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Three-Dimensional Phase-Field FEM Modelling for Fracture

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Abstract. The study of crack growth is very important in Structural Engineering to prevent catastrophic collapses. This task becomes easier through developing software to model and to study the crack propagation, the crack path and the prediction of where these pathologies will emerge in a solid. A promising approach, which lately has been largely used, is the Phase-Field modelling, that transforms the sharp crack of Griffith's criterion into a smoothed crack that spreads on a certain region of the domain. Our research group, located in the Structural Engineering Department (DEES) of the Federal University of Minas Gerais (UFMG), has been studying the Phase-Field modelling since 2019. The computational implementations have been done in INSANE (INteractive Structural ANalysis Environment), an object-oriented software developed by DEES. The two-dimensional modelling of Phase-Field was previously implemented by Leão [1] in which has been proven the benefits of that approach in the study of crack propagation. This paper presents the expansion of Phase-Field FEM models for the 3D version, where it is possible to evaluate the cracks in solids. Preliminary results using 3D modelling are compared with those published in the literature.

Keywords: Phase-Field; 3D Modelling; Finite Element Method; INSANE; Crack propagation.

1 Introduction

Fracture mechanisms have been extensively studied by engineers, since fracture alerts to structural problems that can lead to collapse. Therefore, the study of these pathologies using software indicates the progress of an important area of knowledge of Structural Mechanics.

Among the computational models for crack propagation analysis, Phase-Field modelling is currently emerging. That consists of transforming the discrete Griffith's crack into a smoothed region of degraded material [2]. One of the advances achieved with this new formulation is the independence of a pre-existing crack. In this way Phase-Field detect crack nucleation without the need of a pre-existing crack.

In the Phase-Field approach, is introduced an addition equation to the problem, in order to calculate the Phase-Field parameter that determine a region of the domain where the fracture is going to propagate. Furthermore, length scale (l_0) is introduced to indicate the transition between the fully broken and unbroken parts of the body. The influence of the material parameters, and how they affect the analysis can be found in Leão et al. [3].

The expansion of Phase-Field modelling for the three-dimensional analysis was made in INteractive Structural ANalysis Environment (INSANE), an opensource object-oriented software in which is implemented a few models to deal with cracks, several of them developed in the Structural Engineering Department in the Federal University of Minas Gerais (DEES). This article has the purpose of demonstrating the implementation and the validation of the Phase-Field modelling for three-dimensional analysis. A modelling case that must be done through a 3D analysis will be presented, in order to demonstrate the importance of the implementation.

2 Implementation

All the implementations presented here were made in INSANE, an open-source computational system developed to support the research in the numerical and computational area applied to engineering [4]. Therefore, the implementation presented here is a consequence of the work developed by the DEES researchers. More information about the INSANE software can be found in Pitangueira et. al [4].

The implementation of the Phase-Field modelling was done based on the already existent code developed for the two-dimensional analysis by another researcher of the department. The details regarding the implemented 2D Phase-Field model for fracture can be found in Leão [1].

The INSANE code was developed in Java programing language using the Object-Oriented-Programming (OOP) paradigm. The 3D extension for the Phase-Field model was made considering minimizing the changes in the already existing code, in such way that the classes developed for the 2D Phase-Field were not impacted. New classes were created for solid analysis and some already existent classes were modified in order to generalize and to accommodate the 3D structure.

The 3D version contains four constitutive models: the isotropic model, and the anisotropic models of Lancioni and Royer-Carfagni [5], Amor et al. [6], and Miehe et al.[7]. The constitutive model has the methods that mounts the problem constitutive matrices.

To implement the constitutive models mentioned above, it was necessary to create an extension of the superclass '*PhaseFieldStaggeredConstitutiveModel*'. In the new superclass named '*SolidPhaseFieldStaggeredConstitutiveModel*' were implemented the necessary methods to calculate the stress and to update the constitutive variables of the solid structure.

The diagram of Fig. 1 shows the organization of the classes in the INSANE software. The blue color was used to highlight the created classes and the yellow to the modified classes.

