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Sensitivity analysis and parameter identification to railway bridge structural model updating

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Abstract. One of the major challenges in the management of infrastructure systems is to ensure their safety and structural integrity throughout its useful life. This is due to the fact that the material and structural properties loss and the eventual failure of these structures has catastrophic consequences. In that context, this project aims to contribute to the application of methods that allow the safety and structural integrity assessment of railway bridges based on experimental modal data. The objective is to apply the sensitivity analysis of a real railway bridge structure using finite element model update to its unknown structural parameters. For this purpose, the Sobol' indices are studied to describe how the variability of the model response is affected by the variability of each input parameter or combination thereof. Usually, these indices are computed by Monte Carlo simulation. However, they are practically not applicable to CPU-intensive models such as finite element models. An alternative approach to overcome this scenario is the use of surrogate models to speed-up the calculations, such as a polynomial chaos expansion, a robust framework to compute Sobol' indices. Thus, for the sensitivity analysis of this paper, it brings a connection between these approaches. Finally, the role of the parameter identification is discussed, based on the results of the sensitivity analysis.

Keywords: Sensitivity analysis, Sobol' indices, Surrogate models, Model updating, Railway bridge

1 Introduction

The continued train operations on railway bridges can result in changes of their geometrical and mechanical properties such as stiffness, mass and boundary conditions which reflects in their structural responses [1]. This issue envolves estimates unknowns structural parameters, usually based on experimental responses, thus characterizing an inverse problem [2]. However, in parameters identification and model updating, one parameter may have a greater influence than another for the same response, hence this knowledge allows to detect which parameters are more important in the performance system. In this scenario, the parameter identification and sensitivity analysis based on the dynamic characteristics of structures has gained attention in modern engineering context.

According to Saltelli et al. [3] sensitivity analysis is defined as the study of how the variation in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation and how the given model depends upon the information fed into it. In particular, for nonlinear systems, the parameters is not only affected individually, but also the interaction between then [4]. In the literature, one of the most used methods to perform a sensitivity analysis is the Sobol' variance-based method, generally computed using Monte Carlo simulation, which makes them unfeasible for computationally demanding models, e.g., finite element models in structural engineering [5]. Therefore, a challenge in the sensitivity analysis studies of railway bridges is to perform a mathematical model with high accuracy, but without making the evaluation computationally costly.

In the context of artificial intelligence, surrogate models (or metamodels) attemps to deal with computationally expensive models in order to reduce their cost [6]. Polynomial chaos expansions (PCE) are a powerful surrogate technique that aims to provide a functional approximation of a mathematical model through its spectral representation on a properly constructed base of polynomial functions [7].

This paper aims to assess the influence of four structural parameters on modal responses of a numerical model of Canelas railway bridge, located in Portugal [8]. These are: vertical stiffness of the supports, modulus of elasticity of the concrete, modulus of elasticity of the ballast and ballast density. For this purpose, the Sobol' indices coupled with the polynomial chaos expansion is applied, in order to reduce the computational cost of the solution. The UQLab software framework, implemented in Matlab language, for uncertainty quantification is used in this paper to compute the sensitivity analysis [9]. Finally, this study will be useful in the operational assessment of these bridges such as a basis for studies which envolves to increase the load capacity of trains, and consequently, the productivity of the transport system. Also aims to implementing a damage identification strategy of railway bridge infrastructure referred to as structural health monitoring (SHM) [10].

2 Variance-Based Sensitivity Analysis

The general process to perform a sensitivity analysis consists to select input parameters and assign probabilities distributions to all parameters, evaluate the mathematical model for each set of input parameters and apply the sensitivity analysis to rank the input parameters according to their influence on the output responses. A powerful tool widely used in the literature to compute sensitivity analysis is the Sobol' Index proposed by Sobol [11]. The schematic representation of general sensitivity analysis is illustrated in Fig. 1.

