

# **Reliability analysis of cold formed steel members with plain C-lipped and SupaCee section in shear**

Frederico B. Costa<sup>1</sup>, André L. R. Brandão<sup>2</sup>, Marcilio S. R. Freitas<sup>1</sup>

**<sup>1</sup>***Programa de Pós-Graduação em Engenharia Civil, Universidade Federal de Ouro Preto Campus Universitário Morro do Cruzeiro, 35400-000, Ouro Preto, Minas Gerais, Brasil. frederico.borges@aluno.ufop.edu.br, marcilio@ufop.edu.br* **2** *Instituto de Engenharias Integradas, Universidade Federal de Itajubá - Campus de Itabira Rua Irmã Ivone Drumond, 200, Distrito Industrial, 35903-087, Itabira, Minas Gerais, Brasil. andreriqueira@unifei.edu.br*

**Abstract.** This article presents a procedure for the reliability assessment of cold-formed steel members with plain C-lipped and SupaCee® sections in shear. The SupaCee® sections contain additional return lips and web stiffeners which enhance the bending and shear capacity of the sections. The development of the DSM for designing of coldformed sections in pure shear is available in North American (AISI) and Australian (AS/NZS 4600) standards. However, the Brazilian standard does not provide the application of the DSM for shear case. A test database of 23 cold-formed steel members in shear was assembled and test-to-predicted statistics were obtained for the Direct Strength Method (DSM). The reliability indexes, resulting from the reliability analysis, were determined using the First Order Reliability Method (FORM), First Order Second Moment (FOSM) and Monte Carlo Method (MCM). It was found that the DSM safety level, adapted to the Brazilian standard, satisfies the target reliability index of 2.5 if a resistance factor of 1/1.2 is used.

**Keywords:** Shear, Cold-formed steel, Reliability.

## **1 Introduction**

The cold-formed steel (CFS) members are used in civil construction in residential edifications and commercial and industrial installations. The CFS grant to the edifications qualities like high resistance combined with reduction of its weight and can make the process of fabrication and installation easy (Yu et al. [1]). The Direct strength Method has been employed in design codes like AISI S100:2016 and AS/NZS 4600 (Yu [2]).

The objective of this article is to evaluate the reliability of structural elements of cold formed steel in shear. The profiles were designed following the proposed standards by AISI S100:2016 and NBR 14762:2010. The results were compared to tests obtained in literature being 17 experiments using C-lipped profiles and 6 experiments using SupaCee® profiles as represented in Fig. 1. The comparison between the theoretical results and the experimental ones offered the statistics for the professional factor, which is a variable used to determine the reliability of the elements by the FOSM (First Order Second Moment), FORM (First Order Reliability Method) and Monte Carlo (MCM) methods. The reliability indices were obtained using the resistance factors used in AISI S100:2016 [3] and NBR 14762:2010 [4] and were compared to the target values of AISI S100:2016, on both design philosophies LRFD (Load and Resistance Factor Design) and LSD (Limit State Design).



Figure 1. Tested profiles: (a) C-lipped and (b) SupaCee®.

### **2 Structural reliability**

The structural reliability analysis foreshadows the definition of a limit state function  $G(X)$ . This function is described in function of the random variables X involved on the analysis. The limit state function must be defined by  $G(X) = 0$ , being possible to divide the domain in two groups: the failure domain  $(G(X) < 0)$  and the safety domain  $(G(X) > 0)$  (Hsiao [5]).

So the failure probability can be represented by eq. (1):

$$
P_f = P\big[G(X) \le 0\big] \tag{1}
$$

where:

 $P_f$  = failure probability;

In a structural analysis the failure function can be defined by the random variables of resistance and solicitation, as is shown in eq. (2).

$$
G(X) = R - Q \tag{2}
$$

where:

 $R$  = resistance random variable;

 $Q =$  solicitation random variable.

If we admit that the probability density functions and the cumulative density function of R and Q are known, we can describe the probability of some event as eq. (3):

$$
P_f = P(R < Q) = P[G(X) \le 0] = \int_{G(X) \le 0} f_x(x) dx
$$
 (3)

where:

 $f_x(X) =$  joint probability density function of all variables X involved on the analysis.

As the eq. (3) can have a very complex solving, usually are used reliability methods to obtain the failure probability and its respective reliability index β. Such methods are based on Taylor series expansions, like for example the FORM and FOSM methods, and some are based in the generation of synthetic samples like in MCM (Monte Carlo Method) (HALDAR e MAHADEVAN [6]).

