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Design of optimum viscoelastic dynamic neutralizers by response reanalysis

Gabriel R. do Amaral[1], Kevin M. M. Ribeiro[2], José M. Balthazar[3], Alexandre de Macêdo Wahrhaftig[4], Isabel Gebauer Soares[2], Eduardo M. de Oliveira Lopes[2]

[1]*Institute of Technology for Development – Lactec, Mechanical Testing Laboratory*
BR-116, km 98, 8813 – Jardim das Américas, Curitiba – 80.210-132 – Paraná, Brazil
gabriel.amaral@lactec.org.br
[2]*Dept. of Mechanical Engineering, Federal University of Paraná (UFPR)*
Av. Cel. Francisco H. dos Santos, 100 – Jardim das Américas, Curitiba – 80.060-000 – Paraná, Brazil
eng.kevin.mmr@gmail.com, isabel.gebauer1@gmail.com, eduardo_lopes@ufpr.br
[3]*Dept. of Mechanical Engineering, Engineering School, São Paulo State University (UNESP) – Bauru*
Av. Luís Edmundo Carrijo Coube, 14-01 – Vargem Limpa, Bauru – 17.033-360, São Paulo, Brazil
manoel.balthazar@unesp.br
[4]*Dept. of Construction and Structures, Polytechnic School, Federal University of Bahia (UFBA)*
Rua Aristides Novís, 02, 5th floor – Federação, Salvador – 40.210-910, Bahia, Brazil
alixa@ufba.br

Abstract. In the past decades, response reanalysis techniques have been widely used to predict the dynamic effects of localized structural modifications, for they offer the advantage of circumventing the need to reprocess the whole set of information relative to the system of concern at each modification stage. Some recent works addressed the issue of evaluating the effects of inserting a viscoelastic dynamic neutralizer into a primary system by means of reanalysis, but none of them tackled the problem of finding the optimum modal parameters for the device based on these techniques. In this context, the present work aims to investigate the use of two response reanalysis techniques - in matrix formulation - to iteratively predict the response of the modified system after alterations in the parameters of a single-degree-of-freedom neutralizer, which provides a convenient method to find the optimal modal characteristics for the device. The considered primary system consists of a cantilever steel beam, the vibrations of which are meant to be kept under control. A finite-element model for the beam is implemented and a combination of a genetic algorithm and a local Nelder-Mead technique is used to ensure that global minimum vibration levels are achieved. The results show promising evidence for generalizing the use of the technique to multiple-degree-of-freedom devices and for applying the method to specific, large scale systems, such as overhead transmission line conductor cables.

Keywords: Response reanalysis, Finite-element model, Viscoelastic dynamic neutralizer, Broadband vibration control.

1 Introduction

Keeping vibration levels under control in a specific mechanical system (the primary system) is the main objective of dynamic vibration neutralizers (DVNs). One of its most often quoted applications is in vibration control of overhead transmission line conductor cables, in the form of the so-called Stockbridge damper and other less popular devices ([1], [2]). A special class within the DVNs consists of viscoelastic dynamic neutralizers (VDNs), which, by the addition of viscoelastic elements, provide enhanced energy dissipation properties.

The insertion of a DVN can be regarded as a structural modification, as they are amenable to treatment by reanalysis methods ([3], [4]). These may be classified - according to what they primarily handle - as modal models (with its corresponding modal properties) or response models (via the relevant frequency response functions,