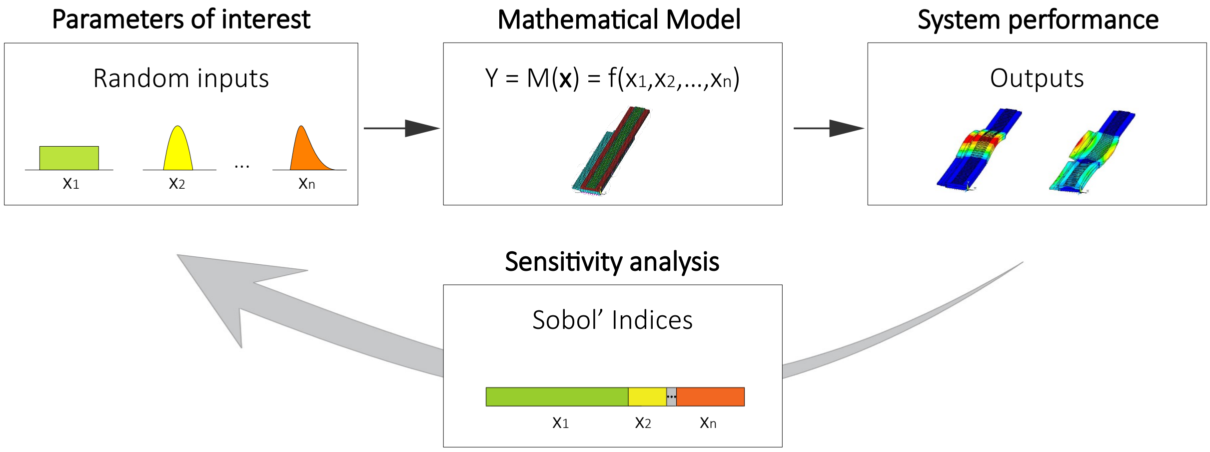

Figure 1. Schematic illustration of general sensitivity analysis

2.1 Sobol' Decomposition

The sensitivity analysis based on Sobol' method uses the decomposition of the model output function $Y = \mathcal{M}(\mathbf{X})$ into summands the variance of increasing dimension using combination of input parameters $\mathbf{X}$ to calculate the Sobol' indices, proposed by Sobol [11]. In order to determine the sensitivity of the output Y to the variation of input parameters, the Sobol' decomposition is reads as follows:

$$Y = \mathcal{M}_0 + \sum_{i=1}^{n} \mathcal{M}_i(X_i) + \sum_{1 \leq i < j \leq n} \mathcal{M}_{ij}(X_i, X_j) + \cdots + \mathcal{M}_{1 \cdots m}(X_i, \cdots, X_n), \quad (1)$$

where $\mathcal{M}_0$ is the mean value or expectance of the vector Y, and the terms of higher order are conditional expectations defined in a recursive way, that characterize an unique orthogonal decomposition of the model response [12]. As a consequence, the model response Y is a random output, whose variance reads:

$$Var(Y) = \sum_{\mathbf{u}} Var(\mathcal{M}_u(\mathbf{X}_u)), \text{for} \neq \mathbf{u} \subset (1, \cdots, n), (2)$$

where $Var(\mathcal{M}_u(X_u))$ denotes the conditional variance for the subvector $\mathbf{X_u}$, containing the variables whose indices are indicated by the subset $\mathbf{u}$. Following this idea, the Sobol' indices are defined as:

$$S_u = \frac{Var(\mathcal{M}_u(\mathbf{X}_u)}{Var(Y)} \tag{3}$$

By definition, the eq.(3) satisfy:

$$\sum_u S_u = \sum_{i=1}^{n} S_i + \sum_{1 \leq i < j \leq n} S_{ij} + \cdots + S_{1\cdots m} = 1, \tag{4}$$

where the first term of the Sobol' index reads:

$$S_i = \frac{Var(\mathcal{M}_i(\mathbf{X}_i)}{Var(Y)},\ i = 1, \cdots, n \tag{5}$$

Thus, the first index denotes the effect for the single parameter X_i for the total model variance, called the first order Sobol' indices. Similarly, to compute the second order Sobol' indices, i.e, the effect of interaction between the parameters X_i and X_j:

$$S_{ij} = \frac{Var(\mathcal{M}_{ij}(\mathbf{X}_{ij})}{Var(Y)}, 1 \leq i < j \leq n \tag{6}$$

Following the formulation, the Sobol indices can be implemented of all orders up to the mth order index $S_{1,\ldots,m}$, which represents the contribution of the interaction between all input parameters in **X** [5].