#### **2.1 FOSM, FORM and MCM methods**

The objective on structural reliability is to determine the failure probability of a structure.Then it uses methods that allows an explicit representation of the uncertainties by the random variables of resistance (R) and solicitation (Q). Some methods are analytical like FORM and FOSM, and some work by simulations like Monte Carlo Method (MCM).

The FOSM method is based on the first order approximation of the Taylor series of the limit state function using two statistic parameters, the mean and the variance. The FORM method is an iterative procedure to determine the failure probability. The reliability index  $\beta$  is the minimal distance between the origin and the limit state surface in the reduced space. The Hasofer*–*Lind and Rakwitz–Fiessler (HL-RF) algorithm, which was proposed by Hasofer and Lind [7] and then extended by Rackwitz and Fiessier [8], was used.

The MCM consists in the generation of values of the involved random variables using their statistical information. In this method a set of the generated variables is used to verify if there is a violation in the limit state function in each cycle. The relative frequency of the fails can be used to estimate the failure probability. This methods precision depends on the number of simulations.

#### **2.2 Failure function**

A failure function can be proposed in terms of the resistance (R) and solicitation (Q) variables if taking the state limit in the standards as reference. The structural resistance is typically a function of the material resistance, the profiles geometry and its dimensions. The solicitation can be expressed in terms of the dead and the live loads. This function can be mathematically represented by eq. (4) (Hsiao [5]):

$$
G(\ ) = R_n MFP - c(D + L) \tag{4}
$$

The M, F and P variables are random and dimensionless. The random variable M, known as "material factor", can be determined by the ration between a tested mechanical property and a nominal value. The "fabrication factor" F is related to the variability of the geometrical properties. The "professional factor" P is a variable that reflects the uncertainties that come from the used analyses methods that will be discussed in the next section. D and L are the variables of the dead and live loads. The statistic parameters and probability distributions of the variables, in this paper, are presented on Tab 1 and were obtained from Ellingwood et al. [9].

Random variable	Mean	Coefficient of Variation	Distribution type
M	1.10	0.10	Lognormal
	1.00	0.05	Lognormal
	1.05	$0.10\,$	Normal
	.00.	0.25	Ext. type I

Table 1. Statistic data

#### **2.3 Professional factor**

The professional factor P is a random variable that reflects the uncertainties on the determination of the resistance capacity on a structural component, and so it is important to be considered on the structural reliability analysis. The variable P will be represented by eq. (5) that shows a comparison between the experimental result  $(V_{Test})$  and the nominal or characteristic value  $(V_n)$ , following the standards procedures:

$$
P = \frac{V_{test}}{V_n} \tag{5}
$$

A database was generated by tests in beams of cold-formed steel, focusing in obtaining the shear resistance and minimizing the effect of the other forces than shear as shown in the Fig. 2. Experimental tests of 17 C-lipped and 6 SupaCee® profiles were obtained in literature.



Figure 2. Tests with the predominance in shear stress. Source: Keerthan and Mahendran [10]

As a result of this analysis is obtained the mean  $(P_m)$  and the standard deviation  $(\sigma_P)$  of the variable P. The ratio between the standard deviation and the mean defines the coefficient of variation  $(V_P)$  of this variable.

#### **2.4 Nominal resistance of beams in shear**

The nominal resistance of the beams in shear was obtained by following the standard procedures of the item 9.8.3 of ABNT NBR 14762:2010 [4] for the Brazilian method. For the Direct strength method (DSM) analysis, the AISI S100:2016 [3] standard recommends on the item G the procedures that were followed. For this method it is needed the elastic shear buckling force  $(V_{cr})$  that is defined in AISI S100:2016 [3] on the item G2.3 for flat webs like C-lipped profiles and in Appendix 2 for other profiles like SupaCee®. The University of Sydney developed a software named THIN-WALL 2 that can provide the elastic shear buckling force  $(V_{cr})$  of a profile by defining the signature curve of the element as shown in Fig. 3. This study used both ways to determine the elastic shear buckling force  $(V_{cr})$  to determine which one has better precision.



Figure 3. Signature curve provided by the THIN-WALL 2 software. The red dot points the  $V_{cr}$ .

