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Model updating using hierarchical Bayesian strategy and error scale factor employing B-WIM calibration data

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Abstract. Data collected during the calibration process of bridge weigh-in-motion (BWIM) systems can be applied both for evaluating the behavior of the structure and for updating models and parameters. These model update techniques aim to adjust parameters of a structural model making predicted responses closer to the experimental behavior. Bayesian modeling is well applied to the present problem, as it makes possible the combination of previous knowledge and experimental data, allowing better parameter estimates. However, in some civil engineering applications the updated parameters may contain inherent variability during the experimental process, due to external factors such as environmental conditions, and may have considerable changes during the process. To consider this inherent variability, a hierarchical Bayesian model was adopted. Sampling from Markov Chain Monte Carlo (MCMC) methods is applied. It was also observed that there is an increase in variability with increasing vehicle weight. The introduction of this effect to the model was then studied, comparing 2 ways of considering this variation, both as a linear function of the expected signals for a given vehicle, and using the area under this predicted signal. Results for both numerical simulations and real bridge calibration data indicate that the hierarchical Bayesian approach proposed for the model update, including the scale factor according to vehicle weight, is able to perform properly, providing confidence intervals for predicted signals by unseen vehicles that best fit within the observed strains.

Keywords: Model updating, hierarchical Bayesian modeling, bridge weigh-in-motion, scale factor, response prediction.

1 Introduction

Model updating is a technique with the objective of adjusting parameters of a given model so that the responses predicted by it are as close as possible to the experimental behavior. Calibrated models, often adopting finite elements, can be used, for example, to improve the strength prediction [1], to obtain a better estimate of lifetime for the structure [2] or predict behavior responses, and detect damage [3]

Bridge Weigh-in-Motion (B-WIM) systems aim to obtain weights of vehicles passing over the bridge from strain measurements [4]. They are efficient tools for providing real time monitoring of traffic, with vehicles traveling at their usual speeds [5]. For this reason, these data have been applied with the objective of structure health monitoring (SHM) [4]. The influence line obtained in the calibration step can also be used to evaluate the structure of the bridge, dynamic amplification factors, and changes in this influence line, that can be related to temperature changes, and indicate loss of stiffness [6]. This calibration step can also be used for model update [7]. It allows the reduction of uncertainties about a parameter that, in the case of bridges whose structural design is unavailable, could be estimated from some geometric and normative data, but with little precision.

Bayesian methods allow updating the knowledge about the parameters of interest, given the experimental results. A parameter is assumed to be an unknown fixed value and the uncertainty related to its value is reduced when increasing the quantity of data [8]. During B-WIM calibration step, depending on the number of planned runs, varying transverse positions, speeds or trucks, the procedure can be long, lasting hours or days, subject to variations in winds and temperatures or any other variability inherent to the process. In general, results for different runs are recorded under different environmental conditions, which indicates that an inherent variability in the quantity of interest (QoI) is expected [7]. To include the inherent variability a hierarchical Bayesian model is proposed. In this case, the theoretical answers are functions of variables that follow probability distributions defined by additional parameters called hyperparameters, considered unknown and that will be updated during the process. The hierarchical framework allows a more flexible consideration regarding the QoI, including, for instance, an unknown mean and covariance matrix. As a result, confidence intervals can be estimated. [9]. This variability can include dynamic effects that are sometimes disregarded in the model, and by including the hierarchical framework, it can be seen that errors tend to be greater in heavier vehicles. This indicates the possibility of include some weighting for this error.

In the present work it is proposed a hierarchical Bayesian model, including a scale factor with error term, to perform model updating employing a set of time-history strain measurements related to the response of a bridge structure due to the passage of heavy vehicles. Such data is available as a result of the calibration process of bridge weigh-in-motion systems. Numerically simulated data, which enables the complete knowledge about the target values of updated parameters, are employed for assessing the suitability of results.