FRFs) of a structure, either based on exact algebra or approximate representations ([5]).
Recently, Soares [6] and Soares and Lopes [7] employed exact response reanalysis methods in evaluating the insertion of a generic VDN (designed with the aid of LAVIBS-ND®, a specialized software for optimizing VDNs via a modal approach) into a cantilever beam, showing that the prediction of the effects of changing the dynamics of the primary system provided by those reanalysis methods is exact and reliable.
In Lopes [8], the matrix product method was adopted for analyzing modifications by viscoelastic devices in an aluminum frame, considering both numerical and experimental responses. On the other hand, in Rodrigues [9], using the matrix partition method, a multi-degree-of-freedom (MDOF) VDN was designed to reduce vibration in a wide frequency range in a cantilever beam. In the latter, a geometry for the VDN was assumed *a priori*, and an optimization procedure provided the optimal values for its relevant physical characteristics.
The present work aims to show that the two response reanalysis methods mentioned above are convenient alternatives to the modal approach, as they can correspondingly and successfully provide the optimal modal characteristics for a single-degree-of-freedom (SDOF) NDV inserted into a given primary system.

2 Designing the neutralizer

In general, for an MDOF mechanical system, the matrices $[\bar{S}(\omega)]$ relating the generalized displacement vector to the generalized force vector are known as dynamic stiffness matrices. They can be defined as

$$[\bar{S}(\omega)] = \left[-\omega^2[M] + i\omega[C] + [K]\right], \tag{1}$$

where ω stands for circular frequency; $[M]$, $[C]$, and $[K]$ are the mass, damping, and stiffness matrices; i is the imaginary unit; and the overbar denotes complex quantities.
Proportional Rayleigh damping is assumed above, so that $[C] = \alpha[M] + \beta[K]$, where α and β are the coefficients of proportionality with the mass and stiffness matrices, respectively. Although the present work focuses on that kind of damping, the presented design method can be extended to systems with hysteretic - as well as non-proportional viscous - damping.
The inverses of $[\bar{S}(\omega)]$ are the receptance matrices, $[\bar{R}(\omega)]$, so that

$$[\bar{R}(\omega)] = [\bar{S}(\omega)]^{-1}. \tag{2}$$

Experimentally, it may be convenient to use another set of frequency response functions (FRF) - inertances $[\bar{I}(\omega)]$ - related to the receptances by

$$[\bar{I}(\omega)] = -\frac{[\bar{R}(\omega)]}{\omega^2}. \tag{3}$$

2.1 Generalized equivalent parameters

Generalized equivalent parameters (GEP) are a convenient way of introducing an auxiliary system (such as a VDN) to a primary system with no increment in the order of the corresponding matrices. Figure 1 shows how any SDOF VDN can be represented by a system composed of equivalent spring (with an equivalent stiffness k_e) and damper (with an equivalent damping c_e) elements.
It can be shown ([10]) that, for a given SDOF VDN at a given temperature, as frequency functions, k_e and c_e can be computed respectively as

$$k_e(\omega) = real\left[\frac{\omega^2 m_a \bar{k}(\omega)}{\omega^2 m_a - \bar{k}(\omega)}\right], \tag{4}$$

$$c_e(\omega) = \left\{imag\left[\frac{\omega^2 m_a \bar{k}(\omega)}{\omega^2 m_a - \bar{k}(\omega)}\right]\right\}/\omega, \tag{5}$$

where $real[\cdot]$ and $imag[\cdot]$ denote respectively real and imaginary parts of the quantity in brackets (i.e., the dynamic stiffness at the base of the VDN), m_a is the mass of the VDN, and $\bar{k}(\omega)$ is given by

$$\bar{k}(\omega) = L\bar{G}(\omega), \tag{6}$$

where L is named the design factor of the viscoelastic element, and $\bar{G}(\omega)$ is the corresponding complex shear modulus, which can be obtained experimentally. In the absence of damping, $\bar{G}(\omega) = G(\omega)$.

