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Topology Optimization of Plane Trusses Employing the Progressive Directional Selection Method.

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Abstract. More and more, inspiration is sought from nature to solve complex problems in modern society. Whether observing the morphological characteristics of animals and plants or even the behavior of living beings in the environment they live. The naturalist Charles Darwin studied this behavior in depth and proposed a theory that explains how life became what it is today, the Theory of Natural Selection. The theory of natural selection of the directional type inspires the topological optimization method proposed in this work. The Progressive Directional Selection (PDS) method seeks to optimize a structure by selecting the parts that contribute the most to support the mechanical loads and eliminating the parts that contribute the least in different stages of removal. Numerical applications of PDS were performed in two-dimensional truss structures, starting from a ground structure. The matrix analysis of plane trusses and the direct stiffness method are employed to analyze the plane truss in the linear regime. The obtained optimal topologies are structurally stable and efficient and very similar to the results of other topological optimization techniques presented in the literature. The results demonstrate that this method can be employed for the topological optimization of two-dimensional trusses.

Keywords: Progressive Directional Selection; Topology Optimization; Ground Structure; Trusses.

1 Introduction

Man has always sought inspiration in nature to solve problems that appear in society, and engineering is no different. The beak of the kingfisher bird (Alcedo atthis) needs to dive into the water to feed, quickly changing from a low resistance environment (air) to one with a lot of resistance (water). The shape of the bird's beak made possible the aerodynamic solution for the bullet train's nose, which, in addition to being quieter, also became faster and more economical, Arruda [1].

However, it is not enough just to be inspired by the morphology of animals or plants, it is necessary to understand how life works and how the environment changes it so that we can solve much more complex problems. Darwin's hypothesis that living beings are in a frequent struggle for survival and that those who survive are not necessarily the strongest, but those that best adapt to the changes imposed by the environment, is the basis of the theory of Natural Selection.

In structural engineering, we need efficient designs and economic and safety structures. A topological optimization is a tool used to achieve this goal, which is already a reality for some designers. It is a type of structural optimization that changes the topology of the structure in order to obtain a better performance and, at the same time, an economy of the employed material in its manufacture. A new topological optimization approach is presented in this work, inspired by the Theory of Natural Selection, more specifically the directional natural selection, arriving at a new method named Progressive Directional Selection or PDS. The method consists of removing elements (individuals) from an initial design domain formed by several elements (population) that contribute less to support the loads acting on the structure. The process of removing elements that contribute less to the structure, meeting the pre-established boundary conditions, results in an optimized structural design that can support loads more efficiently.

This work focuses on two-dimensional truss structures, in which the initial design domain is a grid of points interconnected by bars, named ground structure (GS). Each member is an individual of the initial population. The members that contribute the least to the structure are removed from the GS, resulting in an optimized truss topology that supports the load and satisfies the displacement constraint conditions that are previously defined.

2 Literature review

2.1 Natural selection

The naturalist Charles Darwin concluded that primitive living beings that inhabited the Earth thousands of years ago underwent modifications until they evolved into the live beings that we know today. This evolutionary process is attributed to an intense and successive struggle for the survival of living beings in the environment in which they live. The changes in the environment imposed on a particular population lead to the survival of individuals who best adapt or who have certain specific characteristics that favor them in these changes. Thus, these surviving individuals pass their characteristics on to their descendants. The “selection of individuals” process imposed by nature gave rise to the theory of natural selection. Currently, it is known that there are three types of natural selection: directional selection, in which individuals with one of the extreme phenotypes (set of observable characteristics of an organism) are selected; stabilizing selection, in which individuals with intermediate phenotypes, that is, closer to the mean, are selected; and disruptive selection, in which both extreme phenotypes are favored.

2.2 Matrix structural analysis of plane trusses

A truss is defined as a set of straight members connected at their ends by flexible connections and subject to loads and reactions only at these ends. The members of an ideal truss only develop axial forces when loading occurs, Kassimali [2]. The truss is classified as a plane truss when all members and the applied loads are in a single plane.

