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Dynamic Analysis of an Aluminum Spur Gear pair

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Abstract. Dynamical systems are those that show evolution with respect to time and are generally described by ordinary differential equations with their respective initial conditions. However, the equations of motion obtained are often difficult to solve analytically, which makes it necessary to use one of the methods of computational numerical simulation. This work presents the dynamic modeling of a spur gears pair, considering the torque input of an electric motor. The system is modeled analytically using Newton-Euler equations of motion resulting in a system of equations with four degrees of freedom. The mathematical model results in an ordinary second-order differential equation and is solved using the fourth and fifth order Runge Kutta methods, implemented in MATLAB software; the parameters inertia, damping and gear stiffness are considered. Dynamic system simulations are performed to observe the dynamic behavior in different scenarios. The results clearly show the differences obtained between the transient and steady regimes and the short period of disturbance of the system.

Keywords: Gear Pair, Dynamic Model, Numerical Simulation

1 Introduction

A gear pair is a mechanism primarily responsible for transmitting torque, however, it is often related to applications where movement accuracy is critical. In these circumstances, in order to meet the design requirements, dynamic modeling is often employed. Modeling consists of mastering the behavior of the interaction between gears, involving forces, states and parameters, allowing the simulation and control of movement with precision. The dynamic analysis may have a greater influence on the transient regime, especially in cases where the system transits through the rest condition, which may be when the engine is started or when suddenly changing the direction of rotation. In theory, the transmission between gears occurs through the perfect bearing between the involute profile teeth, although in practice, there are manufacturing and assembly errors, which can lead to irregularities between the input and output movements. In this sense, the present study aims to develop a dynamic model for a pair of gears that can be used in control systems.

2 Bibliographic Review

This topic has been recently reviewed by several authors. Li and Kahraman [1], investigate the interactions between tribological and dynamic aspects in a pair of spur gears, involving a lubrication model. Cooley and Parker [2], highlight the growth in the number of publications in the last two decades related to gear dynamics applied to planetary gearboxes, making a brief introduction about the main authors. Liang, Zuo and Feng [3], cite the importance of the dynamic model employed in the detection of failures in transmission mechanisms. Moraes [4] develops a mathematical model of a pair of gears in order to assist in monitoring the conditions confronting practical vibration data obtained experimentally. Finally, Dai, Long, Chen and Xun present a methodology for calculating the stiffnesses involved in meshing, including the Hetzian contact stiffness and the time-varying stiffness.

3 Model description

A pair of gears is modeled as a rigid body, based on Newton's laws of motion and Euler's equation that describes rotational motion. According to Liang, Zuo and Feng [3], the dynamic model can even be used to identify symptoms and mechanisms of failure generation. A dynamic model of a gearbox is usually simplified, so that the most important variables are considered where only the study variable is detailed, mainly due to the complexity of the system regarding modeling and the computational cost.

The dynamic model was inspired by the Howard, Jia and Wang [5], Lin and Liou [6] and Moraes [4], model shown in Fig. 1.