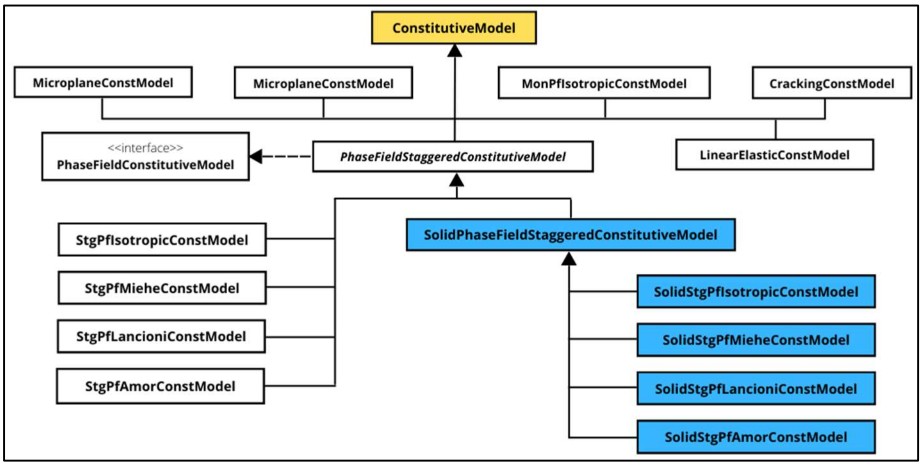


Figure 1. Constitutive Model organization

To represent the analysis model the additional class '*SolidPhaseField*' was created to implement the necessary methods to calculate the internal and state variables operator, i.e. the matrices [N] and [B].

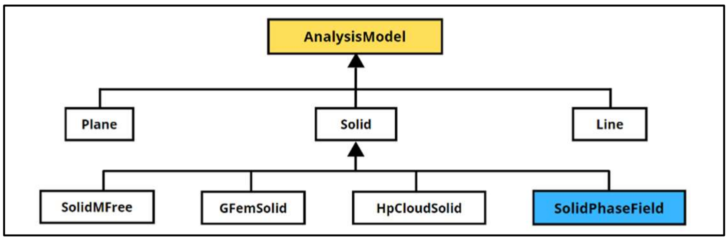


Figure 2. Analysis Model organization

3 Implementation Validation

To validate the implementation some examples were used for each constitutive model. Primarily, a bar under traction was tested under the isotropic constitutive model and the obtained 3D results were compared with the 2D results for the same corresponding mesh found in Leão [1]. Then, other tests were made using other constitutive models and the results were compared qualitatively, since the 3D model generates a very large mesh and the authors do not have access to a hardware able of handle with that. In this way, the parameter l_0 always has to be modified because, according to Miehe et al. [7], the mesh size must be at most half of the length scale parameter value. The results of both 2D and 3D versions are shown to demonstrate the similarities found in the tests.

3.1 Isotropic constitutive model

The example used to test the isotropic constitutive model was a bar under traction, subject to a uniform load. In our literature reference, this test was made in a 2D analysis with quadrilateral elements with size of 0.5 mm. In this way, in the 3D analysis presented here, the mesh was composed by hexahedrons, with 0.5 mm of size. The analysis was performed through direct displacement control, controlling the horizontal displacement of the top right node (see the red node depicted in Fig. 3a) with increments of $1.0\text{x}10^{-4}$ mm. The properties used for this model was: elasticity modulus E_0= 25850.0 N/mm^2, Poisson ratio ν =0.18, length scale l_0= 24.31 mm, fracture energy G_f=0.064 N/mm. All the nodes that contain restrains that are localized in z = 1.0 mm are also restricted in the direction of the z axis.

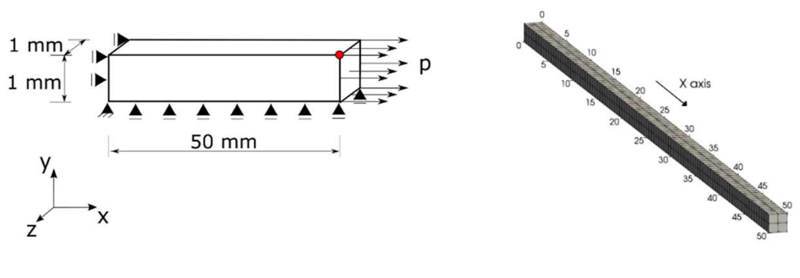


Figure 3. Traction test. (a) Problem setting, (b) Tested mesh

The results obtained in 2D and 3D version of the Phase-Field modelling are presented in Fig. 4. The similarities found in both analyses lead to the validation of the implemented three-dimensional method.

