Structural reliability of strengthened reinforced concrete beams: comparative study of different design methods

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Abstract. There are some methods in the literature for the design of concrete jackets for beams and they differ in the way they represent the structural behavior of the beam cross-section. As a consequence, it is possible to obtain different results for a flexure strengthened beam. This scenario raises doubts about what are the consequences of adopting different hypotheses. In the present study, two methods from the literature for strengthening design with jackets were compared. The first considers a monolithicy factor that reduces the beam resistance, while the second considers the compression steel contribution to resistance. The strengthened sections were analyzed using the First Order Reliability Method. Information about the random variables limit state functions involved were obtained from the literature. Ten experimental strengthened beams available in the literature were selected to apply the design methods and to perform the reliability analysis. The strengthening design and reliability analysis routines were implemented in the Matlab environment. The results indicate that the first method, the one that considers a monolithicy factor, results in a larger steel reinforcement area for the strengthening. Also, the reliability analyses shows that the different parameters considered in each design method have a significant impact in the reliability index.

Keywords: reinforced concrete, structural reliability, jacketing.

1 Introduction

The flexural strengthening of reinforced concrete beams is necessary when there are design errors, execution deficiencies, or changes in the functionality of the structure. One way to execute this strengthening is through the jacketing of the original cross-section of the beam. Although its use is common in national engineering, there is still a certain degree of empiricism in its practice.

There are some methods in the literature for the design of concrete jackets and they differ in the way of representing the behavior of the beam cross-section and verifying secondary stresses. As a consequence, it is possible to obtain different results for a strengthened beam section. This scenario raises doubts about choosing the most appropriate method for each structural situation and what are the consequences of adopting different hypotheses. As there is no national Standard in Brazil that regulates the subject, it is convenient to discuss it from a probabilistic perspective to offer a notion of safety for this type of design.

In the present paper, ten experimental beams available in the literature were selected to conduct both the design and the reliability analysis. These reinforced concrete beams were strengthened with concrete jackets and new layers of reinforcement steel. Experimental data on the failure of the beams was used to design the strengthening jacket by two methods, namely one presented by Gomes and Appleton [1] and the one used by Santos [2]. After that, the strengthened sections were submitted to a reliability analysis using FORM.

The reliability analysis used statistical information available in Santos et al [3] to describe the random variable, also two different limit state functions, representing failure by each of the design methods, were applied to obtain the reliability index for failure due to flexure. As each design method consider different information of the beam section, is expected to observe these results in terms of reliability index.

The remainder of this paper is organized as follows. Section 2 presents the main points that define the structural reliability problem. Section 3 described the two design methods considered for the strengthening jacket. Section 4 presents the data for experimental beams and their strengthening design results. Finally, section 5 resumes the reliability results and section 6 brings some conclusions about the results.

2 STRUCTURAL RELIABILITY FORMULATION

The reliability analysis, in basic terms, aims to evaluate the probability of a given system failing considering its random behavior. This can be best defined by observing the fundamental reliability problem, which, according to Melchers and Beck [4], considers only one load effect S and one resistance R, described by a known probability density function. The failure domain of the structural response is limited by the load-resistance equality (R=S), resulting in the following definition for the failure probability:

$$P_f = \int_{-\infty}^{\infty} f_S(s) F_R(s) ds. \quad (1)$$

In simpler words, eq. (1) can be seen as simply a sum of the failure probabilities over all the cases in which the resistance is inferior to the load.

Although eq. (1) defines the reliability problem, its solution demands numerical methods so that the probability of failure can the evaluated. The structural reliability method applied in this paper is the First Order Reliability Method or FORM. This transformation method translates the problem from a physical to a normalized space and approximates the limit state surface at the design point considering this failure surface as linear, a visual definition is in Fig (1).