2 B-WIM systems and the application of calibration data for model updating

Most BWIM systems in operation today are based on ideas established by Moses [10] in 1979, who suggested that an instrumented bridge with strain sensors could be used to obtain the weight of moving vehicle axles. The axle weights can be estimated as the ones that minimize the error between the measured and the theoretical responses, using the concept of bending influence line to calculate them. The strain signals recorded by sensors installed at the bottom of the beams are converted into bending moment.

$$\boldsymbol{M} = \sum_{j=1}^{J} E_j Z_j \varepsilon_j \,, \tag{1}$$

where **M** is the measured bending moment, E is the elastic modulus of the beam, Z is the section modulus, ε are the strain obtained and J is the total number of beams. In this way, it is possible to consider that the j beams have different stiffnesses given by possible geometric differences in cross section or state of degradation [11].

The theoretical bending moment at each instant, represented here by $\widehat{M}_k$, results from the multiplication of the truck axle weights (W_i) by the ordinate of the influence line (I) related to its position at that moment.

$$\widehat{M}_k = \sum_{i=1}^{n} W_i I_{(K-Ci)} \,, \tag{2}$$

$$C_i = \frac{d_i f}{v} \,, \tag{3}$$

where K is the total number of readings, n is the number of truck axles, C_i is the number of readings corresponding to the distance d_i between the first axle and the axle in question, which depends on the frequency f of acquisition of these readings, and the speed v that vehicle crosses the bridge. The distance between axles and truck speed are calculated from signals from FAD (Free of Axle Detectors) sensors, as shown in Kalin *et al.* [12], installed under the slab.

The error function can be given by least squares and must be minimized. For this study, instead of calculating axles weights or obtaining the influence line, this system will be used to update the model, as in Gonçalves *et al.* [7]. During system calibration, it is possible to obtain the influence line for BWIM and also to update knowledge about a structural parameter, allowing to predict the response induced by any vehicle. The QoI is called θ, and here it will be defined as the constant EZ, elastic modulus multiplied by section modulus. The constant depends on beam dimensions and material. For some bridges where we don't have access to the design, it is difficult to define the characteristic strength of concrete or inertia and neutral line that depend on the amount of steel inside the beams, so θ includes all stiffness information. Therefore, eq. (1) and eq. (2) can be rewritten as:

$$y = \sum_{j=1}^{J} \varepsilon_j \,, \tag{4}$$

$$\hat{y}_k = \frac{1}{\theta} \sum_{i=1}^{n} W_i I_{(K-Ci)} \,, \tag{5}$$

where **y** is the measured strain and $\hat{\boldsymbol{y}}_{\boldsymbol{k}}$ is the theoretical response at each instant.

Although the theoretical model represents reality well, it is expected that there will be some difference between the two answers. Therefore, the relationship between them is given by:

$$y_{k,i} = \hat{y}_{k,i}(\theta) + \epsilon_{k,i} \quad or \quad y_k = \mathcal{M}(\theta, W, Ci, k) + \epsilon_{k,i} \tag{6}$$

where $\boldsymbol{\epsilon_{k,i}}$ is an error between the theoretical and measured signal, at each reading k, at each truck pass i, here adopted as Gaussian, independent and equally distributed, with zero mean. There are theoretical simplifications, of course, that ignore important aspects such as dynamic effects, which end up being absorbed by this error. The theoretical influence line was adopted for the double-supported beam, fixed for the entire bridge.

The error in eq. (6) indicates that a vehicle with different configurations from another should have deviations of the same order and magnitude. In this case, as the weight of the vehicle is directly proportional to its response, it is expected that the deviations between theoretical and experimental models are greater for vehicles that induce larger signals. A purely additive error may not be the best option for modeling. Therefore, it is proposed here to include the effect of the vehicle configuration on the variability between theoretical model and experimental results. An extra constant is then added to the error:

$$y_{k,i} = \hat{y}_{k,i}(\theta) + \gamma_i \epsilon_{k,i} \tag{7}$$

where γi is constant at all instances and varies for each vehicle i.