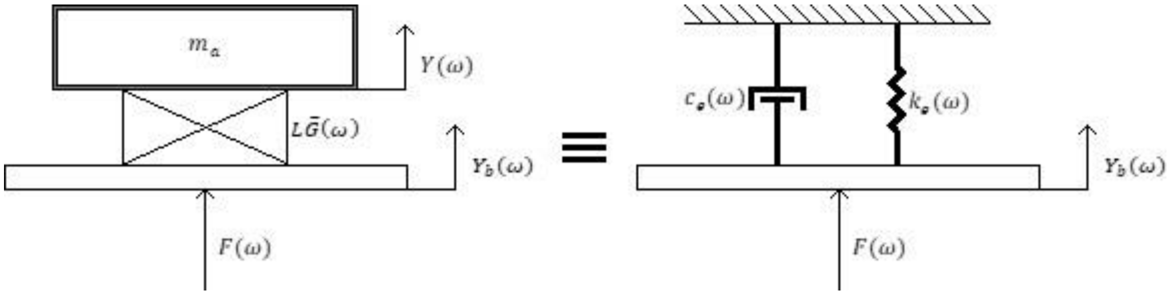

Figure 1. Equivalent generalized parameters.

The anti-resonant - or characteristic - frequency, ω_a, of the SDOF VDN is defined as the one such that, in the absence of damping, the denominator of the dynamic stiffness at its base is equal to zero ([11]), i.e.,

$$\omega_a = LG(\omega_a)/m_a. \tag{7}$$

The modified damping $[\tilde{C}]$ and stiffness $[\tilde{K}]$ matrices of the primary system can now be expressed respectively as

$$[\tilde{C}(\omega)] = [C] + [C_{eq}] = [\alpha[M] + \beta[K]] + [C_{eq}], \tag{8}$$

$$[\tilde{K}(\omega)] = [K] + [K_{eq}], \tag{9}$$

where $[C_{eq}]$ and $[K_{eq}]$ are full-order diagonal matrices, the entries of which are the equivalent damping and stiffness, respectively, of as many SDOF VDNs as are located in the corresponding primary system's DOFs.

2.2 The modal approach

In the context of the modal approach (such as that of LAVIBS-ND®), the design of a VDN is entirely carried out in the modal space ([11]). By this approach, the VDN optimal mass can be computed by

$$m_a = \sum_{j=Mi}^{Ms} \frac{\mu_j m_j}{NM\left(\sum_{i=1}^{p} \psi_{k_i j}^2\right)}, \tag{10}$$

where μ_j and m_j are respectively the mass ratio (1/10) and the modal mass of j^{th} mode under control; $\psi_{k_i j}$ is the entry of the modal matrix in the k_i^{th} row and j^{th} column, where k_i is the position (DOF number) of the i^{th} VDN, and *Mi* and *Ms* are the inferior and superior modes – whereas *NM* is the number of modes – to be controlled.

2.3 The matrix partition method

The dynamic stiffness matrices for a general modification can be expressed as ([8], [12])

$$[\Delta\bar{S}(\omega)] = [-\omega^2[\Delta M(\omega)] + i\omega[\Delta C(\omega)] + [\Delta K(\omega)]], \tag{11}$$

where $[\Delta M(\omega)]$, $[\Delta C(\omega)]$, and $[\Delta K(\omega)]$ are the modifications in mass, damping, and stiffness, respectively.
For a localized modification, matrices $[\Delta\bar{S}(\omega)]$ assume the following form:

$$[\Delta\bar{S}(\omega)]_{n\times n} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & [\Delta\bar{S}(\omega)]_{r\times r} \end{bmatrix}, \tag{12}$$

where n is the number of DOFs of the system, and r is the number of DOFs of the modification.
Then, the modified receptance matrices, $[\bar{R}^*(\omega)]_{n\times n}$, are given by

$$[\bar{R}^*(\omega)]_{n\times n} = ([\bar{S}(\omega)]_{n\times n} + [\Delta\bar{S}(\omega)]_{n\times n})^{-1}. \tag{13}$$

