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Human-Structure Interaction during jumping on Rectangular Plates

Phablo V. I Dias[1], Zenón J. G. N. Del Prado[1]

[1]*School of Civil and Environmental Engineering, Federal University of Goiás*
Avenida Universitária, 1488, Setor Leste Universitário, 74605-200, Goiânia, GO, Brasil
phablo@discente.ufg.br, zenon@ufg.br

Abstract. In this work a Spring-Mass-Damping system (SMD) of one degree of freedom is used to represent the human-structure interaction during jumping on a thin rectangular plate under a time-dependent base excitation. The plate is considered simply supported and its strains relations are described by the Von Karman nonlinear theory. A parametric analysis using the piecewise-smooth contact dynamics theory is performed to study the influence of loss of contact during the flight phase of jumping cycles. The different stable jumping strategies of the human body for incremental values of the human damping ratio are found. Obtained results show that, depending on the values assumed for the biodynamic parameter, chaotic responses, hysteresis and coexisting attractors can be observed in the bifurcation diagrams when the human degree of freedom is controlled.

Keywords: Human-structure interaction, Rectangular plates, Nonlinear dynamics.

1 Introduction

Structures as footbridges, buildings floors and stadia are susceptible to vibrations induced by human activities, such as: walking, jumping and bobbing (Bachmann et al. [1]). In order to prevent the human discomfort caused by excessive levels of vibrations and to predict the respective structural damage, the vibrations serviceability in these structures has received a many attention in the literature, mainly due to the fact that new design conceptions allow to use lightweight, long spans and slender structures (Kala et al. [2] and Jones et al. [3]). As a result, extensive studies have been carried out on dynamic systems with low damping and more prone to interacting with human actions (Shahabpoor et al. [4]), where resonant states can be easily perceived which can severely affect the structural integrity (Chen et al. [5] and).

Although loads due human activities are recurrent in different types of structures, rigorous studies are still necessary for their correct mathematical characterization, in order to represent sufficiently the effect of human-structure interaction. This is because the dynamic actions generated are directly related to the unique characteristics of each person, as weight, age, height, knee flexion and even the way each person performs an activity on the structure (Varela et al. al [6]). In this aspect, many studies carried out experimental analyzes to discuss the biodynamic effect and the human load potential while performing different activities on structures (Shahabpoor et al. [7]).

In the design practice of floors subjected to vibrations induced by human jumping, the modelling of structures with force-only models is recurrent, where deterministic loads capable of reproducing asymmetric experimental responses are used to characterize human behavior (Racic and Pavic [8]). In the analysis using force-only models, the influence of individual biodynamic properties, such as stiffness, mass and damping, on the modal parameters of the structure is neglected. With that approach, the beneficial effect of human-structure interaction (HSI) is not perceived, which can result in expensive structural projects.

Several studies have been dedicated to representing the HSI through biodynamic models with few degrees of freedom, such as: Inverted Pendula (Milton et al. [9] and Kwon et al. [10]) and Spring-Mass-Damping – SMD (Caprani and Ahmadi [11] and Shahabpoor et al. [12]), which are coupled to the structure and allow capturing the biodynamic behavior of the human body. About floors subjected to human jumping, Gaspar et al. [13] carried out numerical and experimental analyzes of the dynamic behavior of a flexible concrete plate during periodic jumps.

Through experimental analyses, the biodynamic parameters of the human body were obtained for an equivalent SMD model. In the numerical analyses, initially a force-only model was used, which was compared with a biodynamic model of SMD type without considering the loss of contact between the structure and the jumper's feet. The responses obtained showed that the force-only model provided higher floor acceleration peaks when compared to the SMD model for the quasi-resonant case. On the other hand, the force-only model has a better approximation with the experimental response when the excitation frequency moves away from the resonant region. In the work of White et al. [14], dynamic patterns for human behavior when periodic jumps are performed on a rigid platform was studied, where was noticed that the leg stiffness of the jumper can change depending on the jumping frequency and has a non-linear dependency. Next, White et al. [15] using a simple model with one degree of freedom, studied the effect of loss of contact between the feet and the structure during jumping on an oscillating base. The findings of this work showed that chaotic vibrations and the coexistence of multiple solutions for human jumping strategies can be found in a numerical modeling for a given parameters combination.