In this study, 2 ways of obtaining this constant will be compared:

(a) The maximum expected strain, considering the bridge with unitary elastic modulus and section modulus.

$$\gamma_i = argmáx_k\left(\sum_{i=1}^{N} W_i I_{(K-Ci)}\right) \tag{8}$$

(b) The area under the signal obtained with unitary E and Z, where Δt_k is the time interval between 2 measurements.

$$\gamma_i = \sum_k^K (\Delta t_k \sum_{i=1}^{N} W_i I_{(K-Ci)}) \tag{9}$$

It would be possible to use the experimental data for these 2 values, that is, for each measured signal after crossing the vehicle, the maximum strain could be obtained or the area below that signal could be calculated. In the calibration step, this would lead to more accurate results, or closer to reality, since the measured signal varies a little with the transverse position and includes several variables not considered in the theoretical equation. However, it should be noted that the objective of this study is to update the model. From the distributions obtained for QoI, we can predict the signal that could be generated with the passage of any other vehicle with known characteristics. That is, if the objective is to estimate the signal of a vehicle before it crosses the bridge, it is important that the adopted value of γi could be previously calculated.

2.2 Hierarchical Bayesian Framework and Sampling Strategy

Let us consider that θ is the QoI to be estimated. For each calibration event there is a different value θ_i in a total of N events. The values of θ approximately follow a normal distribution. As the values of this constant are never negative, we decided to work with a lognormal distribution with parameters mean (μ_θ) and variance (σ_θ^2). The error term in eq. (6) and eq. (7) can be described as a normal function of μ_ϵ and σ_ϵ^2. Here μ_ϵ is zero and will be suppressed from the equations. The signal recorded in each event will be described as R_i, with a variable number of readings K_i, forming a set of values $\mathbf{R} = \{R_1, .. R_i, ...R_N\}$. Applying Bayes' theorem, the posterior distribution of parameters μ_θ, σ_θ^2 and σ_ϵ^2 given all measured values R can be written as:

$$\underbrace{p(\mu_\theta, \sigma_\theta^2, \theta, \sigma_\epsilon^2 \big| \boldsymbol{R})}_{posterior} = \underbrace{p(\boldsymbol{R}|\theta, \sigma_\epsilon^2)}_{likelihood} \underbrace{p(\theta \big| \mu_\theta, \sigma_\theta^2)}_{prior} \underbrace{p(\mu_\theta, \sigma_\theta^2, \sigma_\epsilon^2)}_{hyperprior} \tag{10}$$

The hyperparameters are considered independent of each other, so the probability of the hyperprior is obtained from the multiplication of the independent distributions. For $p(\mu_\theta)$ no prior knowledge is added. For the probabilities $p(\sigma_\theta^2)$ and $p(\sigma_\epsilon^2)$ Inverse Gamma distributions were chosen, with their respective shape (α) and scale (β) parameters, as they are positive and conjugated to the Gaussian likelihood. For both prior and likelihood, the measures are independent. These distributions can be consulted in Gonçalves *et al.*[7].

To consider the scale factor parameter γ_i, in all places where the term σ_ϵ appears, it must be replaced by $\gamma_i\sigma_\epsilon$.

The equation of the posterior is given by:

$$p(\mu_\theta,\sigma_\theta^2,\theta,\sigma_\epsilon^2|R) \propto \overbrace{\frac{1}{(\gamma_i\sigma_\epsilon)^{\sum Ki}} exp\left(-\frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\left(\frac{y_{i,k}-\hat{y}_{i,k}(\theta)}{\gamma_i\sigma_\epsilon}\right)^2\right)}^{p(R|\theta,\sigma_\epsilon^2)} \times \overbrace{\frac{1}{(\sigma_\theta)^{\sum N}\prod\theta_i} exp\left(-\frac{1}{2}\sum_{i=1}^{N}\left(\frac{log(\theta_i)-\mu_\theta}{\sigma_\theta}\right)^2\right)}^{p(\theta|\mu_\theta,\sigma_\theta^2)}$$
$$\times \underbrace{\left(\frac{1}{\sigma_\theta^2}\right)^{\alpha_\theta+1} exp\left(-\frac{\beta_\theta}{\sigma_\theta^2}\right)}_{p(\sigma_\theta^2)} \underbrace{\left(\frac{1}{(\gamma_i\sigma_\epsilon)^2}\right)^{\alpha_\epsilon+1} exp\left(-\frac{\beta_\epsilon}{(\gamma_i\sigma_\epsilon)^2}\right)}_{p(\sigma_\epsilon^2)} \quad (11)$$

This posterior does not have a closed solution. To access this information, the Gibbs sampler will be used. This is a special case of the Markov Chain Monte Carlo (MCMC) sampling strategy, which comprises a series of techniques for iteratively generating samples from approximate distributions until the desired distribution is reached. Full conditional distributions can be obtained by deriving the posterior with respect to the desired parameter [13]. The following set of equations is used:

$$p(\mu_\theta|.) \sim \mathcal{N}\left(\sum\frac{log(\theta_i)}{N},\frac{\sigma_\theta^2}{N}\right) \quad (12)$$

$$p(\sigma_\theta^2|.) \sim InverseGamma\left(\frac{N}{2}+\alpha_\theta,\ \frac{\sum_{i=1}^{N}(log(\theta_i)-\mu_\theta)^2-2\beta_\theta}{2}\right) \quad (13)$$

$$p(\sigma_\epsilon^2|.) \sim InverseGamma\left(\sum_{i=1}^{N}\frac{K_i}{2}+\alpha_\epsilon,\frac{1}{2}\left(\frac{\sum_{i=1}^{N}\sum_{k=1}^{K}\left(y_{i,k}-\hat{y}_{i,k}(\theta)\right)^2}{\gamma_i}\right)+2\beta_\epsilon\right) \quad (14)$$

$$p(\theta_i|.) \propto \frac{1}{\theta_i}.exp\left(-\frac{1}{2}\left(\frac{\sum_{i=1}^{N}\sum_{k=1}^{K}\left(y_{i,k}-\hat{y}_{i,k}(\theta)\right)^2}{(\gamma_i\sigma_\epsilon)^2}+\frac{\sum_{i=1}^{N}(log(\theta_i)-\mu_\theta)^2}{\sigma_\theta^2}\right)\right) \quad (15)$$

As conjugate prior distributions were used, the full conditional distribution of most parameters results in standard distributions that are easy to sample [13]. The exception is p(θ|.). It is possible to approximate it to a normal distribution. The parameters of this approximate normal distribution can be obtained in Gonçalvez *et al.*[7].

2.3 Uncertainty Propagation

Uncertainties quantified through the hierarchical model can be propagated, allowing estimates of the monitored quantity for vehicles other than the calibration ones [14]. In this study, this implies the possibility of estimating confidence intervals for strains for any vehicle [8]. This can be applied to assess the feasibility of the model update strategy for real cases, where there is no prior knowledge about the parameters. It is possible to do a cross validation, using some vehicles to update the model and others to obtain strains. In addition, the propagated uncertainties can be used for reliability analysis of the necessary parameters, which can be important in the safety assessment of structures and decision-making like authorizations for special loads, for example.