The matrix structural analysis is employed to understand how structures formed by bars behave when requested, systematizing the analysis. Through matrix structural analysis, it is possible to determine the internal forces, reactions in the supports, displacements, stresses, and strains. The displacements are associated with the nodes of the elements that compose the truss. Concentrated loads act at nodes where displacements are unknown, and support reactions must be determined at nodes with prescribed displacements. The possible displacements of a given node are called degrees of freedom. Naturally, each structure has constraints (supports) that should, at the very least, prevent rigid body movements. The number of equations that will be solved equals the total number of degrees of freedom minus the number of constraints, Sennett [3].

The force-displacement relationship in structures with multiple degrees of freedom employing matrix notation:

$$\{F\} = [k]\{u\} \tag{1}$$

where $\{F\}$ is the force vector, $[k]$ is the stiffness matrix and $\{u\}$ is the displacement vector.

For the case of two-dimensional truss elements, nodal displacements can occur in any direction in the *x-y* plane. In general, the axial axis of the element is not parallel to the global coordinates of the structure, so the displacement at each node will be decomposed in two to be parallel to the global *x-y* axes, resulting in two degrees of freedom per node. Initially, assembling the element stiffness matrix in the local coordinate system, which is parallel to the element axis, is necessary. We then transform this local stiffness matrix by employing the structure's global system. The direct stiffness method is employed to assemble the global system of the structure, Eq. (1), where nodes will connect the various elements, and global indexing must be associated with the different degrees of freedom of the structure. The boundary conditions of the structure must be considered in the solution of the global system, informing the prescribed forces and displacements:

$$\begin{Bmatrix} \{F_p\} \\ \{F_s\} \end{Bmatrix} = \begin{bmatrix} [K_{pp}] & [K_{ps}] \\ [K_{sp}] & [K_{ss}] \end{bmatrix} \begin{Bmatrix} \{u_p\} \\ \{u_s\} \end{Bmatrix} \tag{2}$$

where $\{F_p\}$ and $\{u_p\}$ are the prescribed nodal forces and the corresponding nodal displacements, respectively, and $\{F_s\}$ and $\{u_s\}$ are the unknown support reactions and the corresponding prescribed displacements, respectively. After solving the global system of equations, it is possible to determine the nodal displacements and, subsequently, the stress and strain in each bar.

2.3 Topological structural optimization

Structural optimization consists of determining the best distribution of material within a physical volume domain able to safely transmit or support the loads applied to the structure, Querin et al. [4]. According to Klarbring and Christensen [5], there are three types of structural optimization: size optimization, shape optimization, and topological optimization. In topological optimization, in the case of discrete structures such as trusses, the optimized solution is obtained by adopting the cross-sectional areas of the truss members as design variables and then allowing these variables to take on a value equal to or close to zero, i.e., the members are removed from the truss. Thus, the connectivity of the nodes can change, and it can be stated that the truss topology undergoes a modification.

3 PDS

The Progressive Directional Selection (PDS) method was developed by Véras and Cavalcante [6] as a new approach to the topological optimization of two-dimensional continuous elastic structures. As with any topological optimization method, PDS seeks to achieve the best topology for the structure through the best distribution of the material in a physical domain, considering an objective function and constraints. The initial design domain is treated as the initial population of individuals, where each truss element is an individual. When boundary conditions are imposed, each element must contribute so that the structure can support the applied loads. The least contributing elements in the structure are gradually eliminated, just as in nature, so the structure evolves into a configuration that supports loads more efficiently.

In nature, when directional selection acts on a population, a specific characteristic can guarantee the survival of these individuals. Therefore, the PDS method optimizes the structure by minimizing the objective function (flexibility, strain energy, or von Mises stress) and defines which structural elements will remain until the end of the selection. The process is simple, and once the desired final volume of the structure is defined, the main idea is to gradually remove elements from an initial configuration as many times as necessary. In each selection stage, the number of removal steps increases, and the number of elements removed per step is reduced until this process results in the same set of individuals, i.e., in the same selection.