Based on this, in this work, the nonlinear human response is studied when jumps are performed on a thin rectangular plate subjected a time-dependent base excitation. The touch-down and take-off effects caused by the loss of contact events during jumping are investigated. The Kirchhoff non-linear thin elastic plate theory is used to model the plate and the nonlinear Von-Kármán relations are used to describe the deformation relations. A SMD model with one degree of freedom is used to represent the human body coupled onto the plate. In order to capture the mechanisms of the loss of contact between the SMD model and the plate during the flight and contact phase, the coupled system is solved as a piecewise-smooth contact dynamics problem (Di Bernardo et al. [16]) which is integrated in time by the fourth-order Runge-Kutta method. The dynamic responses of the SMD model for various system parameter combinations of human damping ratio and forcing frequency are evaluated.

2 Mathematical formulation

Consider a simply supported elastic rectangular plate subjected to a time-dependent base excitation with amplitude A_b and excitation frequency Ω, as shown in Fig. 1. The plate has coordinates $(O; x; y; z)$ and displacement fields u, v and w with length a, b, thickness h, density ρ and Young modulus E. In order to represent the human interaction on the plate, a spring-mass-damper (SMD) model with one degree of freedom and modal mass m_h, equivalent stiffness k_h and damping coefficient c_h is considered. The SMD model is located at coordinates (x_1, y_1) and the application of human potential gravitational force ($G_h = m_h\, g$) is considered directly at jumper degree of freedom (w_h). In this section, the formulation presented is based in the work of Dias e Del Prado [17].

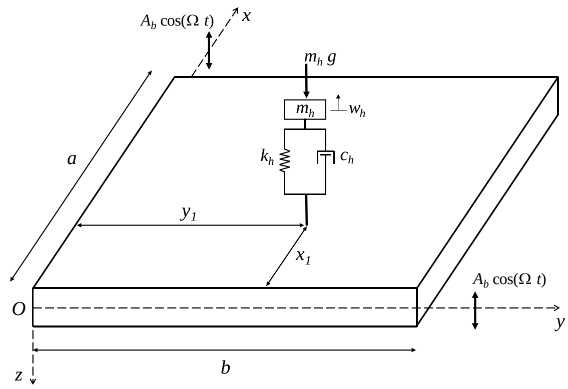

Figure 1. Rectangular plate with the SMD model and base excitation

Neglecting the rotatory inertia and the shear deformation and using the nonlinear Von-Kármán theory, the elastic potential energy of the plate can be written as:

$$
\begin{aligned}
U_P = {} & \frac{Eh}{2(1-\nu^2)} \int_0^a \int_0^b \left(\varepsilon_{x,0}^2 + \varepsilon_{y,0}^2 + 2\nu \varepsilon_{x,0} \varepsilon_{y,0} + \frac{1-\nu}{2} \gamma_{xy,0}^2 \right) dydx \\
& + \frac{Eh^3}{2(12(1-\nu^2))} \int_0^a \int_0^b \left(k_x^2 + k_y^2 + 2\nu k_x k_y + \frac{1-\nu}{2} k_{xy}^2 \right) dydx.
\end{aligned} \tag{1}
$$

The jumper stiffness k_h is considered by the elastic potential energy of the SMD model (U_H) given by:

$$U_H = \frac{1}{2} k_h \left(w_h - w|_{x=x_1, y=y_1} \right)^2 = \frac{1}{2} k_h \left(w_h^2 - 2 w_h w_{x=x_1, y=y_1} + w_{x=x_1, y=y_1}^2 \right), \tag{2}$$

where w_h is the displacement of the SMD model (see Fig. 1), which refers to the displacement of the body centre of mass of the jumper and w is the transverse displacement of the plate. In this way, the total elastic potential energy of the coupled system (plate and SMD model) results in sum: $U = U_P + U_H$.