2.2 Polynomial chaos expansion

Consider the input random vector **X** (dim **X** = M) with given probability density function (PDF) $f_x(x) = \prod_{i=1}^{M} f_{x_i}(x_i)$. Assuming that the random output $Y = M(\mathbf{X})$ has finite variance, it can be cast as the following polynomial chaos expansion:

$$Y = \mathcal{M}(\mathbf{X}) = \sum_{\alpha=0}^{\infty} y_a \Psi_a(\mathbf{X}), \tag{7}$$

$$Y \approx \mathcal{M}^{PC}(\mathbf{X}) = \sum_{\alpha=0}^{P} y_a \Psi_a(\mathbf{X}), \tag{8}$$

where $\Psi_\alpha(X)$ are basis functions, y_α are coefficients to be compute and P is the number of terms in the polynomial expansion, which depends on the number of input parameters m and the maximum degree allowed for the expansion p, according to $P + 1 = (m + p)!/(m!p!)$. In this work, the number m is equal to the number of input parameters n of the model, because all random inputs follows a Gaussian distribution [5].

The PCE basis $\{\Psi_\alpha(X), \alpha \in N^M\}$ is made of multivariate orthonormal polynomials chosen according to the model input distribution, in order to minimize the number of terms needed in the expansion and perform a good computational simulation of the model. For input parameters with Gaussian probability distribution, applied in this paper, the family of Hermite polynomials with a Hilbertian basis is commonly used [13].

In this paper, the idea is apply the polynomial Chaos expansion to compute the Sobol' indices. For this purpose, the conditioned variances of the Sobol' indices can be calculated by the sum of the squared of all the PCE coefficients:

$$S_{\mathbf{u}} = \frac{\sum\limits_{\alpha \in u} y_\alpha^2}{\sum\limits_{\alpha=1}^{P} y_\alpha^2} \tag{9}$$

According to Sudret [5], the use of PCE provides a computational cost that is 2–3 orders of magnitude smaller than the traditional Monte Carlo-based evaluation of the Sobol' indices. A more detailed explanation of sensitivity analysis using Polynomial chaos expansion can be see in [7].

3 Case study: Numerical model of the Canelas railway bridge

The Canelas railway bridge is selected for the case study. This bridge is located on the Northern Line of the Portuguese railway network, near the Aveiro city, in Portugal, and has a total length of 72 m between the axes of the extreme supports. This structure is composed of 6 simply supported spans, each 12 m long. The bridge is a mixed filler beam type structure, composed of two independent concrete decks, each supporting a railway track. Both decks are 6.2 m wide, with 4.5 m of solid slab 0.7 m thick and nine HE500B metal beams 0.475 m apart. The ends of the transversal slab decks are composed of corbels of width 1.7 m and a linear thickness that varies between 0.3 m and 0.7 m. The bridge slab deck is also provided with a ballast guard consisting of a beam 0.60 m high and 0.3 m wide, located between the concrete slab deck and the corbel side. On the board are arranged the gravel ballast, with an average height of 60 cm, the concrete sleepers and the rails. The guard ballast makes the ballast structure confined on the deck. All deck spans are fixed on one side and displaceable on the other, made of elastomers involving stainless steel sheets and supported under each HE500B profile , totaling 9 support devices. A more detailed description of the geometry and material properties of the Canelas railway bridge can be found in [14].

In the finite element numerical model, developed by Silva [15] using the software ANSYS® Parametric Design Language (APDL), a half slab deck is simulated with two successive spans, the span under study and the consecutive span, and an extension of the track to simulate its continuity over the embankment. The deck structure has no connection between the spans, given by a 6 cm joint, this connection being made by the elements of the track. The numerical model was discretized in 23709 elements and 29150 nodes. Figure 2 presents general view of Canelas bridge (left) and its respective finite element numerical model (right).

