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Interactive graphic software for structural analysis of slabs using Grid Analogy with consideration of column's stiffness

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Abstract. For the dimensioning of reinforced concrete slabs, the appropriate determination of internal forces and displacements is vital. These parameters are subject of structural analysis and, in the case of slabs, the equation that mathematically describes such elements' behavior is the Lagrange Differential Equation, which analytical solution has a high degree of complexity, that increases considerably depending on the boundary conditions involved. However, in modern days, thanks to the computational revolution, numerical solutions for this problem have become more and more feasible and accessible, and are now widely used as basis for the structural analysis of slabs. Among the several viable methodologies used with this purpose, this paper describes the use of the Grid Analogy Method as the core method for developing a Python interactive-graphical software focused on the structural analysis of concrete slabs. Furthermore, based on the observation that a solidarization process occurs in the slab-column interface regions, the effect of the direct employment of the columns' bending stiffnesses on the stiffness matrix of the structural system is also analyzed. For verification matters, the recorded grid analogy results are compared with data present in the Literature, while the influence of the columns' stiffness is studied by comparing the results of the analysis of the same structure with and without this consideration. The results were very satisfactory, both numerically and regarding the graphical environment created, that can be used for handling the structural model design and presenting the results of internal forces and displacements.

Keywords: Structural analysis, Grid Analogy, slabs, graphic user interface, pillars' stiffness.

1 Introduction

For a long time, the structural analysis of slabs has been done in a simplified way, using abacuses and tables as a basis, since its integral contemplation is associated with the resolution of the Lagrange Differential Equation (Eq. 1), which is derived from the theory of elasticity of thin plates (Silva, Figueiredo & Carvalho) [1].

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{-q(x,y)}{D}. \tag{1}$$

Such simplifications, when related to the use of reinforced concrete, in Brazil governed by NBR 6118:2014 [2], adopt a deterministic perspective in the face of the uncertainties involved, which is based on the premise of the increment of active efforts combined with the reduction of structural resistance parameters, favoring the safety aspect of the system at the expense of associated economic factors, which implies the design of robust structures and optimizable efficiency. Thus, considering that slabs correspond to a high portion of the total consumption of concrete in a building (França & Fusco) [3], such over-dimensioning of these elements leads to problems concerning the economic and environmental viability of civil engineering projects.

Currently, such barriers are overcome thanks to the computational revolution, which brought to light the possibility of analyzing slabs through numerical approaches, which give rise to a number of methods of proven effectiveness in this field, such as the Finite Differences Method, the Boundary Element Method, the Finite Element Method and the Grid Analogy Method.

This last methodology, idealized by Marcus in 1932 (apud Timoshenko) [4], consists of replacing a slab by a grid of beams, called equivalent grid, in which the distributed loads are divided among the beams and the concentrated loads are applied directly on the beams or nodes (Figueiredo Filho) [5]. However, it is worth noting that such replacement carries with it the need for adjustments on the stiffness parameters of the beams, especially regarding the phenomenon of torsion. Such considerations are the object of study of several authors, such as Hambly [6] and Stramandinoli [7], who contemplate different methods of weighting this parameter, in order that such analogy satisfactorily approximates the real situation.

At the same time, the advantage of using this method lies precisely in the discretization of a two-dimensional element into a series of one-dimensional elements, which makes it possible to analyze the system by using techniques directed at reticulated systems, which are usually simpler. Among the methods available for this purpose, the Direct Stiffness Method stands out, which, in addition to the possibility of contemplating any structural system composed of bars (isostatic or hyperstatic), also has a generalized matrix formulation that makes it suitable for a computational implementation of low relative complexity (Martha) [8].

Moreover, in order to make the structural analysis results more convergent in relation to the real situation of these structural systems, the computation of the influence of the bending stiffnesses of the columns directly connected to the slab becomes relevant. This is an issue addressed by NBR 6118:2014 [2], which, in its items 14.6.6.1 and 14.6.6.2, emphasizes that when the exact calculation of this influence over beams and slabs is not calculated, the bending moment in the region of intersection of the column with these elements should be reduced by applying a coefficient based on the bending moments of inertia and lengths of the columns and beams involved.