## **3 Results**

The profiles studied in this article are 11 C-lipped found in Keerthan and Mahendran [10], 6 C-lipped found in Pham and Hancock [11] and 6 SupaCee® profiles found in Pham and Hancock [12]. The nominal shear strength resistance of each tested beam  $(V_n)$  was obtained by the design method described by ABNT NBR 14762:2010 [4] and AISI S100:2016 [3]. The data was arranged in three sets: (I) P factor measured using ABNT NBR 14762:2010 [4] methods, (II) P factor based on the software Thin-Wall 2 to get the elastic shear buckling  $(V_{cr})$  with AISI S100:2016 [3] methods and (III) P factor measured using AISI S100:2016 [3] methods.

In the Fig. 4, the factor P data was compared to the relation  $h/t$  with h been the depth of flat portion of web measured along plane of web and t been the web thickness. It can be noted that on Fig. 4 (a) the dispersion got higher as the relation h/t got higher, meaning that the NBR standard lose precision when h/t get higher. It also can be noted that most of the points in Fig. 4 (a) are above the unitary line showing that the standard is underestimating the profiles resistance. When the other methodologies are used, the relation h/t does not have influence in the dispersion as shown in Fig. 4 (b) and (c).



Figure 4. Influence of the relation h/t on the factor P in: (a) Set I, (b) Set II and (c) Set III

The Anderson Darling test made with the help of the software Minitab 19 showed that the lognormal probability density function (pdf) was the one that had the best adjustment for variable P. The statistic data of the professional factor are described on the Tab. 2.

Statistic data	Set I	Set II	Set III
r m	. .346	.038	0.986
Oр	0.541	0.069	0.075
Vр	0.402	0.067	0.076

Table 2. Statistic data of the professional factor.

The reliability indices β were obtained from FORM, FOSM and Monte Carlo Method (MCM) with 100000

simulations, using the Resistance factor  $\phi = 0.95$  (AISI S100:2016 [3] LRFD),  $\phi = 0.80$  (AISI S100:2016 [3] LSD) and  $\phi = 1/1.1$  (NBR 14762:2010 [4]). The MCM results were used to verify the precision of FORM and FOSM analytical methods. The Fig. 5 (a) represent the reliability indices β obtained for the combinations of 1.2Dn+1.6Ln and the relation between the nominal actions  $L_n/D_n$  of 5 (LRFD). The Fig. 5 (b) uses the combination of 1.25Dn+1.5L and the relation between the nominal actions  $L_n/D_n$  of 3 (LSD).



Figure 5. Reliability indices using the LRFD (a) and LSD (b) philosophies.

Using the Brazilian standard procedures for the reliability indices, the combination of 1.25Dn+1.5L and the relation between the nominal actions  $L_n/D_n$  of 5 were used. The Fig. 6 represents the β index with the three calculation methods presented in this article.



Figure 6. Reliability indices using NBR standard procedures.

The reliability indices  $\beta$  obtained are inferior to the target 2.5 in LRFD for all the cases but were close enough to the target, especially when using the Set II calculation method. For LSD the β obtained were very close to the target 3.0 but bellow for all the cases unless in the FOSM analysis in group II.

For NBR, all cases of β were below the target 2.5, but the Sets II and III were close to the target. This suggests that the resistance factor  $\phi$  need to be inferior to the specified (1/1.1) in the standard. So, Set I needed to have  $\phi$  of 1/1.55 using Set I calculation methods, 1/1.15 using Set II and 1/1.2 using Set III.

## **4 Conclusions**

According to the obtained results, the following conclusions can be obtained:

- The  $\beta$  values found using the MCM method were closer to the FORM results than the ones using FOSM method. This means that the FORM method presents a good precision;
- The factor P analysis demonstrate that the precision of the calculation method of NBR (Set I) decreases when the relation h/t increases, because the factor P got further away from 1;
- The LRFD and LSD philosophies regardless of the reliability method used (Sets II and III), presented reliability indexes close to the target;
- The results using the Brazilian standard procedures showed that the reliability indexes based on the Direct Strength Method (DSM) (Set II and Set III) were close to the target of 2.5. When  $\phi$  was calibrated for a target  $\beta_0 = 2.5$ , the results for  $\phi$  were lower than 1/1.1, being 1/1.15 for Set II and 1/1.2 for Set III;
- The results using the Brazilian standard procedures showed that the reliability indexes based on the NBR 14762:2010 [4] methods did not reach the target of 2.5. In this case, it is necessary  $\phi = 1/1.55$ ;
- It is suggested to incorporate the Direct Strength Method (DSM) of AISI S100:2016 in ABNT NBR 14762:2010 standard to design elements in shear.

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### **6 Authorship statement**

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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