As an illustrative example of this process, consider Figure 1. Suppose a building floor in which the designer needs to define the position of ten columns, knowing there are twenty possibilities for their location. In this case, the initial population of twenty columns will undergo a progressive directional selection. Once loads are applied to the initial configuration of the structure, the first selection stage consists of removing the ten least contributing columns in a single step. The number of removed elements is partitioned in the following steps to exclude them from the initial set progressively. For example, the second stage is divided into two steps. First, load the structure and remove the five columns that contribute the least, then load the structure again on the remaining fifteen elements, then remove the five columns that contribute the least, leaving the desired number of ten columns at the end of the stage. The third stage has three steps for removing individuals. One possibility would be to remove 4, 3, and 3 columns in each step, respectively. By rearranging the internal forces along the removing process, the final population of columns at each stage may differ. The PDS continues until the last stage has the same elements as the previous stage. In the pavement case, the selected columns at the end of the third stage are the same as in the second stage, so it can be considered that convergence was reached.

In the analysis of continuous two-dimensional elastic structures, it is necessary to apply a penalty factor (PF) to the stiffness of eliminated elements from the discretized analysis domain, thus avoiding remeshing and the singularity of the global stiffness matrix. Special attention must be given to the population ranking. To overcome numerical accuracy and symmetry issues during the selection process, a selection tolerance (*ST*) must be applied for each removal step, which can add an element with a performance criterium value very close to the last selected element in the ranked vector.