In this study the predicted signals are obtained as in eq. (7). At each iteration, a value for θ_i is sampled from a normal distribution using mean μ_θ and variance σ_θ^2 obtained in the model update step using the calibration vehicles. Adopting the theoretical influence line, vehicle weights and σ_ϵ^2, strain signals are generated, obtaining probability distributions and confidence intervals

3 Application in numerical simulations

To evaluate the proposal of including a constant proportional to the weight of the vehicles together with the error between the theoretical and measured strain, a numerical simulation was adopted, where the bridge structure is modeled as a simply supported Euler-Bernoulli girder with total length of 15 m. Seven types of vehicles, from 2 to 8 axles, were modeled as a sprung mass system described in Carraro *et al.* [15]. Vehicle axle weights and positions in relation to the first axle are in Table 1. Damping and stiffness of each axle were also included in the model. For each vehicle, 50 signals were generated, considering 10 runs for a speed of 16 m/s, 30 runs for 20 m/s and 10 runs for 24 m/s. The exact velocity varies around these averages, with a coefficient of variation of 5%. It was assumed that the vehicle's speed remains constant throughout the time it crosses the bridge.

Table 1. Weights (in ton) and axle positions in relation to the first (in m) for the calibration trucks

Vehicle ID	GVW	W1	W2	W3	W4	W5	W6	W7	W8	d1	d2	d3	d4	d5	d6	d7	d8
1	16	6	10							0	4,25						
2	23	6	8,5	8,5						0	3,7	5,5					
3	33	6	10	8,5	8,5					0	3,7	7,4	9,2				
4	43	6	8,5	8,5	10	10				0	3,7	5,5	9,2	12,9			
5	53	6	8,5	8,5	10	10	10			0	3,7	5,5	9,2	12,9	16,6		
6	57	6	8,5	8,5	8,5	8,5	8,5	8,5		0	3,7	5,5	9,2	11	14,7	16,5	
7	67	6	8,5	8,5	8,5	8,5	8,5	8,5	10	0	3,7	5,5	9,2	11	14,7	16,5	20,2

For the bridge, mass per unit length was considered equal to 1000kg/m and damping coefficient of 0.05. For each simulation, a different θi was considered, sampled from a Gaussian distribution with mean 5GPa and standard deviation 0.5GPa. In addition, class B roughness [16] and a white noise (SNR) of 20 were considered.

For Gibbs sampler, 10000 iterations and 20% burn-in were used. The variables μ_θ, σ_θ, and σ_ϵ were considered.

The actual values for μ_θ and σ_θ used in the simulation are precisely known and therefore are used as calibration criteria. On the other hand, as the parameter σ_ϵ comprises all the reasons for it to deviate from the model, its exact value is not known. For the hierarchical models $\alpha_\theta = \alpha_\varepsilon = 0{,}5$, $\beta_\theta = 10^{-3}$ and $\beta_\varepsilon = 0$.

4 Results

Adopting the hierarchical approach, it can be seen that the values of the mean and standard deviation of QoI achieved are quite close to the exact values as presented in Figure 1, with $\gamma_i = 1$. There is no rule about how the variability of θ is affected by the weight of vehicles. It is important to note that for shorter vehicles the generated signal has less data and this can contribute to the results deviating from the real value. It was noticed that the greater the weight of the vehicle, the greater the values of the standard deviation of the error. This may indicate, for example, that heavier vehicles may cause greater vibrations on the bridge, absorbed by this error variable. It is understood, then, that the hierarchical model, without any correction for σ_E, is adequate to estimate μ_θ and σ_θ, however the estimates for σ_E may be inadequate between different vehicles, especially if the model is used to propagate uncertainties.

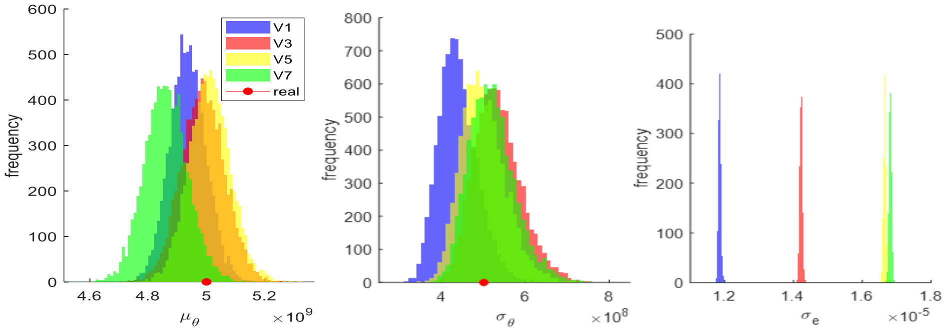

Figure 1. Histograms for the MCMC samples of each variable considering γi=1, as a function of the vehicle ID

