) is based on discrete representations of stress fields caused by applied loads and boundary conditions present in reinforced concrete structural elements. Structures can be divided into two types of regions: B region is a structural part for which Bernoulli's hypothesis is valid; D regions or discontinuous are regions where distribution of deformations is not linear due to static or geometric discontinuities, so Bernoulli's-hypothesis is not valid. The Strut-and-Tie modelling (STM) needs to be used when the structural element under consideration requires geometric or static discontinuities. The STM method is an effective tool for engineers to design disturbed regions (so-called D-regions) of reinforced concrete (RC) structures [4].

3 Incompatible mode method in 2D

The method of incompatible modes was introduced in 1973 by Wilson et al. [5] to improve the performance low-order elements. The simple four-node isoparametric element does not produce accurate results for many applications. To illustrate this deficiency, consider the rectangular element, shown in Fig. 1.

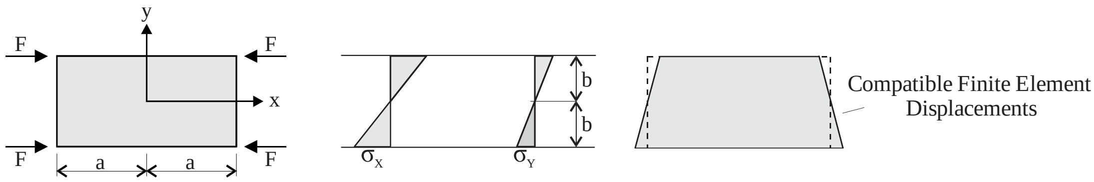


Figure 1. Basic equilibrium errors in four-node plane element.

This deficiency might be compensated with the addition of the displacements of quadratic type:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} N_i \\ N_i \end{bmatrix} \begin{bmatrix} u_i \\ v_i \end{bmatrix} + \begin{bmatrix} M_j \\ M_j \end{bmatrix} \begin{bmatrix} \alpha_j \\ \alpha_j \end{bmatrix} \tag{1}$$

where N_i are element shape functions; u_i and v_i are nodal displacements; α_j are the incompatible mode parameters to be determined; M_j the incompatible modes shape functions; i = (1, …,4) and j = (1, …,2). The incompatible shape functions are:

$$\begin{aligned} M_1 &= 1 - \xi^2 \\ M_2 &= 1 - \eta^2. \end{aligned} \tag{2}$$

In the case of small displacements and linear elasticity the principle of minimum potential energy can be applied and so the complete element stiffness matrix with the added incompatible modes is:

$$\mathbf{K}_m = t \int_A \mathbf{B}_m^T \mathbf{D} \mathbf{B}_m dA \tag{3}$$

where t is the element thickness; $\mathbf{B}_m$ is a complete matrix with the added incompatible modes; and $\mathbf{D}$ is the elasticity tensor. Then the nodal displacements $\boldsymbol{d}$ are obtained and thus the stresses $\boldsymbol{\sigma}$ at the Gauss points.

4 Numerical examples

In this paper, a Julia algorithm was implemented using the plane stress from a linear elastic analysis in finite elements. The 2D beams models was discretized using low-order quadrilateral (standard Q4 and incompatible Q4-INC) and triangular (CST) elements. Paraview software was used for visualization of tension trajectories.

4.1 Simply-supported beam with opening

In order to validate the ability of the implemented elements to study the tensile and compressive behavior of reinforced concrete beams, used one model of beam with opening, studied by Silva [6]. The simply supported beam RC containing a web opening and the characteristics of the geometry and the boundary conditions is presented in the Figure 2. A uniformly distributed load of 14.0 kN/m is applied. The design value of concrete compressive strength is f_{cd} = 15 MPa and design yield strength of reinforcement is f_{yd} = 434 MPa. The numerical models were developed using the algorithm implemented. The beam was discretized in 237, 948 and 3792 four node bilinear quadrilateral elements (Q4 and Q4-INC), with size 10×10 cm, 5×5 cm and 2.5×2.5 cm, respectively. Fig. 3 shown the stress distribution σ_{xx} in two sections of the span of a beam.