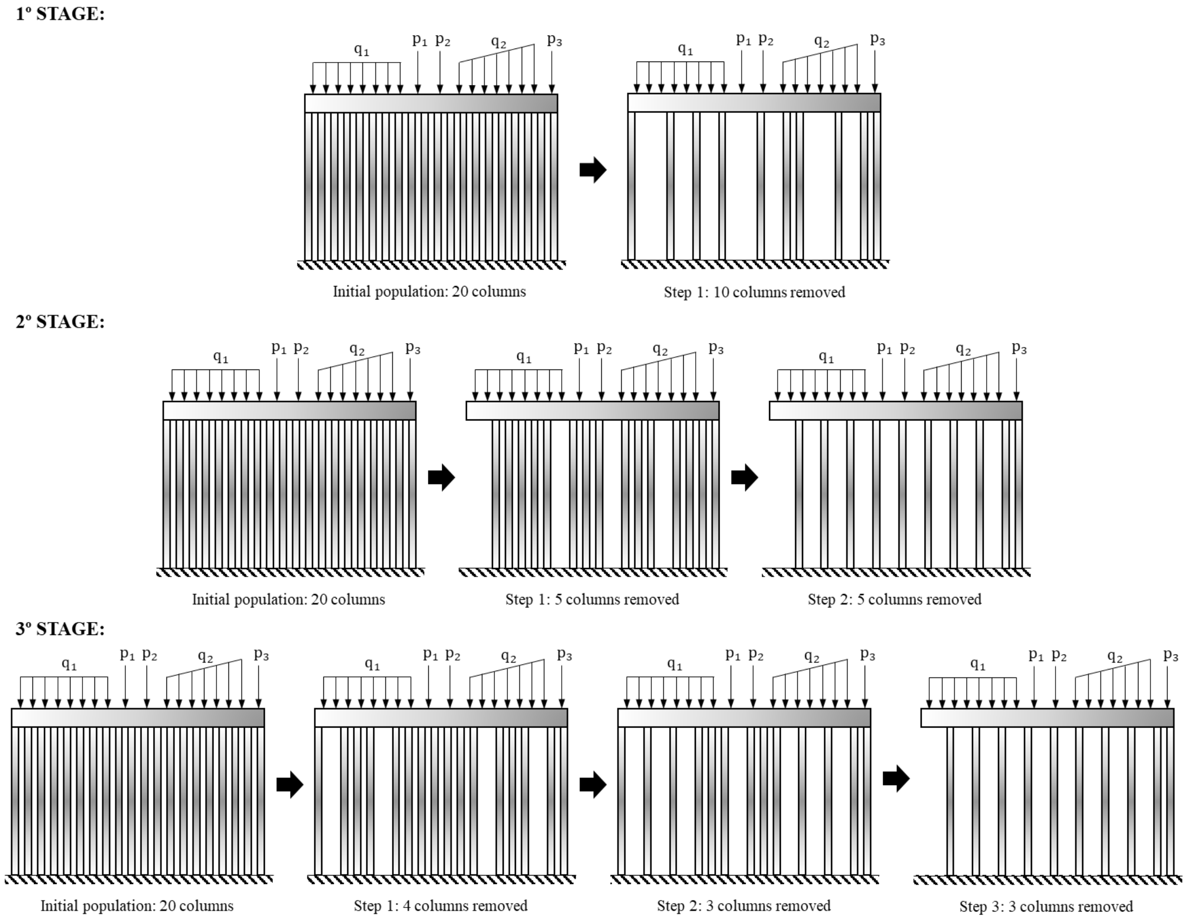


Figure 1. A hypothetical example of PDS application on a building floor with an initial population of 20 columns.

4 Methodology

Initially, the initial design domain is determined. In the case of this work, the proposed domain is a GS. The employed GS for the examples has a rectangular shape. First, the dimensions of the initial domain are determined, in this case, the width, *B*, and the height, *H*, of the rectangle. After that, the numbers of elements present in the horizontal edge (*NHE*) and the vertical edge (*NVE*) are determined; thus, the grid of points is established, and the GS has a total number of nodes equal to *(NHE+1)(NVE+1)*. Fig. 2a shows the initial rectangular domain formed by three elements on the horizontal edge (*NHE = 3*) and two elements on the vertical edge (*NVE = 2*), with a total of 12 nodes.

For connectivity level 1, nodes are connected to their neighboring nodes through bars, Fig. 2b. For connectivity level 2, the nodes are connected to the neighboring nodes of their neighbors, while for connectivity 3, the connection occurs with the neighboring nodes of the neighbors' neighbors. Fig. 2c shows the ground structure with connectivity level 3, with connectivity level 2 in red and connectivity level 3 in green.

Then, the matrix structural analysis is performed. Each bar of the ground structure corresponds to a two-dimensional truss element. The boundary conditions of the initial domain, the cross-sectional area, *A*, and the elastic modulus, *E*, of the elements are adopted. Then the length and angle of inclination in relation to the global coordinates are calculated for each element. With these data, it is possible to determine the stiffness matrix of each element in the local and global system and its contribution to the global stiffness matrix. The stiffness matrix of each element needs to be stored to assemble the GS's global stiffness matrix during the selection process. After assembling the global system of equations in each step of the selection process, Eq. (1), the displacements in the free nodes are evaluated and, consequently, the strain and stress in each element.

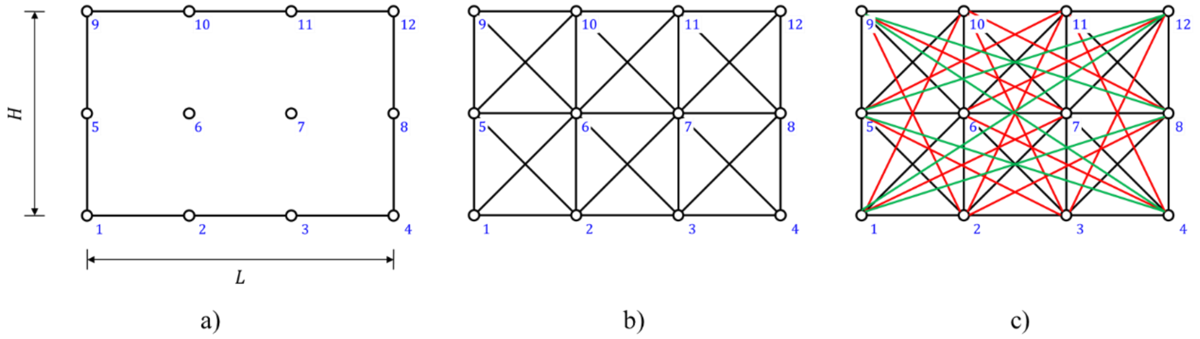


Figure 2. Determination of the GS: a) dimensions of the initial domain; b) connectivity level 1; c) connectivity level 2 and 3.