For this study, the plate is considered simply-supported with a base excitation (CHAI et al. [18]). Then, to satisfy the boundary conditions and to reduce the plate system to finite dimensions, the following u, v and w fields displacements are adopted:

$$u(x,y,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} u_{m,n}(t) \sin\left(\frac{m \pi x}{a} \right) \sin\left(\frac{n \pi y}{b} \right); \tag{3a}$$

$$v(x,y,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} v_{m,n}(t) \sin\left(\frac{m \pi x}{a} \right) \sin\left(\frac{n \pi y}{b} \right); \tag{3b}$$

$$w(x,y,t) = \sum_{m=1}^{M} \sum_{n=1}^{N} w_{m,n}(t) \sin\left(\frac{m \pi x}{a} \right) \sin\left(\frac{n \pi y}{b} \right) + A_b \cos(\Omega t); \tag{3c}$$

where m e n are respectively the half-wave numbers in x and y directions; M and N are the number of terms used in each field displacement and $u_{m,n}(t)$, $v_{m,n}(t)$ e $w_{m,n}(t)$ are the unknown amplitudes; A_b is the amplitude of the transverse base displacement and Ω is the excitation frequency of the base. Thus, the vector of generalized amplitudes of the plate is given by: $\mathbf{q} = [u_{m,n}(t), v_{m,n}(t), w_{m,n}(t)]^T$, where its dimension is given by N_q, which is the number of degrees of freedom considering the plate's field displacement (u, v and w) and excluding the degree of freedom of SMD model; The generic element of the vector $\mathbf{q}$ is referred to as q_j, for $1 \le j \le N_q$. When the generalized amplitude of the SMD model (w_h) is important, such a generic term will be referred as $q_{\tilde{N}}$, with $\tilde{N} = N_q + 1$.

The kinetic energy of the coupled system T is given by the sum of the kinetic energy of the plate T_P with the kinetic energy of the jumper T_H, as described by:

$$T = T_P + T_H = \frac{1}{2} \rho h \int_0^a \int_0^b \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) dy\, dx + \frac{1}{2} m_h \dot{w}_h^2; \tag{4}$$

where ρ is the density in kg/m^3, h is the plate thickness, m_h is the mass of the jumper and the over-dot indicates the time derivative.

The nonconservative damping forces of plate are assumed to be of viscous type and, using the Rayleigh dissipation function, can be written as:

$$F_P = \frac{1}{2} c_p \int_0^a \int_0^b \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2 \right) dy\, dx; \quad \text{with } c_p = 2 \zeta_p \rho h \omega_{m,n}, \tag{5}$$

where c_p, ξ_p, ρ, h and $\omega_{m,n}$ are respectively the damping coefficient, the viscous damping ratio, the density, the thickness and the natural frequency of plate, which is found in the first mode of vibration (m = 1, n = 1). Similarly, the Rayleigh dissipation function for SMD model F_H is given by:

$$F_H = \frac{1}{2} c_h \left(\dot{w}_h - \dot{w}|_{x=x_1, y=y_1} \right)^2 = \frac{1}{2} c_h \left(\dot{w}_h^2 - 2 \dot{w}_h \dot{w}_{x=x_1, y=y_1} + \dot{w}_{x=x_1, y=y_1}^2 \right), \tag{6}$$

where the human damping coefficient is given by: $c_h = 2\, \zeta_h\, m_h\, \omega_h$, in which ξ_h is the human damping ratio, m_h is the mass of human body and ω_h is the jumper body natural frequency. The nonconservative damping forces of coupled system (F) is given by the sum: $F = F_P + F_H$.

The work (W) done by the jumper weight (G_h) refer the human potential gravitational energy acting on the system during the jumping and it can be written as:

$$W = -G_h\, w_h; \quad \text{with } G_h = m_h\, g, \tag{7}$$

where G_h is the human static weight, g is the gravity acceleration and w_h and m_h are, respectively, the displacement and the mass for the body centre of mass of the jumper.

In order to obtain the set of nonlinear dynamics equations, the Rayleigh-Ritz method together with the Hamilton principle are used in order to method given by:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}+\frac{\partial U}{\partial q_j}=Q_j;\ \text{ with: }\ Q_j=\frac{\partial W}{\partial q_j}-\frac{\partial F}{\partial \dot{q}_j},\tag{8a}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_{\tilde{N}}}\right)-\frac{\partial T}{\partial q_{\tilde{N}}}+\frac{\partial U}{\partial q_{\tilde{N}}}=Q_{\tilde{N}};\ \text{ with: }\ Q_{\tilde{N}}=\frac{\partial W}{\partial q_{\tilde{N}}}-\frac{\partial F}{\partial \dot{q}_{\tilde{N}}},\tag{8b}$$

where Q is the generalized forces obtained by differentiation of the Rayleigh dissipation function and of the virtual work done by external forces for $1 \le j \le N_q$ and $\tilde{N} = N_q + 1$.