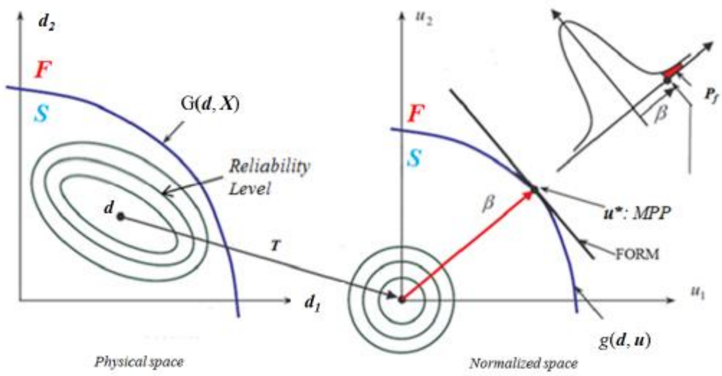

Figure 1 - Transformation and failure surface of FORM, Lopez and Beck [5]

3 STRENGTHENING DESIGN METHODS

The literature suggests several approaches for the design of flexure strengthening jackets. The difference between them lies in the representation of the beam cross-section and the structural parameters taken into consideration.

One can use design methods developed for steel plates as indicated in Souza and Ripper [6], and use the general concept of section balance to find the reinforcement area. These are known by their author names, e.g. method of Bresson, Cánovas and Ziraba, and Hussein. Another useful method is based on the moment-curvature of the beam cross-section, such as applied by Andreolli [7].

In the present paper, two strengthening design methods are applied: one presented in Gomes and Appleton [1] and one used in Santos [2]. They are presented below together with a brief explanation.

The method presented in Gomes e Appleton [1] is summarized in eq. (2). The concept of the method is obtaining the balance of the section considering its flexural resistance is composed of the original steel reinforcement area and a new reinforcement (strengthening) area, the last being the result of the design. For this design is applied the partial safety coefficients, and is also considered a monolithicity factor of 0.9 ($\gamma_{n,M}$) due to the existing difference between a monolithic and composed section.

$$M_{rd} = \gamma_{n,M} * f_{sy}(A_s * 0.9 * d + A_{sr} * 0.9 * d_r). \tag{2}$$

The second method, described in Santos [2], is presented in eq. (3). It uses the same idea of section balance to obtain the strengthening reinforcement area, but it does so by iteratively searching the depth of the neutral axis till the balance is achieved. Also, although the method does not consider the monolithicity factor, it does consider the contribution of the compression steel.

$$M_{rd} = A_s * f_{sy}(d - 0.4 * x) + A_{sr} * f_{sy}(d_r - 0.4 * x) + A'_s * f_{sy}(0.4 * x - d'). \tag{3}$$

In eq. (2) and (3) M_{rd} is the resistance bending moment, f_{sy} is the steel yield strength, As and Asr are the reinforcement steel area and strengthening steel area, d and d_r are respectively the effective height of the reinforcement and strengthening steel, x is the depth of the neutral axis, As' is compression steel area and d' is the height of the compression steel.

4 EXPERIMENTAL BEAMS

The ten experimental beams are listed in Tab. 1 with their respective strengthening reinforcement steel area (Asr), and the ultimate bending moment (Mu) obtained in the experimental failures and can be identified with their reference number. The authors are the following: Clímaco [8], Soto [9], Canaval [10], Cheong and MacAlevey [11], Piancastelli [12], Souza [13], Reis [14], Tsioulou et al [15] and Souza and Appleton [16].

As complementary information: the two beams of the Piancastelli [12] were from the "series 2" and "series 6", and their difference is solely the way concrete was cast.

Table 1. Experimental beams

	[8]	[9]	[10]	[11]	S2 [12]	S6 [12]	[13]	[14]	[15]	[16]
Asr [cm²]	6.0	2.5	2.5	10.0	2.4	2.4	1.0	6.0	2.3	3.4
Mu [kN.m]	70.8	110.0	31.22	202.0	63.36	64.98	24.2	160.0	42.4	42.6

The reinforcement steel area was applied in each of the design method equations (eq. (2) and eq. (3)) to obtain a theoretical ultimate bending moment for each beam. These results were then used to define a simplified random variable called Model Error (θr), defined as the ratio between the ultimate bending moment and its theoretical correspondent. The theoretical value was calculated considering the characteristics of each beam (e.g. geometric and material properties) available in each respective reference.