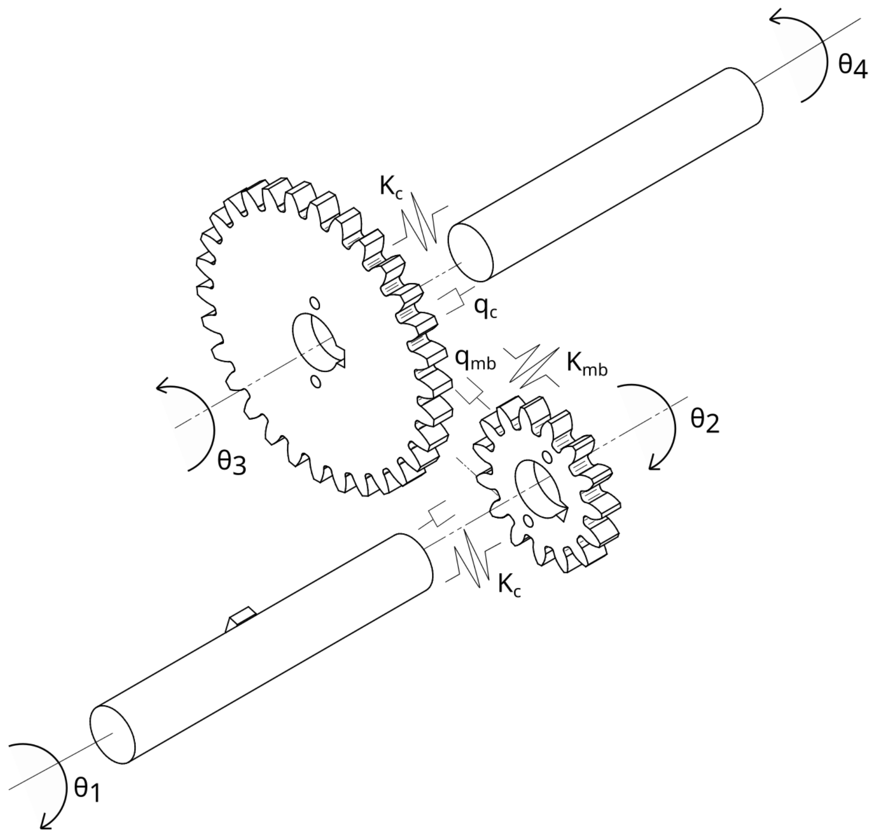

Figure 1. Dynamic model

The mathematical model of Howard, Jia and Wang [5], presented by the eqs. (1), (2), (3) and (4) has not been modified, except for the axial displacements that were canceled considering that the bearings are rigid.

$$I_m\ddot{\theta}_1 + q_c(\dot{\theta}_1 - \dot{\theta}_2) + k_c(\theta_1 - \theta_2) = T_{in}. \tag{1}$$

$$I_p\ddot{\theta}_2 + k_c(\theta_2 - \theta_1) + r_p k_{mb}(r_p\theta_2 - r_g\theta_3) + q_c(\dot{\theta}_2 - \dot{\theta}_1) + r_p q_{mb}(\dot{\theta}_2 - r_g\dot{\theta}_3) + f_c \cdot r_p = 0. \tag{2}$$

$$I_g\ddot{\theta}_3 + k_c(\theta_3 - \theta_4) + r_g k_{mb}(r_g\theta_3 - r_p\theta_2) + q_c(\dot{\theta}_3 - \dot{\theta}_4) + r_g q_{mb}(r_g\dot{\theta}_3 - r_p\dot{\theta}_2) - f_c \cdot r_g = 0. \tag{3}$$

$$I_L\ddot{\theta}_4 + q_c(\dot{\theta}_4 - \dot{\theta}_3) + k_c(\theta_4 - \theta_3) = -T_{out}. \tag{4}$$

In eq. (1), I_m is the input shaft moment of Inertia, $\ddot{\theta}_1$ is the input shaft angular acceleration, q_c is the shafts damping coefficient, $\dot{\theta}_1$ and $\dot{\theta}_2$ is the input shaft and pinion angular velocity respectively and T_{in} is the electrical motor input torque. In eq. (2) I_p is the pinion moment of inertia, $\ddot{\theta}_2$ is the pinion angular acceleration, r_p and r_g is the pinion and gear radius, K_{mb} is the gearing stiffness, θ_2 and θ_3 is the pinion and gear angular displacement respectively, q_{mb} is the gearing damping coefficient, $\dot{\theta}_3$ is the gear angular velocity and f_c is the gearing contact force. In eq. (3), I_g is the gear moment of inertia, $\ddot{\theta}_3$ is the gear angular acceleration, θ_4 and $\dot{\theta}_4$ is the output shaft angular displacement and velocity respectively. In eq. (4), I_L is the output shaft moment of inertia, $\ddot{\theta}_4$ is the output shaft angular acceleration and T_{out} is the output torque.