2 Objectives and Methodology

This paper aims to present the development of a computer program with graphical user interface focused on the structural analysis of reinforced concrete slabs, by employing the grid analogy and taking into account the influence of the bending stiffness of the columns directly connected to the horizontal system.

The proposed software was developed in three programming stages: pre-processing, processing, and post-processing, all of them based on the Python language. During the pre- and post-processing stages, the focus is given to the graphical and interactive aspects of modeling the structure and presenting the results. For this purpose, the PySide2 library was used, which is a binding framework between the Python language and the C++-based framework Qt, designed for the development of Graphical User Interfaces (GUI). On the other hand, the processing stage refers to the implementation of the concepts of Grid Analogy and the Direct Stiffness Method to obtain the results of internal forces and nodal displacements of the horizontal system.

In reference to the Grid Analogy, the necessary adjustment of the stiffness parameters of the adopted beams utilizes standard formulas indicated in the literature. Users are allowed to customize these values, since such refinement causes significant variation in the results.

To implement the Direct Stiffness Method, it was adopted the consideration of bars subject to three local coordinates at each end, referring to torsional, bending, and shear forces, respectively.

The consideration of the bending stiffness of the columns was done by directly inserting the local stiffness matrices of these elements over the global Stiffness Matrix of the horizontal system. The added parameters concerns to the vertical bars directly above and below the floor, which are fixed at their non-contact end with the slab, and are also submitted to two rotations and one vertical translation, which was prescribed as 0.

3 Results

3.1 Graphic User Interface

The graphical interface created has two main screens: a two-dimensional and a three-dimensional one. The first one is directed to the structural model drawing, done via mouse, with the aid of tools such as pan, zoom, offset, snap, mouse selection and magnet for nodes and bars, besides automatic detection and separation of intersecting bars. In this screen are also inserted the physical and geometric parameters of the bars, besides their respective supports and loads, which have visual differentiation according to the colors selected by the user. The three-dimensional screen, in the other hand, includes a three-dimensional representation of the modelled elements, as well as an orbital camera system with mouse control. It is mainly responsible for displaying the results by means of internal forces diagrams and vertical displacement graphs. In order to improve the data analysis experience, the generated diagrams, besides having a color gradient system according to the intensity of the analyzed parameter, can also be rescaled, hidden and directed only to the chosen bars.

3.2 Validação do processamento

For the validation of the data processing, an example of a slab of which the properties are described in Tab. 1 was calculated.

Table 1. Characteristics of the first slab

Type	solid
Boundary Conditions	no border displacements
Measures	$4{,}0x4{,}0\ m$
Uniform load	$10\ kN/m^2$
Longitudinal Modulus of Elasticity	$E = 21000\ MPa$
Transverse Modulus of Elasticity	$G = 0{,}4\ E\ \therefore\ G = 8400\ MPa$
Bending moment of inertia	$I\ =\ bh^3/(12 \cdot (1 - \nu^2))$
Torsional moment of inertia	$J\ =\ 2I$

For this purpose, meshes with three spacings were used, 80x80 cm, 36x36 cm and 19x19 cm (Fig. 1).