Table 2. Model Error statistics

Design method	μ	CV
Gomes and Appleton [1]	1.15	0.178
Santos [2]	0.98	0.162

The mean values obtained show that the method by Santos has a better adjustment to the experimental beams, given that its mean is closer to 1. It can be also seen that Gomes and Appleton tend to underestimate the beam resistance, as the mean is above the unit. The mean values were consistent with similar Model Errors in literature, and although the coefficients of variation are higher (e.g. as seen in Novak and Szerszen [17]) this was expected considering the small population size is discussion.

Table 3 presents the steel area for the strengthening design by the two methods, the values represent the effective, or commercial, steel area. One can observe that the results by Gomes and Appleton tend to be higher. This can be explained mainly by the penalty applied by the monolithicity factor, and also by the fact that Santos considers the compression steel contribution to the section resistance. The model used in Santos also takes into account other parameters of the beam section, such as width and concrete resistance to reach the section balance (see Santos [2] for more details). This feature can make this method take advantage of the positive characteristic of the structure and result in lower values for Asr.

Table 3. Strengthening design and Model Error statistic

Design method	Asr [cm²]									
	[8]	[9]	[10]	[11]	S2 [12]	S6 [12]	[13]	[14]	[15]	[16]
Gomes and Appleton [1]	6	3.5	2	11	3.15	3.5	1.6	6.0	2.4	4
Santos [2]	6	2.5	2	9.45	2.5	3	1.5	4.8	2	3.5

5 RELIABILITY ANALYSIS

The reliability analysis was conducted in Matlab environment. The random variable statistics, as stated in section 1, were obtained in the literature. The characteristic values for variables were taken from each of the references. Model Error distribution was adopted as recommended in the literature, such by Novak and Szerszen [17], therefore considered as lognormal.

Every beam was strengthened considering both design methods, and each design result was submitted to a reliability analysis applying two different limit state functions (LSF). As a result, each beam has four reliability values, two for each design. Equation (4) represents the LSF for failure in flexure considering Gomes and Appleton's representation of the resistance, while eq. (5) presents the LSF for Santos's design framework.

$$g(\boldsymbol{X}) = \theta_R * (\gamma_{n,M} * f_{sy} * \left(A_s 0.9(h - d'') + A_{sr} 0.9(h - d''_r)\right)) - \theta_S * (M_g + M_q). \tag{4}$$

$$g(\boldsymbol{X}) = \theta_R f_{sy}\left(A_s((h - d'') - 0.4x) + A_{sr}((h - d''_r) - 0.4x) + A'_s(0.4x - d')\right) - \theta_S\left(M_g + M_q\right). \tag{5}$$

In eq. (4) and (5) h is the total height of the section, θs is Model Error for the loads, M_g and M_q are the bending moment portions due to permanent and variable loads, d" and dr" are the distance from the steel area to the most tensioned and compressed fiber, respectively. It is worth noting that only $\gamma_{n,M}$, x and d' were taken as deterministic values.

The results of the reliability analysis are resumed in Tab. 4 classified according to the design method and the LSF applied.

Observing the result set of each design method it is clear that the reinforcement steel area provided by Gomes and Appleton leads to superior reliability indexes, no matter the LSF considered. Because Gomes and Appleton ignore the compression steel area contribution to the resistance and apply a penalty factor due to lack of monolithicity, its resulting design steel area is larger. Another contributing factor is the different Model Error for both design methods, where for Santos its mean value is 0.98, reducing the resistance portion in the LSF evaluation and reducing the reliability values in general.