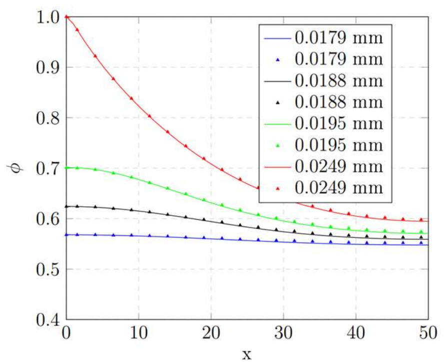
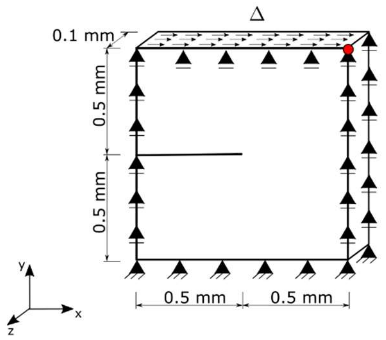


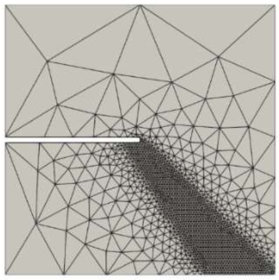
Figure 4. Phase-Field value along the x bar axis, for different displacements of the bar end. The black line indicates the 2D version by Leão [1] and the red triangles represents the 3D version

3.2 Miehe et. al. [7] constitutive model

To test the constitutive model of Miehe et al. [7] the shear test presented in Fig. 5 was used. The mesh is composed by tetrahedrons with mean nodal distance of 0.01 mm in the crack region and 0.5 mm in the remainder domain. The parameters used were: E_0= 210.0 N/mm^2; ν =0.2; l_0= 0.02 mm; G_f =2.7 N/mm. The analysis has controlled the top right node (see the red node depicted in Fig. 5a) with increments of $1\text{x}10^{-4}$ mm. All the nodes that contain restrains are also restricted in the direction of the z axis.

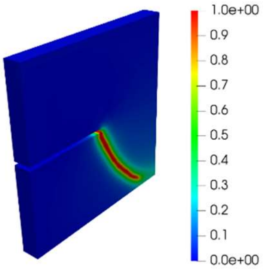
The comparison between the phase-field contour plot of the implemented model and the results already existent in literature are present in Fig. 6. The similarity of the obtained results validates the implementation.

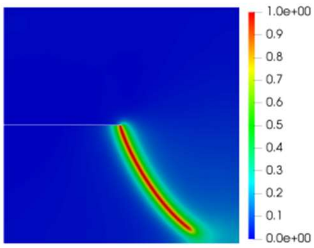




(a) (b)

Figure 5. Problem Setting shear test. (a) Loading of the mesh, (b) Tested mesh





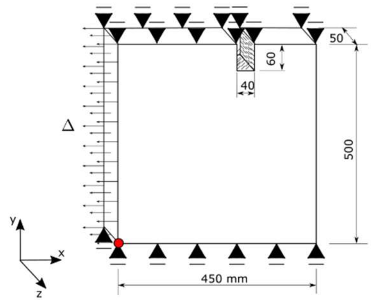
(a) (b)

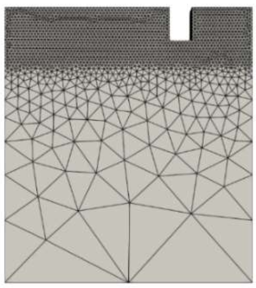
Figure 6. Phase-Field contour plot for shear test analysis. (a) 3D mesh, (b) 2D mesh Leão [1]

3.3 Lancioni and Royer-Carfagni [5] constitutive model

The constitutive model of Lancioni and Royer-Carfagni [5] was tested using the French Panthéon fracture test already modelled and with results existent in literature. The setting of the problem is depicted in Fig. 7. The analyzed mesh is composed by tetrahedrons refined in the top region with main nodal distance of 5.0 mm. The parameters used were: E_0= 1.0 x 10^4 N/mm^2; $\nu = 0.1$; l_0 = 3.0 mm; G_f = 0.025 N/mm.

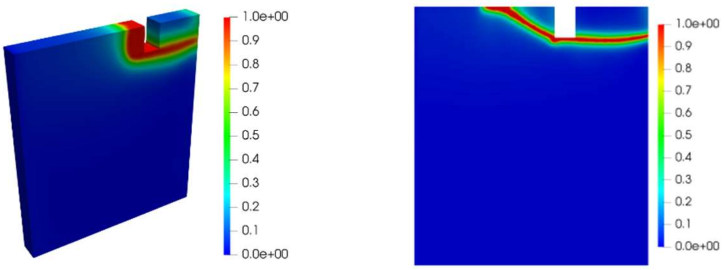
The displacement control method was used by controlling the horizontal displacement of the bottom left node (see the red node depicted in Fig. 7a) with increments of 1x10^{-3} mm. All the nodes that contain restrains are restricted in the direction of the z axis.