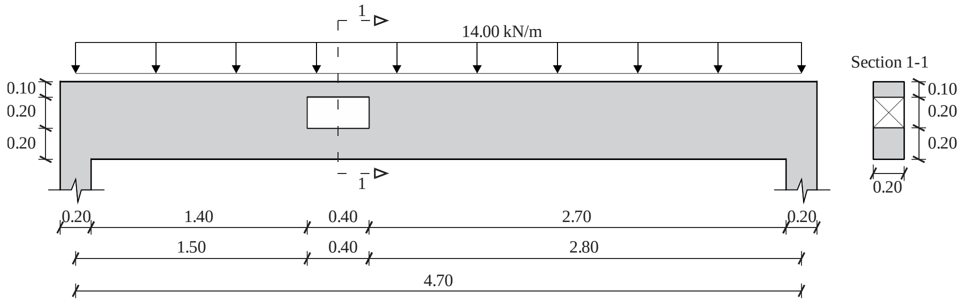
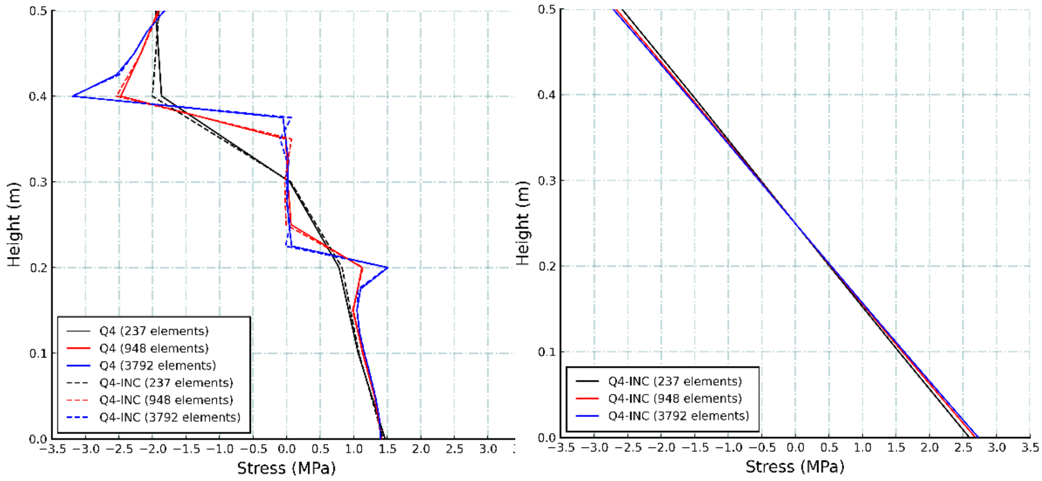


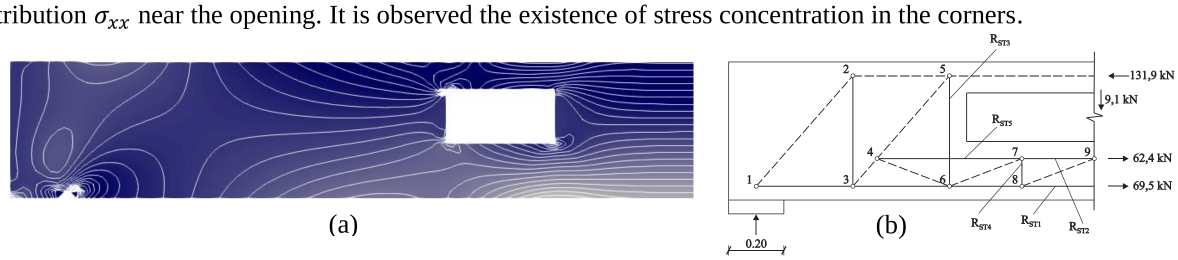
Figure 2. Simply supported beam with opening subject to uniformly distributed load.



(a) (b)

Figure 3. Stress distribution at a distance of the center of the left support: (a) 1.50 m and (b) 2.40 m.

Stress sitting 1.50 m of the center of the left support have nonlinear distribution while stress sitting 2.40 m have linear distribution. This happens because in the first section the stress field is disturbed by the presence of the opening (D region). On the other hand, in the section after opening there is a region B. Both elements have the ability to represent the behavior of stresses along the height of the beam. Figure 4a shows contour lines for stress distribution σ_{xx} near the opening. It is observed the existence of stress concentration in the corners.



(a) (b)

Figure 4. (a) Contour lines for stress distribution σ_{xx} . (b) Strut-and-tie model.

Fig. 4b shows the STM model and, finally, Fig. 5 presents the complete reinforcement resulting from the above model. The geometry of the model is indeed oriented at the elastic stress fields.