As Soares and Lopes [7] noted, the matrix partition method is called 'a local method' because its focus lies on the partition of the receptance matrix that is associated with the DOFs in which the structural modification takes place. As shown in Rodrigues [9], the elements of the receptance matrix of the modified system in the partition associated with the modification DOFs can be obtained - in view of eqs. (2) and (13) - by

$$[\bar{R}^*(\omega)]_{jj} = \left([\bar{R}(\omega)]_{jj}^{-1} + [\Delta\bar{S}(\omega)]_{jj}\right)^{-1}, \tag{14}$$

where *jj* indicates the partition of interest, with dimension $r \times r$. For a SDOF VDN, $r = 1$.
Then, in the matrix partition method, the effects of inserting the VDN are taken into account by computing, for each frequency, the GEPs for each VDN, assembling matrix $[\Delta\bar{S}(\omega)]$, and evaluating eq. (14).

2.4 The matrix product method

The matrix product method is so named because it represents the modification as a product of two matrices ([5], [7]) such that

$$[\Delta\bar{S}(\omega)]_{n\times n} = [U(\omega)]_{n\times r} \times [V(\omega)]_{r\times n}, \tag{15}$$

where $[U(\omega)]$ and $[V(\omega)]$ are called the first and second matrices of the matrix product, respectively.
The modified matrix is given by ([4], [6])

$$[R^*(\omega)]_{n\times n} = [R(\omega)]_{n\times n} - [R(\omega)]_{n\times n}[U(\omega)]_{n\times r}[W(\omega)]_{r\times r}^{-1}[V(\omega)]_{r\times n}[R(\omega)]_{n\times n}, \tag{16}$$

where

$$[W(\omega)]_{r\times r}^{-1} = [I]_{r\times r} - [[V(\omega)]_{r\times n}[R(\omega)]_{n\times n}[U(\omega)]_{n\times r}]. \tag{17}$$

Then, in the matrix product method, the effects of inserting the VDN are taken into account by computing, for each frequency, the GEPs for each VDN, assembling matrices $U(\omega)$ and $V(\omega)$, and evaluating eqs. (16) and (17).
As will be confirmed later, if $r << n$, the matrix product and the matrix partition methods are much more suitable than the modal approach, since they do not demand a thorough analysis of the modified system, providing a reduction in computational efforts.

3 Methodology

In order to investigate the equivalence of the methods presented herein, a cantilever steel beam (see Figure 2) - nominally 1 500 mm long, 50 mm wide, and 8 mm thick - was modeled by the finite-element method (with Euler-Bernoulli beam elements) and tested by experimental modal analysis (EMA) with impulsive excitation 150 mm apart its fixed end and acceleration measurement at its free end. A simple SDOF VDN (also depicted in Figure 2, in illustrative way), positioned at the free end of the beam as well, was then designed by each of those three methods.
In order to properly compare the different techniques, the optimal mass of the VDN, computed via eq. (10) on the modal approach, was also adopted on the computations by the methods of reanalysis. Furthermore, for design purposes, the location of excitation and response was made coincident with the location of the VDN (at the free end of the beam), allowing use of the matrix partition method. Selected viscoelastic material was the EAR™ C-1002, from Aero Technologies LLC, with operating temperature of 293 K (20° C). The frequency range for vibration control was from 45 to 180 Hz (the limits of which are labeled *Li* and *Ls*, respectively).

3.1 Modeling and parametric estimation for the primary system

Mesh convergence of the finite-element model for the primary system was ascertained by both stability on the location of the relevant resonances and comparison of their values with the theoretical ones ([13]). It was found that, by using the Guyan reduction, a mesh of 30 elements would be enough to ensure accurate results.
Parametric identification was then carried out within an optimization process similar to that employed in designing the VDN, as described in section 3.3. Comparison was made between the experimentally-determined inertance I^{exp} and its numerical counterpart I^{num} (see eq. (3)). The objective function for minimization, $f(x)$, was defined as

Figure 2. Computational representation of the primary system and of an instance of VDN.