Figure 2 shows the behavior in cases (a) and (b) indicated in item 2. Inserting the scale factor given by the maximum expected strain (a), results in mean and standard deviation values of θ very similar to the case without it. A greater difference occurs in the variable σ_E, which has lower order, given that the scale factor considers EZ to be unitary, so it is a high value. In this case, lighter vehicles have bigger errors, contrary to what happened before. For case (b), in which the area under the entire expected signal is used, μ_θ and σ_θ are still very close to the exact value. Larger vehicles have more assertive values. The error deviation becomes greater for lighter vehicles.

To illustrate the advantages of using a multiplicative constant for σ_E, Figure 3 is presented. The images on the left side indicate that after a calibration step carried out with heavy vehicles with 8 axles, distributions are obtained for μ_θ, σ_θ and σ_E and these data is used to predict strains for the passage of a 2-axle vehicle. The generated signal would be within the blue region, considering a confidence interval of 95%. However, it can be seen that the real signal of a 2-axle vehicle does not oscillate so much. When applying the scale factor, the predicted region for the signal can be much smaller, represented here by the pink spot, adjusting much better to the real signal.

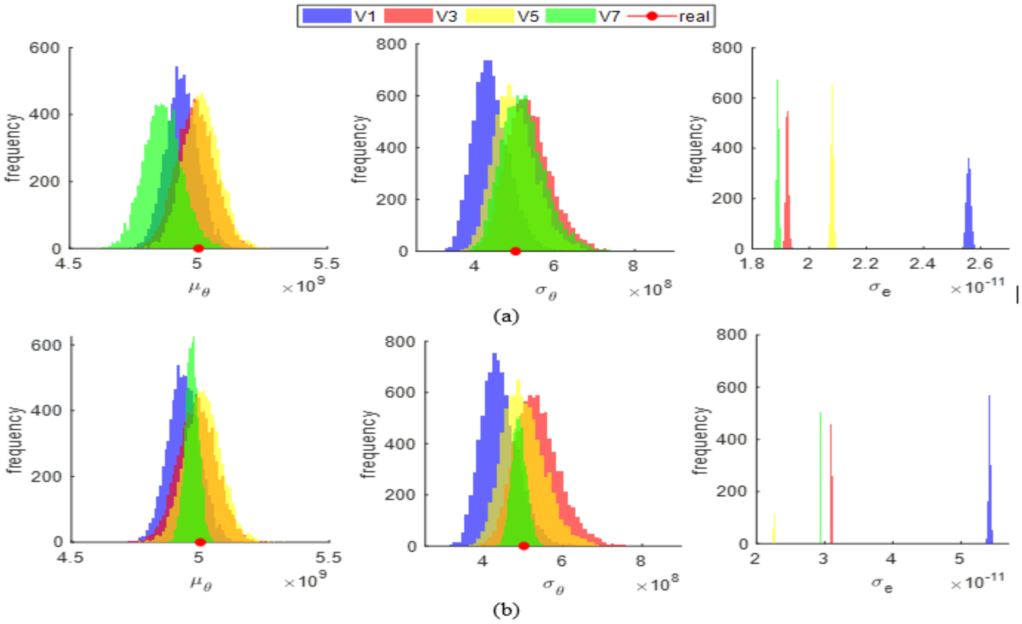

Figure 2. Histograms for the MCMC samples of each variable for cases (a) and (b), as a function of vehicle ID