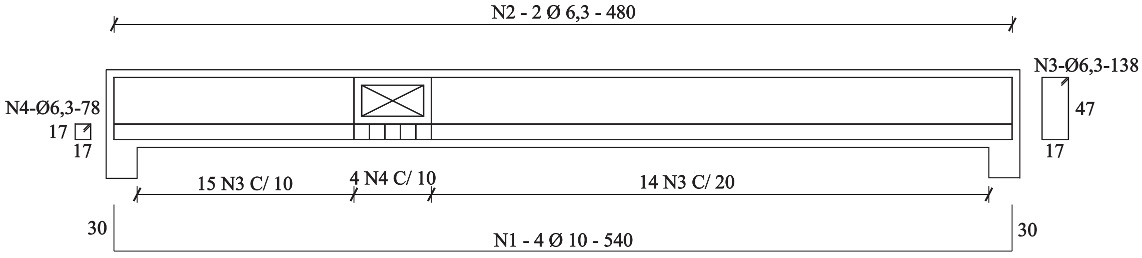
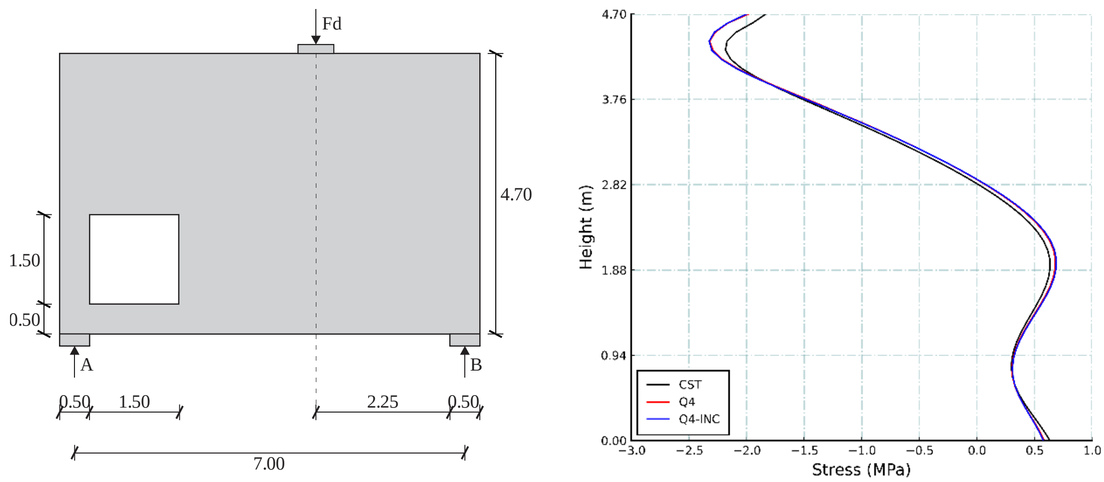


Figure 5. Reinforcement layout.

4.2 Deep beam with a large hole

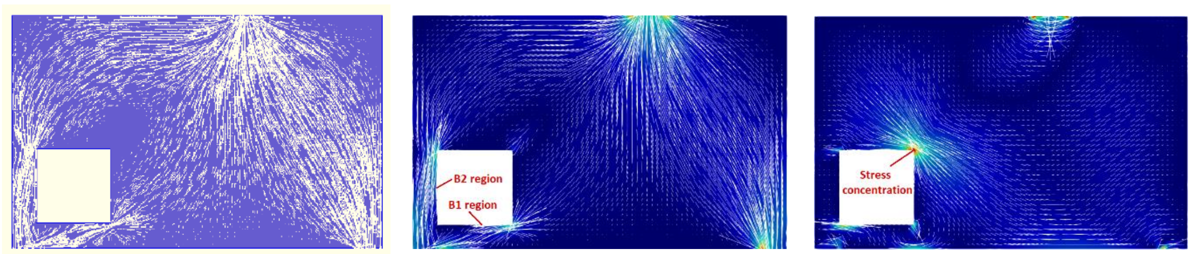
This application example was studied by Schlaich et al. [3]. It is a simply supported deep beam under a concentrated load F_d of 3.0 MN. The geometry and boundary conditions are shown in Fig. 6a. The concrete design strength is f_{cd} = 17 MPa and the reinforcement desing strength is f_{yd} = 434 MPa. This structure has D-region due to the geometric discontinuity. Fig. 6b shown the stress distribution σ_{xx} at mid span. The numerical models were discretized using 3307 (size 10×10 cm) four node bilinear quadrilateral elements (Q4 and Q4-INC) and 6614 (size 10×10 cm – base and height, respectively) three node CST triangular elements.



(a) (b)

Figure 6. (a) Dimensions (m) and load. (b) Stress distribution σ_{xx} at mid span.