Thus, the equations of motion of the coupled system are obtained as:

$$\begin{aligned} &m_j\,\ddot{q}_j+\sum_{i=1}^{N_q}c_{j,i}\,\dot{q}_i+c_h\left(\sum_{i=1}^{N_q}\dot{q}_i\,\psi_i^{\,2}-\dot{q}_{\tilde{N}}\,\psi_j\right)+\sum_{i=1}^{N_q}k_{j,i}\,q_i\\ &+k_h\left(\sum_{i=1}^{N_q}q_i\,\psi_i^{\,2}-q_{\tilde{N}}\,\psi_j\right)+\sum_{i,k=1}^{N_q}k_{j,i,k}\,q_i\,q_k+\sum_{i,k,l=1}^{N_q}k_{j,i,k,l}\,q_i\,q_k\,q_l=F_j;\end{aligned}\tag{9a}$$

$$m_h\,\ddot{q}_{\tilde{N}}+c_h\left(\dot{q}_{\tilde{N}}-\sum_{i=1}^{N_q}\dot{q}_i\,\psi_i\right)+k_h\left(q_{\tilde{N}}-\sum_{i=1}^{N_q}q_i\,\psi_i\right)=F_{\tilde{N}}-G_h;\tag{9b}$$

$$\begin{aligned}F_j&=A_b\,\frac{4^2}{\pi^2}\left(m_j\,\Omega^2\cos(\Omega t)+c_j\,\Omega\sin(\Omega t)\right)\\ &-A_b\left(k_h\cos(\Omega t)-c_h\,\Omega\sin(\Omega t)\right)\psi_j;\end{aligned}\tag{9c}$$

$$F_{\tilde{N}}=A_b\left(k_h\cos(\Omega t)-c_h\,\Omega\sin(\Omega t)\right);\tag{9d}$$

where m_j is the modal mass of the plate given by ¼ $\rho\, a\, b\, h$; $c_{j,i}$ refers to the linear modal damping coefficient of the plate; $k_{j,i}$, $k_{j,i,k}$ and $k_{j,i,k,l}$ are stiffness term giving linear, quadratic and cubic nonlinearities of the plate; ψ_i is the mode shape associated to ith generalized coordinate of plate and applied at points x_1 and y_1; F_j and $F_{\tilde{N}}$ are the base motion forces associated with the plate equations and the SMD model equations, respectively.

The obtained system in Eqs. (9a) to (9d) refers to the contact phase, where the jumper is coupled with the plate. In order to consider the flight phase, i.e., the time when the jumper loses contact with the plate, the interaction force onto the plate and the SMD model is assumed to be zero and the only force that acts in the SMD model is the $G_h = m_h\, g$. In this work that interaction force will be called as Plate Reaction Force (*PRF*), which refers to the reaction force that the jumper applies to the plate. From Eq. (9b) it is observed that the Plate Reaction Force (*PRF*) can be written as:

$$PRF=F_{\tilde{N}}-c_{\tilde{N}}\left(\dot{q}_{\tilde{N}}-\sum_{i=1}^{\tilde{N}-1}\dot{q}_i\,\psi_i\right)+k_{\tilde{N}}\left(q_{\tilde{N}}-\sum_{i=1}^{\tilde{N}-1}q_i\,\psi_i\right).\tag{10}$$

In the flight phase (i.e., $PRF = 0$) the nonlinear vibrations of the plate are only due to the base excitation and a new plate equations system decoupled from the SMD model is used, which is given by:

$$m_j\,\ddot{q}_j+\sum_{i=1}^{\tilde{N}-1}c_{j,i}\,\dot{q}_i+\sum_{i=1}^{\tilde{N}-1}k_{j,i}\,q_i+\sum_{i,k=1}^{\tilde{N}-1}k_{j,i,k}\,q_i\,q_k+\sum_{i,k,l=1}^{\tilde{N}-1}k_{j,i,k,l}\,q_i\,q_k\,q_l=F_j;\tag{11a}$$