Before initiating the PDS selection process, it is necessary to define the performance criterion and the convergence criterion, where the number of stages with the same selected elements is determined to end the optimization process, besides the parameters *PF* and *ST*. The number of selected elements must also be defined. The elements are ranked according to the performance criterion, which in this case is the absolute stress magnitude, in descending order. The maximum number of stages is equal to the number of elements that will be removed. Finally, after obtaining the optimized structure, the deformed configuration and its stress distribution are analyzed.

5 Results and discussions

Two examples of PDS application for two-dimensional trusses are shown below. The first example is a Michell-like structure, a classic example found in the structural optimization literature. The second is a cantilever beam, a very common type of structure in designs. Both examples present GS's with connectivity levels 1 and 3 in their initial design domains. All members have the same cross-sectional area, $A = 0.01\ m^2$, and the same Young's modulus, $E = 70\ GPa$. The values of $ST = 10^{-6}$ and $PF = 10^{-6}$ are employed in the PDS selection process. In order to achieve convergence, the same topology needs to be repeated in seven consecutive stages.

5.1 Michell structure

Fig. 3a shows the initial design domain for the Michell structure with GS with connectivity level 1. The load applied has a value of $P = 1000\ kN$. Fig. 3b shows the optimized structure obtained when selecting 101 elements. The sum of the lengths of the bars in the optimized structure is 62.51 m.

Fig. 4a shows the deformation of the optimized Michell structure with connectivity level 1 and displacement magnification factor equal to 100. The resulting displacement at the point of load application is equal to 5.90 mm. Fig. 4b shows the stress distribution in the same optimized structure. The stresses vary between -33.42 MPa and 50.12 MPa.

Fig. 5a shows the initial domain with connectivity level 3, and Fig 5b the optimized structure. The structure has 37 elements with a total length of the bars equal to 38.66 m. The deformation in the optimized structure, as shown in Fig. 5a, has a displacement at the point of load application of 9.33 mm. The stresses range between -74.28 MPa and 32.42 MPa, as shown in Fig. 5b.

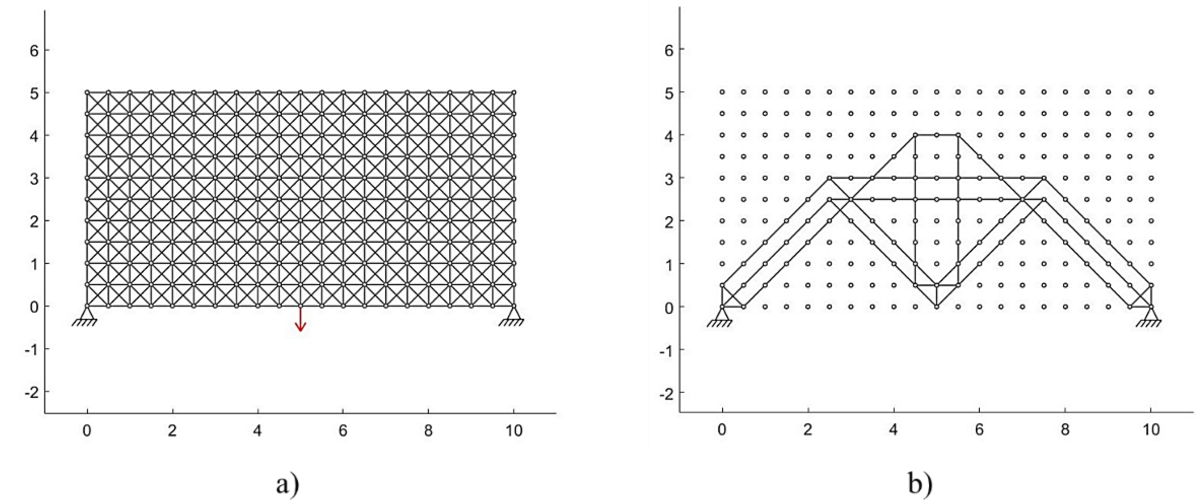


Figure 3. Michell structure with connectivity level 1: a) initial domain; b) optimized structure.