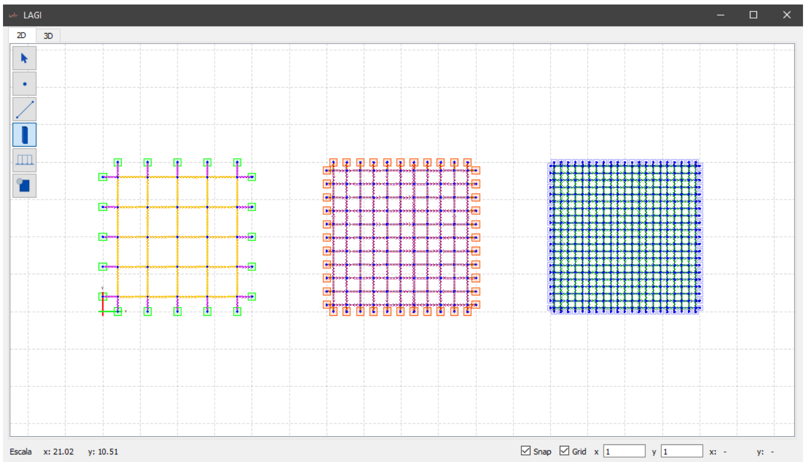

Figure 1. Meshes modeled in the user interface

The bending moment and vertical displacement results obtained were then compared with those obtained by Stramandinoli [7] for the same example, using the Bares' Tables, which are based on the theory of thin plates. The comparison of the results obtained is shown in Tab. 2.

Tabela 2. Bending moments and deflections obtained (J/I = 2)

Type	Bending (kN.m/m)	Deflection (cm)
80x80 cm	8,302	0,807
36x36 cm	7,359	0,721
19x19 cm	7,034	0,675
Bares' Tables	7,05	0,57

It can be observed that, as the mesh is refined, the differences obtained between the values found by the simulator and the results from the Bares' Tables are increasingly smaller, reaching only -0.2% (smaller value) for the 19x19 cm mesh, which is considered satisfactory. Regarding the displacements, significant differences are noticed even for the most refined mesh. This is due to the fact that the parameter J=2I adopted, in this case, was not adequate for displacement analysis. When processing the same structure using J=2.8I, the differences in displacement became satisfactory, reaching 2.46% (the largest value) for the most refined mesh, as can be seen in Tab. 3.

Table 3. Deflections obtained (J/I = 2.8)

Type	Deflection (cm)	Difference (%)
Bares's Table	0,57	-
80x80 cm	0,732	28,49
36x36 cm	0,63	10,53
19x19 cm	0,585	2,46

Fig. 2 presents, respectively, the bending moment and vertical displacement diagrams obtained by the developed tool for the cases in which the differences obtained were the smallest.

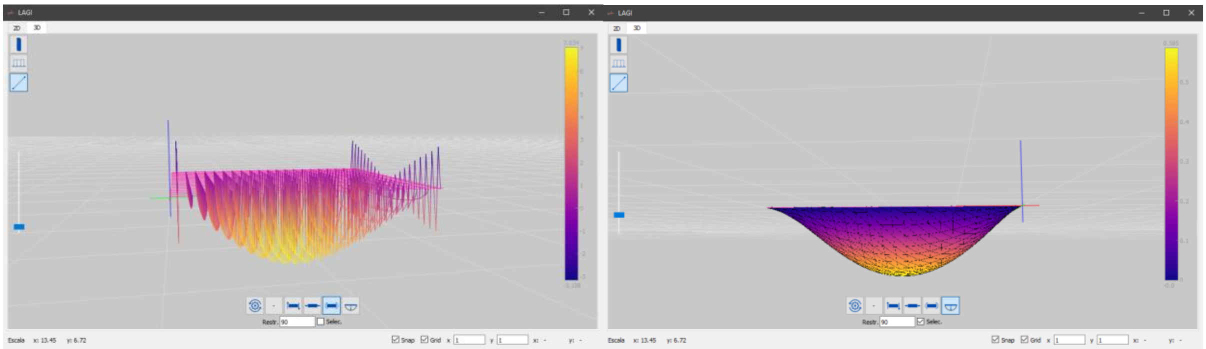

Figura 2. Diagramas de momentos fletores e deslocamentos verticais obtidos

3.3 Análise da influência das rigidezes à flexão de pilares

P To analyze the influence of the columns over the horizontal system, the same slab will be observed under a series of different column combinations. A square slab, simply supported at the corners, with dimensions of 5x5 m, extracted from Barboza (1992), was used. The border beams have cross section dimensions of 12x50 cm and will have their bending stiffness considered, as well as their torsional stiffness, the latter being reduced to 36% of its calculated value. The spacing adopted for the mesh was 35.7x35.7 cm. The remaining data concerning the slab used are shown in Tab 4.