$$F_j=A_b\,\frac{4^2}{\pi^2}\left(m_j\,\Omega^2\cos(\Omega t)+c_j\,\Omega\sin(\Omega t)\right).\tag{11b}$$

where the system of Eqs. (11a) and (11b) is an adaptation of the Eqs. (9a) and (9c) by neglecting human biodynamic parameters, which results in a set of nonlinear equations that describe the plate in forced vibration state by the base excitation. On the other hand, also during the flight phase, the SMD model goes into free fall motion and its dynamic equation is obtained decoupled from the plate, given by:

$$m_h\,\ddot{q}_{\tilde{N}}=-G_h.\tag{12}$$

Assuming that $\alpha = -G_h/m_h$, the solutions of Eq. (12) for the human displacements and velocities during the flight phase are given by Eqs. (13a) and (13b), where the response for the displacements is evaluated as a parabola of the form.

$$q_{\tilde{N}}(t) = \left(\frac{1}{2}\alpha\right)t^2 + \left(\dot{q}_{\tilde{N}}(t_0) - \alpha\, t_0\right)t + \left(q_{\tilde{N}}(t_0) + \frac{1}{2}\alpha\, t_0^2 - t_0\, q_{\tilde{N}}(t_0)\right); \tag{13a}$$

$$\dot{q}_{\tilde{N}}(t) = \alpha\, t + \left(\dot{q}_{\tilde{N}}(t_0) - \alpha\, t_0\right) \tag{13b}$$

Thus, two set of nonlinear equations are obtained for the problem studied here, the first (Eqs. 9a-d) is about coupled equations between the plate and the SMD, which represent the contact phase. On the other hand, the second set is given by the free-falling SMD model system (Eqs. 13a-b) and the empty plate system (11a-b) decoupled from each other in order to represent the flight phase. Then, the sets are solved as a piecewise-smooth contact dynamics problem (White et al. [15] and Di Bernardo et al. [16]) and are numerically integrated by the Runge-Kutta fourth order method. For this, *PRF* is controlled in order to determining the precise touch-down ($PRF > 0$) and take-off ($PRF = 0$) events.

3 Numerical results

For the numerical simulations, a thin concrete rectangular plate has been considered. The physical and geometrical properties of the plate are given by: $a = 14\ m$, $b = 10\ m$, $h = 0.10\ m$, $E = 27\ GPa$, $\rho = 2500\ kg/m^3$, $\nu = 0.2$ and $\zeta_p = 0.02$, which were chosen in order to the natural frequency of the plate ($\omega_{1,1}$) is close to the human body frequency (ω_h). To model the simply supported with fixed edges condition for the plate, 16 degrees of freedom ($N_q = 16$) for the displacements field (u, v and w) in the expansions of Eqs. (3a), (3b) and (3c) were considered using the following generalized coordinates (Amabili [19] and Dias [20]): $u_{2,1}$, $u_{2,3}$, $u_{4,1}$, $u_{4,3}$, $u_{6,1}$, $u_{8,1}$, $v_{1,2}$, $v_{1,4}$, $v_{1,6}$, $v_{1,8}$, $v_{3,2}$, $v_{3,4}$, $w_{1,1}$, $w_{1,3}$, $w_{3,1}$, $w_{3,3}$. The SMD model is allocated at mid-span of the plate ($x_1 = a/2$ and $y_1 = b/2$) and their biodynamic properties are (White et al. [14]): $m_h = 76.64\ kg$ and $\omega_h = 2.48\ Hz$, which implies in a leg stiffness of $k_h = 18.6\ kN/m$. A nondimensionalization of variables is introduced for computational convenience: the displacement of the body centre of mass of the jumper (w_h) has been divided by $\eta = g/\omega_h^2$, the plate vibration amplitudes are divided by the plate thickness (h) and the time is multiplied by human body frequency ω_h. The natural frequency for the empty plate is 2.297 *Hz*. However, when the SMD model is positioned on the plate, occurs an interaction between the two systems and a small change in natural frequencies is observed, where the frequency for the occupied plate and for the human body is now given respectively by: 2.243 *Hz* and 2.537 *Hz*.