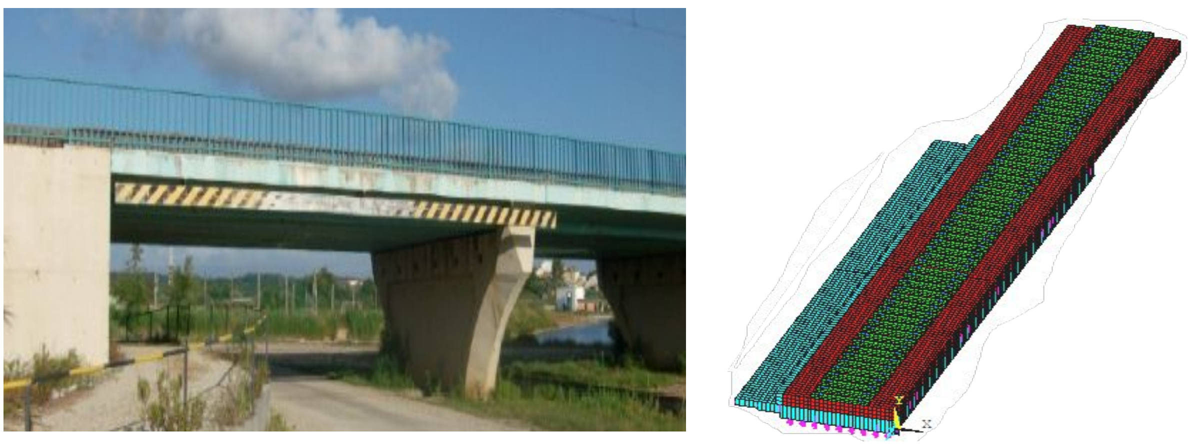

Figure 2. Numerical model and modal responses of the Canelas railway bridge

A study carried out by Rodrigues [16] addressed the modal identification of the Canelas bridge through experimental field measurements and served as a basis for the numerical model updating. In this study, results were obtained that refer to the first vertical bending mode and the first torsional mode of a bridge span. A disadvantage of using modal data to update numerical models is the limited number of modal responses, such as in the case of the Canelas railway bridge. This is due to the fact that identification from measurements of higher vibrational modes may involve data noise, and, as a consequence, do not represent the real behavior of the structure [17]. In this context, natural frequency data corresponding only to the first vibrational modes of the structure can lead to poor conditioning of the algorithm in the process of parameter identification [18].

4 Sensitivity analysis of Canelas railway bridge

The sensitivity analysis of Canelas railway bridge evaluating the influence of four parameters of interest of the structure in five natural frequency responses, the first vertical bending mode and the first torsional mode, studied by Rodrigues [16], and, in addition, the second vertical bending mode, second torsional mode and longitudinal mode. The four structural parameters are: the vertical stiffness of the support, the modulus of elasticity of the concrete, the modulus of elasticity of the ballast and the ballast density. Figure 3 shows the five numerical modal responses applied in the sensitivity analysis of this paper.

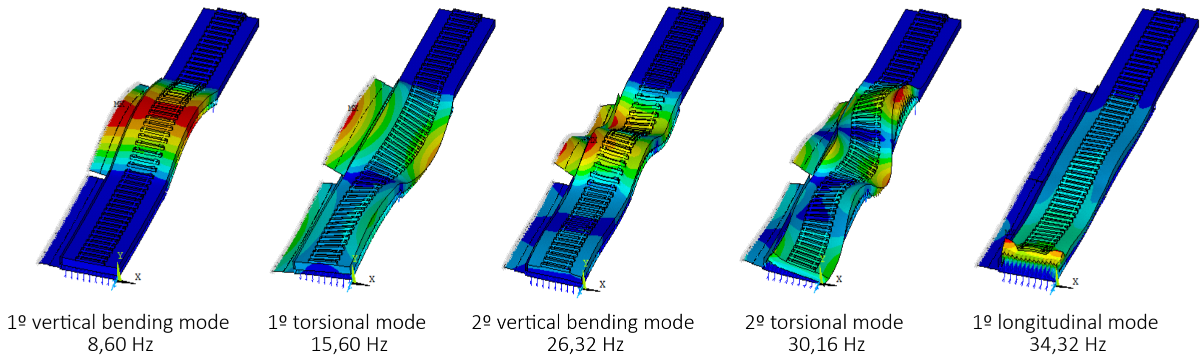

Figure 3. Modal responses of numerical model

The input parameters of the numerical model are evaluated in terms of their probability distribution. In order to compute the Sobol' Indices, the influence of parameters is determined up to their second order with a total number of $N_s = 10e5$ samples. The coefficients is performed through the surrogate model based on the polynomial Chaos expansion of five order and a total number of 200 samples. Using an Intel 9-core 2.90GHz (16 GB of RAM), the PCE method required approximately 100 minutes to compute Sobol' indices.