$$f(x) = \sum_{k=0}^{400\,Hz} \left[\Pi_k \left(|I_k^{num}| - |I_k^{exp}| \right) / |I_k^{exp}| \right]^2, \tag{18}$$

where Π is a weighting function. The vector of design variables x was

$$x = (\alpha, \beta, t, E, \rho)^T, \tag{19}$$

subject, in the second part of the optimization procedure, to the constraints

$$\alpha_{min} \leq \alpha \leq \alpha_{max} \qquad \beta_{min} \leq \beta \leq \beta_{max} \qquad t_{min} \leq t \leq t_{max}$$

$$E_{min} \leq E \leq E_{max} \qquad \rho_{min} \leq \rho \leq \rho_{max}$$

where t, E, and ρ are the beam thickness, the beam material elastic longitudinal modulus, and the beam material density, respectively; and subscripts *min* and *max* refer to minimum and maximum values, respectively.
It is worth noting that a linear normalization scheme was adopted throughout the optimization procedure. Also, experimental estimation of the modal parameters of the beam was carried out via standard EMA techniques: peak picking and half power bandwidth.

3.2 Optimizing the VDN

By each of those three methods, designing the VDN consisted in finding its characteristic frequency in such a way that response of the primary system was minimized by one of two criteria: (1) FRF's peak value, or (2) reduction, in dB relative to original system's FRF, of broadband energy content of the modified system's FRF, within the control range. The chosen FRF was the point receptance of the primary system at its free end.
Then, the objective function for minimization, *f*(*x*), was given, depending on the selected criterion, by

$$\text{Criterion 1: } f(x) = \max\left[20 \log_{10} \left| \bar{R}^*(\omega)_{Li}^{Ls} \right| \right], \text{ or} \tag{20}$$

$$\text{Criterion 2: } f(x) = 20 \log_{10} \left[\frac{\sqrt{\sum_{k=Li}^{k=Ls} |R^*{}_k|^2}}{\sqrt{\sum_{k=Li}^{k=Ls} |R_k|^2}} \right], \tag{21}$$

where the vector of design variables x is

$$x = \omega_a, \tag{22}$$

subject to the constraint that it should lie anywhere within the frequency range of concern (0 to 400 Hz).

3.3 Optimization strategy

The optimization strategy was a hybrid one, with non-linear techniques sequentially associated ([14]). Initially, the genetic algorithm technique was applied (to approximate the values of the variables in the design vector), and then, the SQP (Sequential Quadratic Programming) technique was applied (to refine the values of the variables). Their respective *ga* and *fmincon* implementations in the MATLAB® computational environment were employed.

4 Results and discussions

4.1 Parameters of the primary system

Estimated parameters for the beam are given in Table 1, whereas the corresponding experimental and regenerated inertances are presented in Figure 3. The results are quite close.

Table 1. Estimated primary system parameters.

α	β (10^{-6})	t [mm]	E [GPa]	ρ [kg/m³]
0.3586	9.158	8.097	204.2	8 050

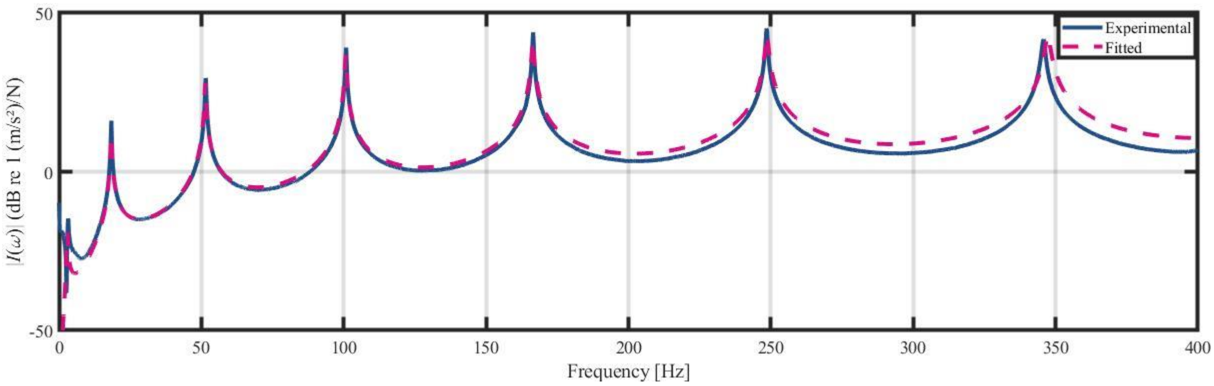

Figure 3. Experimentally-determined and numerical inertances.