Now, parametric sweeps of the human damping ratio will be discussed for increases from 0.1 to 0.5 (White et al. [15]) with excitation frequency values (Ω) around the linear resonance between the jumper body and the base movement, i. e., $\Omega = 1.0\ \omega_h$. For this case, a large base excitation amplitude of $A_b = 0.13\ h$ was chosen. All bifurcations diagrams were obtained using the brute force method (Del Prado [21]). Figure 2 displays the responses for the SMD model due to incremental values of the human damping ratio (ζ_h) with excitation frequency values of $\Omega = 0.9\ \omega_h$ (Fig. 2a), $\Omega = 1.0\ \omega_h$ (Fig. 2b) and $\Omega = 1.1\ \omega_h$ (Fig. 2c).

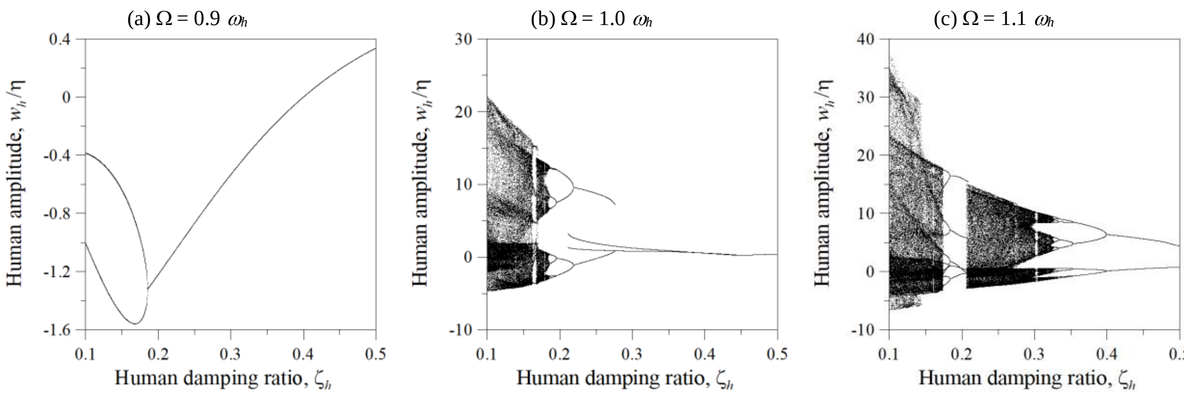

Figure 2. Nondimensional SMD vibration amplitude for increases of the human damping ratio (ζ_h) with $A_b = 0.13\ h$ and (a) $\Omega = 0.9\ \omega_h$, (b) $\Omega = 1.0\ \omega_h$ and (c) $\Omega = 1.1\ \omega_h$.

Figure 2a depicts that, for small values of human damping ratio, 2T periodic vibrations are found for the SMD response. This occurs when a frequency allows the linear resonance is chosen as the jumping strategy, as was also observed by White et al. al. [14] and White et al. [15]. As the damping is increased, at a critical value, the human response is characterized by a 1T stable solution. This is displayed in phase portraits and Poincaré maps of Fig. 3a and 3b, illustrating the stable attractors respectively for 2T and 1T solutions. Figure 2b shows that chaotic solutions can also be found for the jumper response. This finding suggests that is impossible to periodically jump over the plate when the certain parameters combination is considered. For higher values damping, the chaotic solutions become in 4T stable solutions. Afterwards, at ζ_h = 0.28, 2T stable solutions can be perceived for the jumper response. This is depicted in Fig. 3c, 3d and 3e, representing chaotic, 4T and 2T respectively solutions. Figure 2c also points chaotic solutions for small values of the human damping ratio with the base excitation amplitude studied. When the damping continues to be increased, also can be found windows with quasi-periodic and high order periodic solutions. For example, at ζ_h = 0.32 the jumper response shows quasi periodic oscillations and at ζ_h = 0.36 the response suggests 4T periodic vibrations. For high damping values with decreasing damping, the Feigenbaum 2T cascade can be noticed. This is reflected in Fig 3h, 3g and 3f, illustrating the transition in 4T, quasi-periodic and chaotic solutions.