Table 1 shows the parameters of interest and their respective design bounds from the Gaussian probability distribution, applied to sensitivity analysis. This values are extract based on Silva [15], where the mean is approximately the central value of the upper and lower bounds of the parameters of interest and standard deviation between 5-15%. Figure 4 shows the first and second order Sobol' indices and the total Sobol' indices, which are configured by the sum of the first order Sobol indices and the second order Sobol indices, for the natural frequencies corresponding to the five vibrational modes.

Table 1. Parameters of interest based on input Gaussian distribution

Input	Parameter	Unit	Mean	Standard deviation
Kv	Vertical stiffness of supports	MN/m	300	30
Ec	Modulus of elasticity of concrete	GPa	34.5	2
El	Modulus of elasticity of ballast	MPa	160	10
ro	Ballast density	kg/m^3	2170	300

According to Figure 4, it is noted that the ballast density parameter is the parameter that most influences practically all modal responses of the structure. This may be explained because this parameter assumes a large design interval in its input Gaussian distribution, due to its uncertainty in the estimation.

For the natural frequency response of the first vertical bending mode its noted that the ballast density ro and the modulus of elasticity of the concrete Ec are the parameters that most influence the natural frequency response of the first vertical bending mode of the structure. In addition, the parameters vertical stiffness of supports Kv and

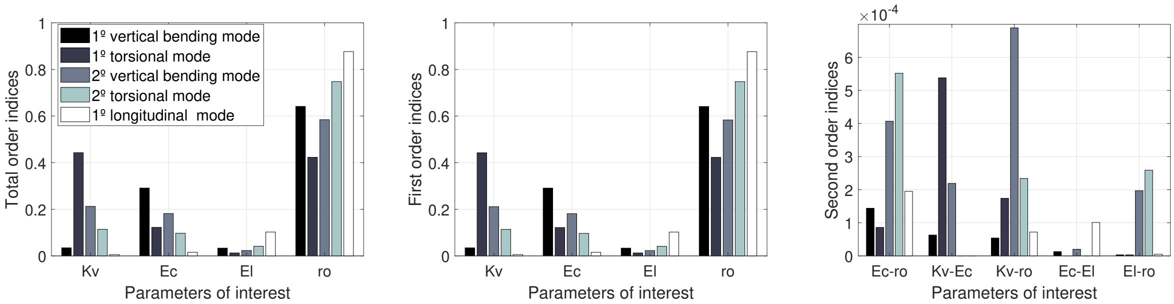

Figure 4. Sobol' indices to numerical model based on its Gaussian distribution of the parameters of interests

modulus of elasticity of the ballast El are not very significant in this response. Regarding to the natural frequency response of the first torsional mode, the support stiffness Kv and ballast density ro are the most influential. In terms of the second vertical bending mode, the parameters Kv and Ec have an influence on this response, while the parameter El is still not very significant. On the other hand, the parameter El has some significance in the natural frequency response of the first longitudinal vibrational mode of the numerical model.

Another way to analyze sensitivity is to assess the influence of coupling parameters on the response to higher Sobol' indices. In this case, for the second-order indice, the coupling of the parameters $Ec - ro$ has a greater influence on the natural frequency response of the second vertical bending mode and on the response of the second torsional mode. Regarding of coupling $Kv - Ec$, the response more affected is the first torsional mode. Finally, the second vertical bending mode is very significantly affected in the coupling of the parameters $Kv - ro$.

5 Conclusions

In this paper, the application of polynomial Chaos expansion to compute Sobol' indices showed efficient in the sensitivity analysis of structural parameters of the numerical model of Canelas railway bridge. The parameter of interest ro was the parameter that most influence in all five modal responses of the evaluated structure. On the other hand, the parameters Kv and Ec had a significant influence on the modal responses addressed, while the parameter El showed little or no significance. From the perspective of parameter identification and model updating, the great influence of the ballast density ro on all of the five modal responses may become a drawback, in order to characterize a non-identifiable problem.

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