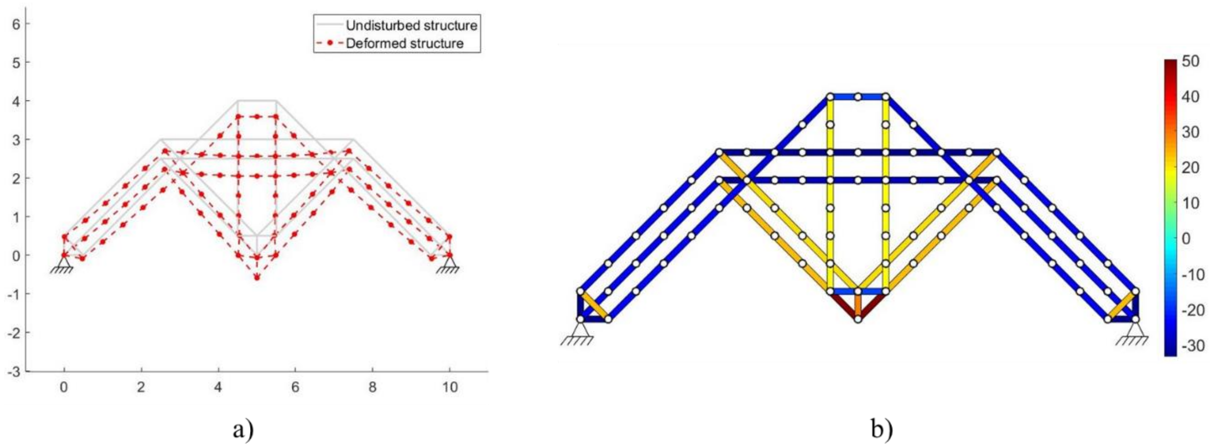


Figure 4. Behavior of the optimized Michell structure with connectivity level 1: a) mesh deformation; b) stresses distribution.

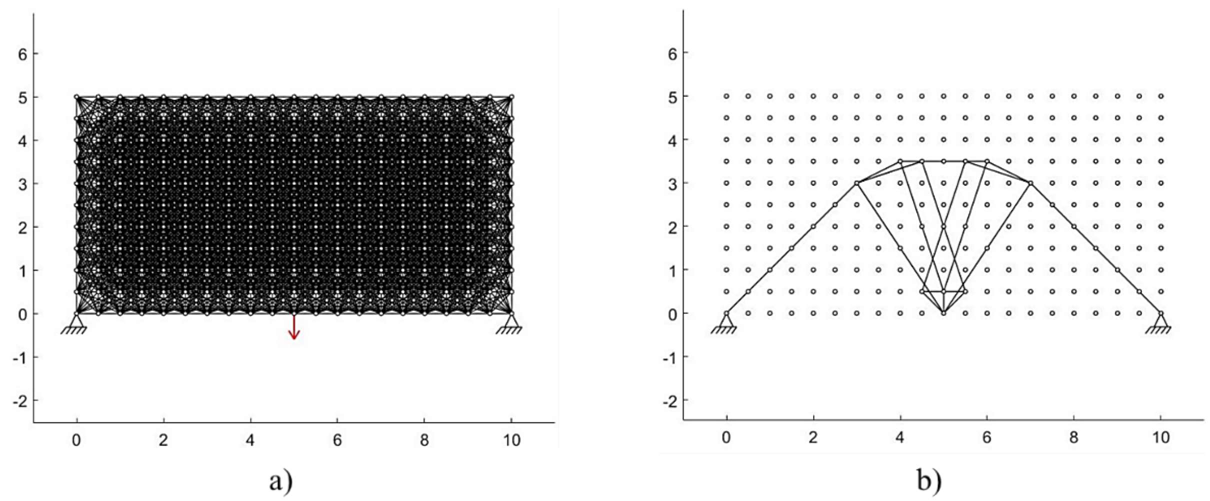
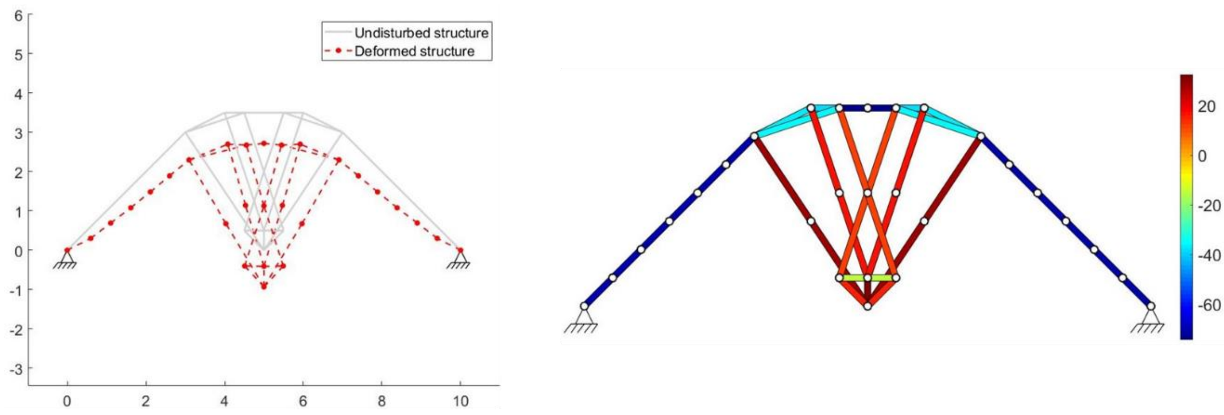