4.2 Design of the VDN

It was found that the VDN’s characteristic frequency is always identical, irrespective of the method used. The optimal mass and corresponding frequency for each of the control criteria are given in Table 2. It should be noted that the methods of reanalysis demanded considerably less computational effort, resulting in a 95% (partition) and 77% (product) decrease in time consumption relative to the modal approach.

Table 2. VDN's characteristic frequency for each control criterion [Hz].

Mass m_a	Criterion 1	Criterion 2
172 g	37.4	48.1

As the VDN’s characteristic frequency is the same, the effects are therefore identical when the device is inserted into the primary system, as it can be seen in Figure 4. In that figure, concerning ‘C1’ (criterion 1), curves for the three methods are plotted, resulting in the same curve. As to ‘C2’ (criterion 2), just one curve is plotted, since the other ones result in identical plots and were omitted for the sake of clarity.
Regarding the two control criteria, the distinction is noticeable. It is clear that criterion 1, focused on the reduction of the largest FRF’s peak value, leads to smaller vibration level reductions on the other resonances within the control band. On the other hand, criterion 2 is more suitable when broadband control is of primary concern, for it properly leads to a greater overall reduction in the area below the FRF curve.

5 Conclusions

As expected, the results show that the optimal modal parameters for a viscoelastic dynamic vibration neutralizer obtained by the three distinct methods considered in the present paper are identical, since both response-reanalysis methods, though one local and the other global in nature, are exact ones. Furthermore, it is verified that the reanalysis methods are considerably less time-consuming than the modal approach, an important advantage when optimization processes are of concern. Finally, the differences in results stemming from the different control strategies are evidenced, as they tackle different aspects of the problem.

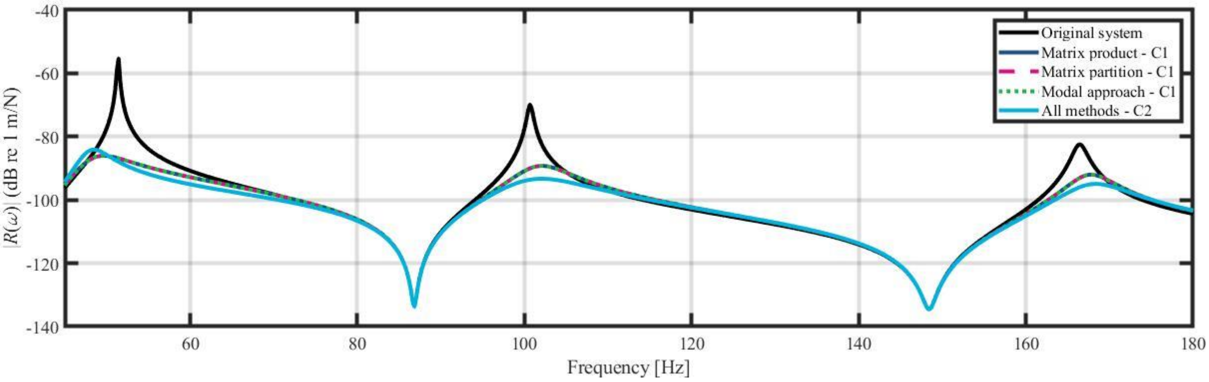

Figure 4. Comparison between design methods and control criteria, “C1” and “C2”.

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