The deep beam shows in Fig. 6a was linearly analyzed using the finite element program implemented in this work. From the obtained elastic principal stress trajectories in Fig. 7b and 7c for beam, the strut-and-tie model shown in Fig. 8a is obtained.



(a) (b) (c)

Figure 7. Stress principal trajectories: (a) σ_2 Abaqus. (b) σ_2 present work. (c) σ_1 present work.

From the elastic principal stress trajectories in Fig. 7b and 7c, it could be noticed that the opening affects the beam's stress trajectories drastically, where zones of tension stresses are formed around the right-upper corner of the opening (load side) and the corner on the same diagonal and compression zones are formed around the two other corners. The reaction A enters the structure vertically and remains in this direction until it has passed the hole. The B2-region is thus a centrically loaded column. In fact, part of the reaction A could also he transferred via the B1-region by bending moments and shear forces. However, comparing the axial stiffness of B1, with the bending stiffness of B2, this part is negligible. Finally, the tension and compression stress trajectories in Fig. 7b and Fig. 7c were followed to develop the STM shown in Fig. 8a. The compression stress trajectories are replaced by compression elements (struts) and the tension stress trajectories are replaced by tension elements (ties). Fig. 8b shows reinforcement resulting from the model.

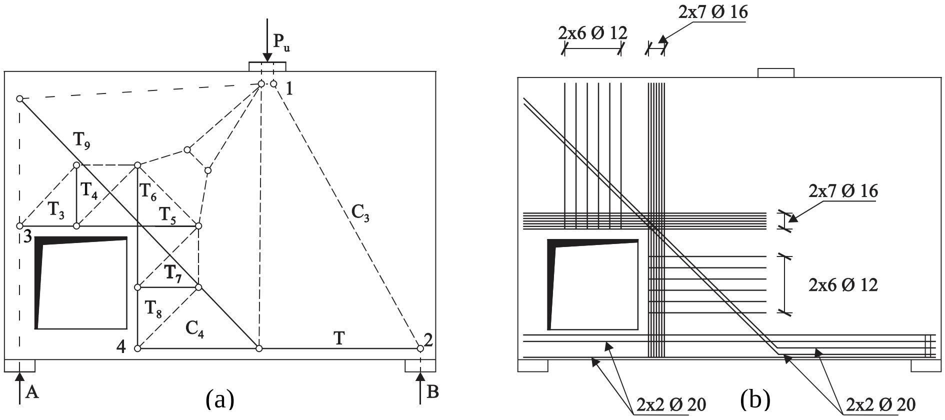


Figure 8. (a) Strut-and-tie model. (b) Reinforcement layout.

5 Conclusions

In this work, it was studied the influence of opening in RC beams and deep beams. A Julia algorithm was implemented using the plane stress from a linear elastic analysis in finite elements. Two classical numerical models were developed using the algorithm implemented. The beams was discretized with four node bilinear quadrilateral elements (Q4 and Q4-INC) and triangular CST element. The beams was linearly analyzed using the finite element program implemented. The opening affects the beam's stress trajectories drastically. The stress field is disturbed by the presence of the opening (D region). From the obtained elastic principal stress trajectories for the beams, the strut-and-tie models were established. The results obtained attest to the quality of the implemented formulation.

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References

[1] Ritter W. Die bauweise hennebique. Schweizerische Bauzeitung. 1899.
[2] Mörsch E. Der Eisenbetonbau, seine Theorie und Anwendung, 2nd ed. concrete-steel construction. New York: Good rich, McGraw Hill; 1909.
[3] Schlaich, J.; Schafer, K.; Jennewe, M. Toward a consistent design of structural concrete. PCI-Journal, vol. 32, nr.3, p. 74 – 150, May/June, 1987.
[4] Xia Y.; Langelaar M.; Hendriks M. A. N. Automated optimization-based generation and quantitative evaluation of Strut-and-Tie models. Computers and Structures. 2020; 238. 106297. https://doi.org/10.1016/j.compstruc.2020.106297
[5] Wilson, E. L., Taylor, R. L., Doherty, W. P., Ghaboussi, J.: Incompatible displacement models, In Fenves, S.J., Perrone, N., Robinson, A.R., Schnobrich, C. (editors), Numerical and Computer Methods in Structural Mechanics, Academic Press, pp. 43-57, 1973, https://doi.org/10.1016/B978-0-12-253250-4.50008-7.
[6] Silva, R. C.; Giongo, J. S. Reinforced concrete: applications of strut-and-tie models. Dissertation of Master – University of São Paulo, São Carlos, 1991.