Table 4. Reliability indexes

Design method	LSF	β									
		[8]	[9]	[10]	[11]	S2 [12]	S6 [12]	[13]	[14]	[15]	[16]
Gomes and Appleton [1]	Gomes and Appleton [1]	3.36	2.72	3.32	2.6	2.6	2.84	2.18	2.66	2.93	2.31
	Santos [2]	3.49	3.22	3.59	3.15	3.26	3.51	3.01	3.26	4.09	3.27
Santos [2]	Gomes and Appleton [1]	2.81	1.52	2.76	1.41	1.2	1.7	1.34	1.5	1.86	1.23
	Santos [2]	2.95	2.06	3.06	2.0	1.9	2.41	2.23	2.13	3.12	2.26

Focusing on the results set dependent on the LSF, the application of the function defined with Santos leads to larger reliability indexes. This LSF considers the contribution of the compression steel to the resistance of the beam, so the obvious result is an increase in the overall reliability. The choice of LSF format impacts the reliability results obtained, even when analytical models are used.

Figure 2 presents the reliability indexes of all beams and helps to visualize the data spread. In the subtitle, the first name refers to the design method (e.g. "G.A." for Gomes and Appleton), and the second term refers to the LSF applied (e.g. "LSF: Santos" for the function of this author). The Fig. 2 shows more clearly the variation in the reliability indexes, with close results for all four reliabilities for some beams and more scattered values for others. Beams with significantly more compression steel area or larger width, such as Reis [14] or Souza and Appleton [16] had reliability indexes more scattered since there are features considered in only one of the methods.

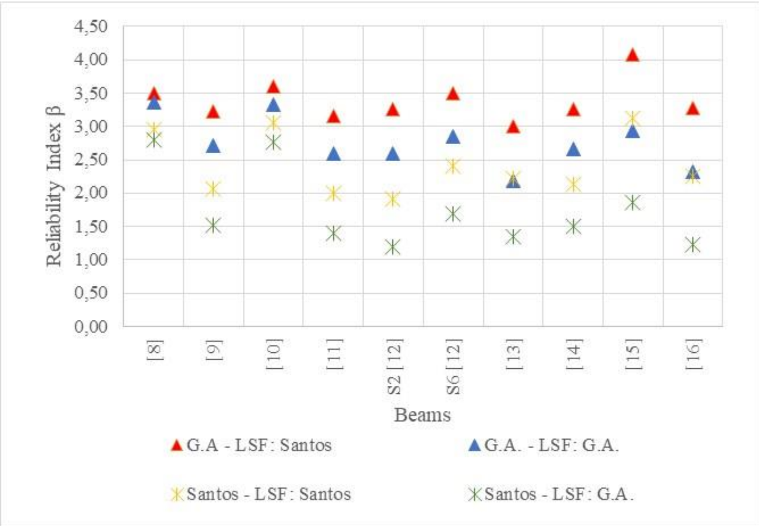

Figure 2. Reliability indexes

An important remark is that the consideration of both LSF for each design result comes from the understanding that none of the models considered for the design of the strengthening is an indisputable perfect model. So, one can only produce relative comparisons between the design methods and between the reliability result. But to draw an absolute conclusion concerning the safety of each method demands a more precise model such as a finite element model.

6 Conclusions

Although the model error analysis was limited in the present paper, due to the small number of experiments considered, it is possible to note a better agreement between the results obtained via the method by Santos [2] and the experimental results.

In what concerns the strengthening design, the method by Gomes and Appleton [1] tends to result in a larger steel area, with the consideration of the monolithicy factor leading to a more critical scenario during the design.

In terms of the reliability index, in general, beams strengthened following the method by Gomes and Appleton method achieved higher reliabilities indexes, no matter the failure function employed.

Also, it was seen that the different parameters taken into account by each method have significant impact in the reliability results.

Future studies should account for a series of phenomena and details which may significantly affect the results, such as material nonlinearity and slip between concrete layers. Numerical models such as those based on finite element method could be developed in an attempt to better represent the structural behavior of the strengthened beams. This paper is part of an initial study to bring contributions to jacketed strengthened beams and future research will be conducted.

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