Table 4. Characteristics of the second slab

Longitudinal Modulus of Elasticity	$Ec = 30 \cdot 10^6 \ kN/m^2$
Poisson coefficient	$\nu = 0{,}2$
Transverse Modulus of Elasticity	$G = Ec/2{,}4$
Bending moment of inertia (edge beams)	$J = \dfrac{3b^3h^3}{10(b^2+h^2)}$
Momento de inércia à torção (slab)	$J = 2I$

For the cases in which the stiffness of the columns was considered, columns with a height of 300cm following the combinations below were used:

0) No consideration of column stiffness;
1) All columns with 20x20 cm and Fck of 20MPa, with consideration of column stiffness;
2) All columns with 20x20 cm and Fck 30Mpa, with column stiffness considered;
3) All columns with 30x30 cm and Fck of 20Mpa, considering the stiffness of the columns and
4) All columns with 30x30 cm and Fck 30Mpa, with column stiffness considered.

The bending, torsion, shear and deflection results obtained for configurations 0-4 are shown in Tab. 5.

Table 5. Resuts of configurations 0-4

Configuration	Bending (kN.m/m)	Torsion (kN.m/m)	Shear (kN/m)	Deflection (cm)
Configuration 0)	**10,663**	**6,859**	**15,338**	**1,537**
Configuration 1)	10,189	7,67	15,371	1,377
Configuration 2)	10,121	7,785	15,37	1,355
Configuration 3)	9,623	8,637	15,36	1,187
Configuration 4)	9,564	8,738	15,36	1,167

Table 5 shows that the inclusion of column bending stiffness directly affects bending and torsional forces, as well as deflection: while the values for deflection and bending moment are reduced, the values for torsional effects are increased. On the other hand, there was no significant variation in the shear force values. The intensity of this influence, negative or positive, increases proportionally to the stiffness of the columns involved. In this sense, it can be observed the fact that the increase in stiffness through additions to the characteristic strength of concrete presents less significant changes in relation to the increase in the dimensions of the cross section of the column. The Fig. 3 shows the bending forces diagrams obtained for the configurations 0) and 4), respectively.

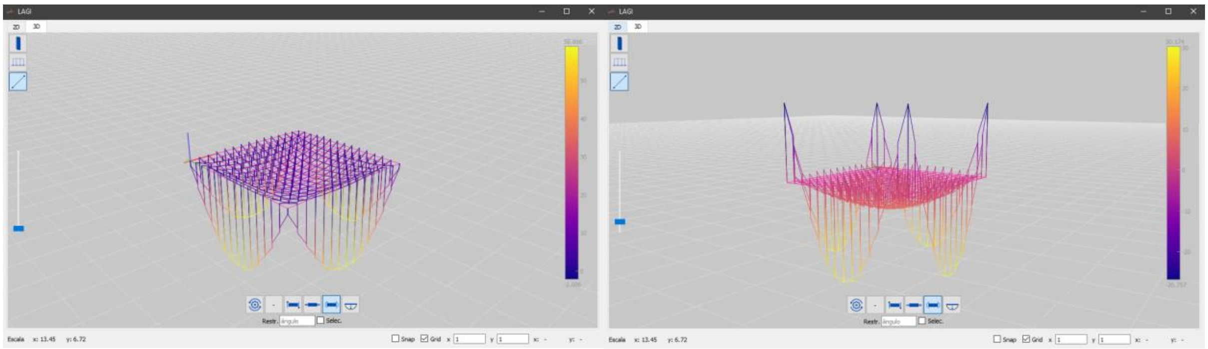

Figure 3. Bending forces diagrams obtained for configurations 0) and 4)

4 Conclusions

It is evident the effectiveness of the Direct Stiffness Method as a means of applying the Grid Analogy, both for the simplicity of its formulation and computational implementation, and for the consistency of the results obtained. The examples presented point to the success of creating a tool for structural analysis of slabs with the use of the grid analogy. In relation to the consideration of the influence of the columns stiffness on the horizontal system, the effects of reducing the maximum bending and deflection, as well as increasing the torsion, were found to be coherent with what structural mechanics suggests.

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