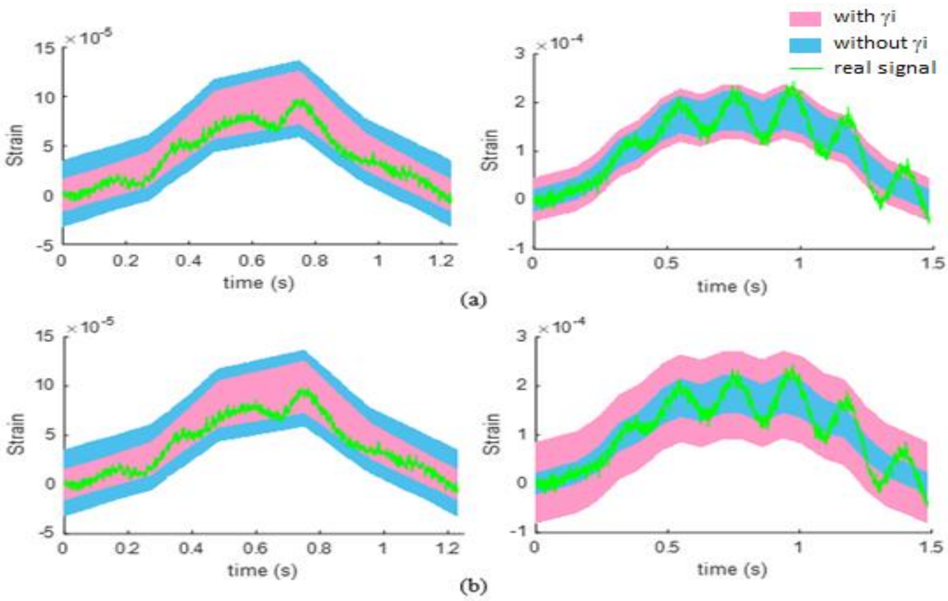

Figure 3. Uncertainty propagation for two distinct cases. Confidence interval of 95%.

In the images on the right, the opposite occurs. In case the system calibration is carried out with 2-axles vehicles, predicted signals will consider that there will be little variability. However, when passing a truck with 8 axles, the region predicted for the occurrence of the signal, with a confidence interval of 95%, would be blue. In this case, the heavy vehicle causes greater oscillation, so when disregarding the weighting of the error, estimated signals may not represent well the behavior of this bridge. Applying the scale factor, the pink interval is obtained.

When the light vehicle passes over the bridge calibrated with the heavy vehicle, model (b) seems to make a more correct fit to the signal behavior than model (a). However, in the case where a light vehicle is used to calibrate and a heavy vehicle crosses the bridge, it is noted that when considering γ_i as the area below the expected signal, it generates a region with too much variability, also not representing the signal so well. It is understood that using the maximum expected strain (a) leads to a better fit, especially in the initial part.

5 Conclusions

A bridge model update technique was presented, adopting hierarchical Bayesian modeling. The study proposed to introduce a multiplicative error constant, adjusted according to the weight of the vehicles. Synthetic BWIM data were adopted and the objective is to obtain the EZ value, which is often unknown. This value can be used to predict strain signs that occur on the bridge during the passage of other vehicles. The hierarchical Bayesian approach allows including confidence intervals and reliability analysis.

The effects of including this scale factor were more relevant on the variable σε, with minor changes in mq and sq. It is noticed that, disregarding the scale factor, when the calibration is performed with a type of vehicle and the strain signal generated by a vehicle with different weight is predicted, the region where the signal would be does not represent the reality. The adoption of a factor proportional to the estimated signal resulted in a region that better represents the signal than adopting the entire area under the signal.

It is important to be aware about the difference between experimental and observational data. This numerical model is useful for testing the method, but it must be applied in dada obtained in real bridges. It is recommended that more quantitative evaluations be carried out, using several signals, since here the analysis presents only 1 deformation signal and the evaluation was visual. Although more studies are necessary, it is understood that the adoption of this error weighting is very important for the model update.

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