(a) (b)

Figure 7. French Panthéon fracture test. (a) Problem setting, (b) Tested mesh



(a) (b)

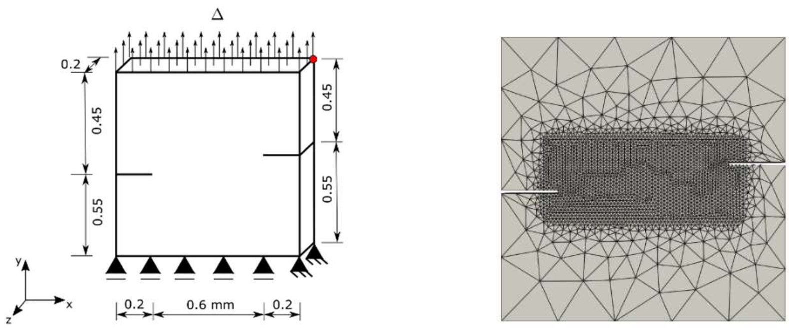
Figure 8. Phase-Field profile. (a) 3D mesh, (b) 2D mesh Leão [1]

3.4 Amor et al. [6] constitutive model

For the Amor et al. [6] constitutive model, the asymmetrical traction test depicted in Fig. 9 was used. The mesh is composed by tetrahedrons with main nodal distance of 0.0125 mm, refined in the crack region with 0.24 mm. The parameters used were: E_0= 1.0 kN/mm^2, ν = 0.3, l_0= 0.05 mm, G_f=0.001 kN/mm.

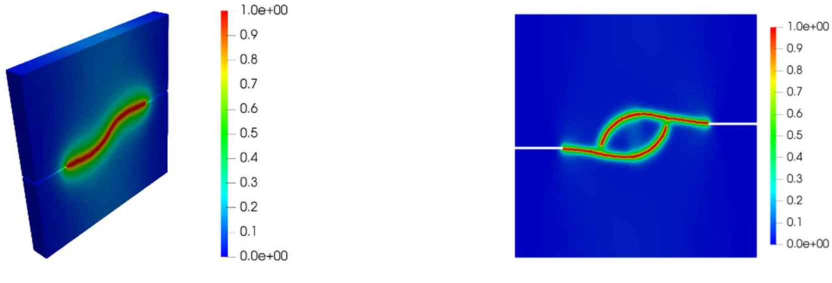
In this test, the vertical displacement of the top right node (see the red node depicted in Fig. 9a) was controlled, with increments of 5x10^{-4} mm. Also, all the nodes that contains restrains are restricted in the direction of the z axis.

Figure 10 shows a comparison between the obtained results with an existing in literature, modelled in 2D. The difference among the results can be attributed due to the refinement of the mesh. With the hardware used to make this analysis is not possible modelling elements with a compatible size, in such way that the 3D mesh was 2.5 times larger than the two-dimensional version. In this case the refined region should extend for a large area, which in computationally very expensive.



(a) (b)

Figure 9. Asymmetric traction test. (a) Problem setting, (b) Tested mesh



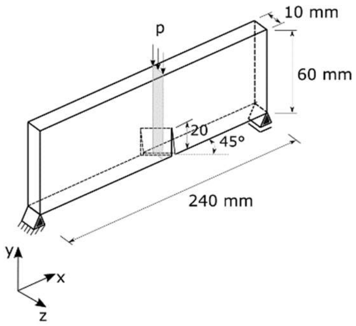
(a) (b)

Figure 10. Contour plot for asymmetric traction test analysis. (a) 3D mesh, (b) 2D mesh Leão [1]

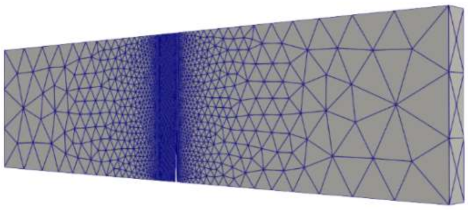
4 Specific 3D

The importance to develop the Phase-Field modelling to analyze three-dimensional structures relies on the particular cases that can't be simplified into a two-dimensional structure and, therefore, it is necessary to perform the test with a 3D analysis.

The three-point bending test, presented by Wolff et al. [8], and depicted in Fig. 11, shows an example where the analysis can't be simplified to a bidimensional one. That example has an initial crack that is not orthogonal with the plane xy of the beam. In this test a uniform force applied to the top surface of the beam and the vertical down central load node was controlled with increments of $1x10^{-2}$ mm.