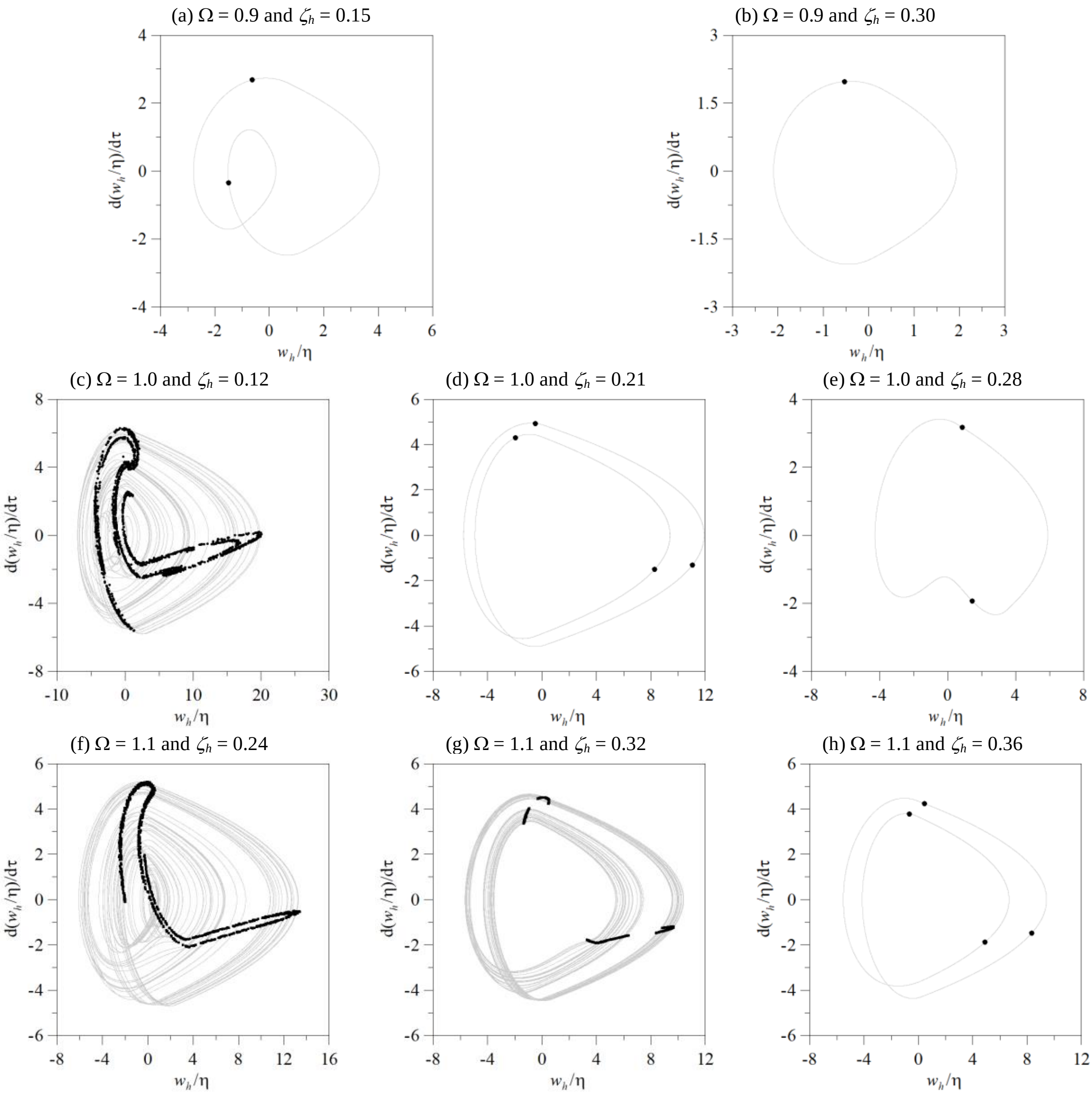

Figure 3. Phase portraits and Poincaré maps for various system parameters.

4 Concluding remarks

A piece-wise nonlinear system for human rhythmic jumping is shown in this work. The system is based on coupling a SMD model with a thin rectangular plate excited by a base movement. In this study, the main objective was on the human response, where the results obtained showed that excitation frequency values before linear resonance provide jumps with 2T periodic vibrations. For small values of human damping considered in the analyses, chaotic and quasi-periodic solutions were also found, which proves that, for certain sets of system parameters, it may be impossible to periodically jump over the plate.

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