3.1 Numerical solution

The mathematical model associated with the physical parameters describes the dynamic behavior of the geared pair, however, as this is an initial-value problems of differential equations, the solution of these equations requires the use of a numerical method, implemented computationally. In this case, it was decided to use the variable-pitch Runge Kutta method alternating between fourth and fifth order, implemented in MATLAB, through the ODE45 function.

3.2 Parameters

Inertia

Polar moment of inertia I, is calculed by eq.(5), where ρ is the density of material and l is the length of the shaft.

$$I = \frac{\pi l \rho r^4}{2}. \tag{5}$$

Stiffness

According to Lin and Liou [6], the stiffness of the shaft can be calculated through the eq. (6).

$$k_c = \frac{\pi G r^4}{2}. \tag{6}$$

Transverse modulus of elasticity G, as presented in eq. (7), where E is the elasticity modulus of material and ν is the Poisson coefficient.

$$G = \frac{E}{2(1+\nu)}. \tag{7}$$

Gear stiffness will be simplified in order to consider, instead of time-varying stiffness, constant stiffness based on the Hertz contact stress principle, as cited by Moraes [4].

$$k_{mb} = \frac{\pi E l}{4(1-\nu)}. \tag{8}$$

In case other contributions to the gear stiffness are considered, an association of springs in series can be made, eq. (9), taking into account the different stiffnesses involved, according to Liang, Zuo and Feng [3] and Dai, Long, Chen and Xun [7].

$$k_{mb} = \frac{1}{\frac{1}{k_p} + \frac{1}{k_g}}. \tag{9}$$

According to Li and Kahraman [1] experimental studies show that the dynamic model of a geared pair is a nonlinear system whose gear stiffness and backlash, which vary in time, are the main causes of this nonlinearity.

One way to reduce the backlash is by modifying the tooth profile of the gears in order to reduce the backlash between the teeth. Cooley and Parker [2] cites that the profile change in gear teeth can bring advantages such as the reduction of transmission errors.

Damping

According to Lin and Liou [6], the damping coefficient for the shafts is approximately $q_c = 0.006$ whereas the damping coefficient relative to the gearing is defined as presented in eq. (10).

$$q_{mb} = 0.2\sqrt{\frac{K_{mb}}{\frac{r_p^2}{I_p} + \frac{r_g^2}{I_g}}}. \tag{10}$$

4 Results

In the implementation of the model, translational displacements were disregarded, so the values of x and y are considered zero. Such a hypothesis is equivalent to saying that the deflections in the axes are equal to zero, and as the axes are made of steel and are relatively short, it is understood that such displacements are irrelevant, since the interest is to obtain the angular positions, and for that , shafts distortions are already being considered. The physical parameters used in both models are shown in Table 1.

Table 1. Parameters of the model

Parameters	Dimension
r_1/r_4	0.006 m
q_c	0.06
k_c	163.33
I_m	0.0012 m^4
I_p	0.0012 m^4
I_g	0.0020 m^4
I_L	0.0016 m^4
r_p	0.015 m
r_g	0.03 m
q_{mb}	9429.55
k_{mb}	485303730
f_c	2333.33 N
T_{in}	35 Nm

The simulation was performed considering a transient regime when the time is less than $0.005s$, and permanent regime when the time is longer than $0.005s$. The response obtained for the steady-state simulation for position and velocity is shown in Fig. 2

As noted, the simulation for steady-state shows that the displacement and angular velocity of the drive gear is always twice that of the input gear, which was expected, since the radius of the output gear is twice as large. Analyzing such behavior of the motion in steady state, we can compare the answer with the eq. (11), commonly used for kinematic relationships between gears, where n_p and n_g is the pinion and gear angular speed, respectively.

$$\frac{n_p}{r_g} = \frac{r_p}{n_g}. \tag{11}$$

The simulation response for the transient regime, both for the position and for the velocity, are presented in Fig. 3.