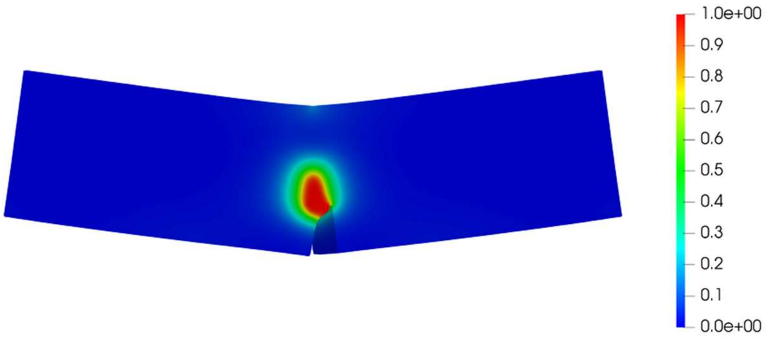
(a)



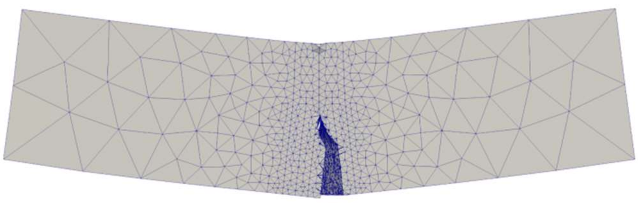
(b)

Figure 11. Three-point bending test. (a) Problem setting, (b) Tested mesh

The parameters used in this analysis was adapted from the literature reference and E_0= 2800.0 N/mm^2, ν =0.38; l_0= 4.0 mm, G_f=0.5 N/mm, was used.



(a)



(b)

Figure 12. Three-point-bending test results. (a) Phase-Field profile, (b) Wolff et al. [8]

The similarities shown in Fig.12 demonstrates the accuracy of the three-dimensional phase-field analysis and validates the implementation.

5 Conclusions

The main objective of this paper was to present the implementation of 3D Phase-Field modelling done in the *INSANE* software. Throughout this research, it was necessary to set some examples, gathered from the literature, to validate the implemented method. Even though some parameters were changed in the three-dimensional analysis, the crack path obtained were similar enough to validate the implemented extension of Phase-Field.

Even though the implementation of the 3D Phase-Field was successful, it also admits some challenges in the analysis. Since the structure is three-dimensional, it is necessary to set up more nodes and elements in the mesh leading to a much great computational effort required to the modelling in such way that, in the analysis presented here, the meshes has to be defined coarser than the literature examples.

In conclusion, it has been proven that the Phase-Field has the capacity to solve structural problems that involves the particular cases of three-dimensional bodies. Therefore, it is possible to confirm that the generalization of the Phase-Field has enlarged the capacity of structural analysis of the INSANE that is a software with a great potential to solve fracture problems.

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References

[1] H. M. Leão. A Critical Study on Phase-Field Modelling of Fracture. Master dissertation, Federal University of Minas Gerais, 2021.

[2] B. Bourdin, G. A. Francfort, and J.-J. Marigo. The variational approach to fracture. *Journal of Elasticity* vol. 91, 5–148, 2008.

[3] H. M. Leão, R. L. Pitangueira, L. Gori, S. S. Penna. Phase-feld modelling of size efect on strength and structural brittleness. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 43(484), 2021.

[4] R. L. Pitangueira, da F. T. Fonseca, J. S. Fuina, L. S. Camara, da M. T. Fonseca, R. L. Ferreira, R. N. Moreira, S. S. Saliba, and S. S. Penna. Insane - versao 2.0. XXIX Ibero-Latin American Congress on Computational Methods in Engineering (CILAMCE), 2008.

[5] G. Lancioni and G. Royer-Carfagni. The variational approach to fracture mechanics. *Journal of Elasticity*, vol. 95, pp. 1–30, 2009.

[6] H. Amor, J.-J. Marigo, and C. Maurini. Regularized formulation of the variational brittle fracture with unilateral contact: Numerical experiments. *Journal of the Mechanics and Physics of Solids*, vol. 57(8), pp. 1209– 1229, 2009.

[7] C. Miehe, M. Hofacker, and F. Welschinger. A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits. *Computer Methods in Applied Mechanics and Engineering*, vol. 199, pp. 2765–2778, 2010b.

[8] K. P. Wolff, R. L. S. Pitangueira, R. G. Peixoto. A displacement-based and explicit non-planar 3D crack propagation model in the generalized/extended finite element method. *Theoretical and Applied Fracture Mechanics*, v. 108, 2020.