Figure 5. Michell structure with connectivity level 3: a) initial domain; b) optimized structure.

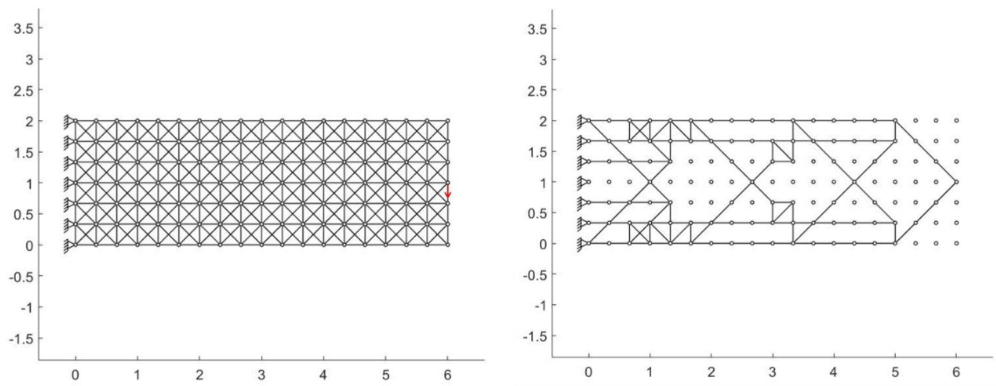


a) b)

Figure 5. Behavior of the optimized Michell structure with connectivity level 3: a) mesh deformation; b) stress distribution.

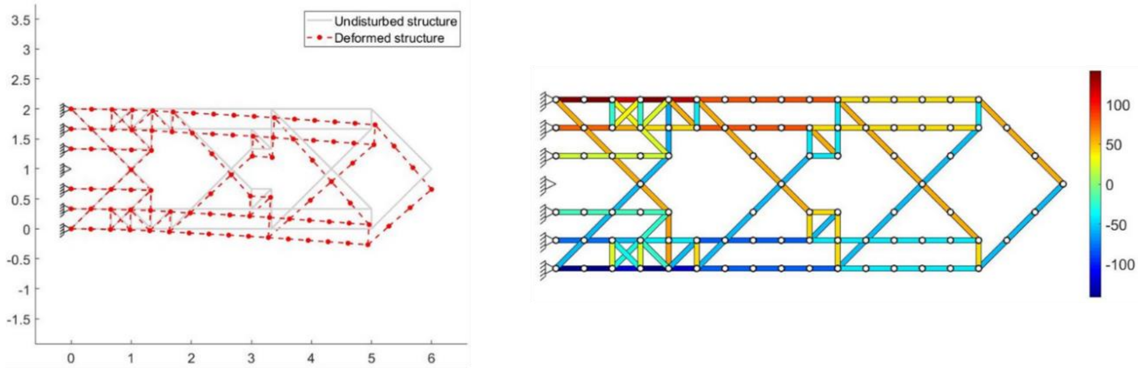
5.2 Cantilever beam

The initial design domain (Fig. 7a) with an applied load of P = 800 kN and the obtained optimized structure (Fig. 7b) are shown below. The optimized structure has a total of 132 elements and a total length of 50.08 m. The load point displacement has a value of 34.22 mm, as shown in Fig. 8a, with a displacement magnification factor equal 10. Fig. 8b shows the stress distribution, which varies between -141.83 MPa and 141.83 MPa.