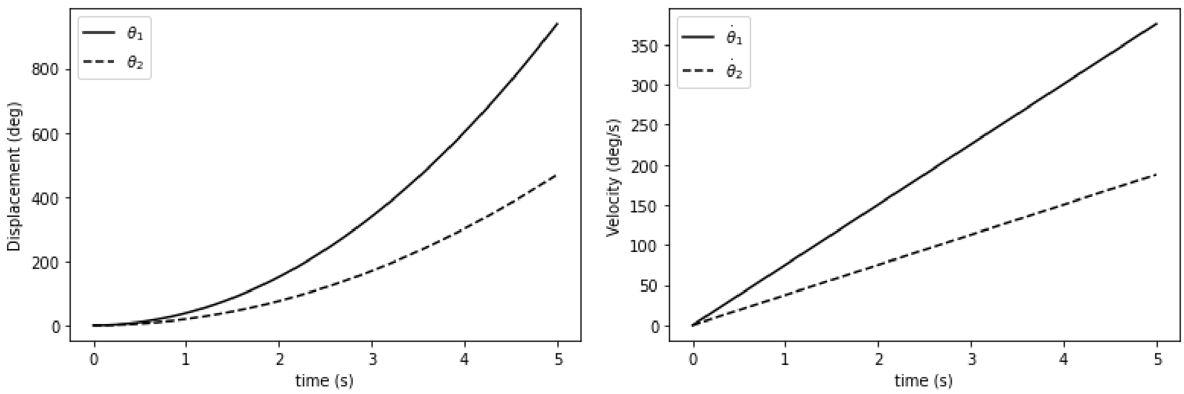

Figure 2. Simulated response for steady state

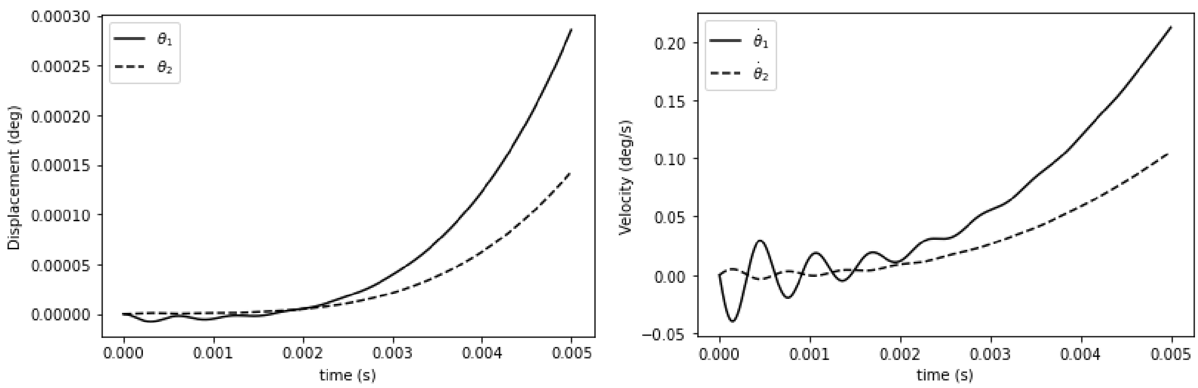

Figure 3. Simulated response for transient regime

In the transient regime, it is possible to initially observe the disturbance of the system under the influence of the parameters and then, after the damping, the transition to the steady state. It is important to highlight that, unlike the case of the steady-state, the motion cannot be described by the eq. (11).

5 Conclusions

In general, it was possible to perform simulations and compare the behavior of the movement of a gear pair for the transient and permanent regimes, confirming the short period of the transient regime. Therefore, we have as a contribution of this analysis, a dynamic model that represents the evolution of displacement and angular velocity of a pair of gears under the influence of parameters such as damping, rigidity and moment of inertia. For future work, the expectation of experimental results and possible comparisons remains, so that it can converge to a dynamic model with responses consistent with the values obtained in the tests.

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