a) b)

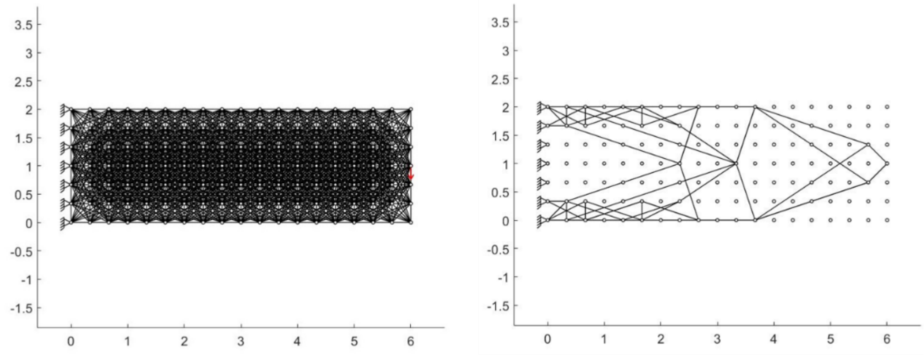
Figure 7. Cantilever beam with connectivity level 1: a) initial domain; b) optimized structure.



a) b)

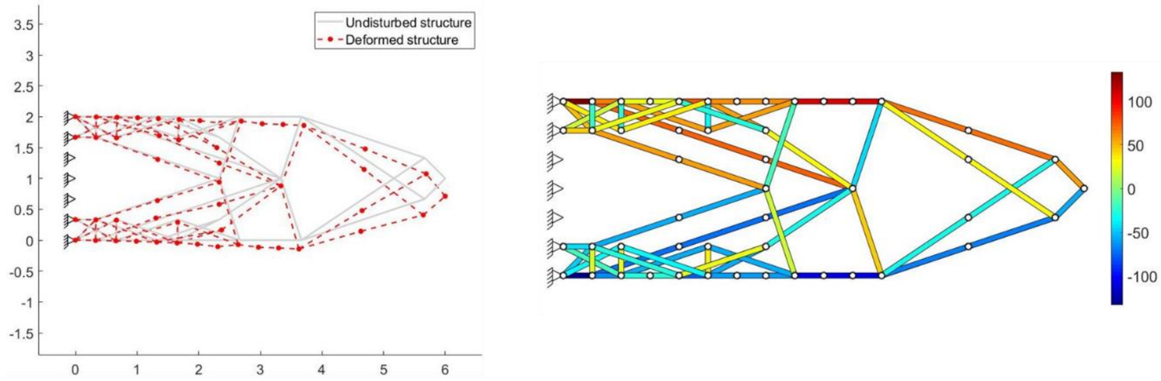
Figure 8. Behavior of the optimized cantilever beam with connectivity level 1: a) mesh deformation; b) stress distribution.

For connectivity level 3 (Fig. 9a), the optimized structure is shown in Fig. 9b with 74 elements and a total length of 53.08 m. The resulting displacement at the point of load application is equal to 29.17 mm (Fig. 10a), and the stress distribution ranged from -133.22 MPa to 133.22 MPa (Fig. 10b).



a) b)

Figure 9. Cantilever beam with connectivity level 3: a) initial domain; b) optimized structure.



a) b)

Figure 10. Behavior of the optimized cantilever beam with connectivity level 3: a) mesh deformation; b) stress distribution.

6 Conclusions

The PDS method proved effective in the topological optimization of two-dimensional trusses. The Michell structure obtained by the method is quite similar to the structure obtained analytically in the literature. On the other hand, the cantilevered beam presented good behavior when deforming, proving to be effective in supporting the applied load.

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