XLIII CILAMCE
21-25, November, 2022
FOZ DO IGUAÇU - PR - BRAZIL

NUMERICAL STUDY OF A MULTI-DEGREE OF FREEDOM STRUCTURE UNDER THE INFLUENCE OF WIND EXCITATION.

Géder G. L. Cunha[1],Marcus Varanis [2],Murilo Cesar Filipus [1] , Clivaldo de Oliveira [1],José Manoel Balthazar [3]

[1]*FAEN,Federal University of Grande Dourados*
gedergabriel@gmail.com
[2] *Instituto de Física, Universidade Federal de Mato Grosso do Sul*
Marcus.varanis@ufms.br
[3] *Universidade Tecnológica Federal do Paraná*

Abstract. Nonlinearity is always present in the most diverse real structures, so the more reliable the computational numerical modeling of these structures, the more one must consider the nonlinearities present. This paper aims to study the nonlinear dynamics of a structure under the action of wind force based on a structure with multiple degrees of freedom (shear building) with the addition of a duffing spring, which introduces nonlinearity to the system, where the external force will be implemented through the wind force applied to one of the floors of the structure. The entire analysis will be performed through the numerical integration of the ordinary differential equations, thus obtaining the response in the time domain, and through transforms, such as the continuous wavelet transform (cwt), extract the signal in the frequency domain, allowing to observe phenomena that are not possible to observe in the time domain, thus having a complete analysis of the system behavior.

Keywords: Shear Building, Wind force, wavelet, Equações Diferencias Ordinárias, Nonlinear.

1 Introduction

The dynamic analysis of mechanisms and structures is one of the most important areas in the education of several engineering disciplines [1–3]. For several of these dynamic analyses, computer simulations are used and implemented in several programming languages. This study has been growing in several areas, from industry to research, mainly with the scientific and computational advances, which made it possible for the study and simulation of the dynamics of mechanisms and structures to become more widely available to the general public. In real structures there will always be nonlinearity associated with their dynamics. With this in mind, nonlinearities must be considered in the analysis of systems in order to make them more similar to reality. This paper studies the nonlinear dynamics of a structure under the action of the wind force applied to a structure with multiple degrees of freedom (shear building) with the addition of a nonlinear spring, which brings nonlinearity to the system, and using standards to add an external force based on wind speed, thus bringing it even closer to a real system. All the analysis will be performed through the integration of the ordinary differential equations (ODE) and using tools such as continuous wavelet transforms (CWT) to transform the signal to the frequency domain with the goal of a complete analysis of the signal.

2 Shear Building model

The Shear building [3–7] is an idealization of a building model for shear strength analysis widely used in studies because it presents an easy interpretation and implementation of ordinary differential equations, also known as ODEs. The model proposed in this paper has 4 floors (3 stories), where the slabs and beams are undeformable and the mass is concentrated only in the slabs, with the first block being the first floor and the last block under the action of an external force (Wind force), seen in Fig. 1.

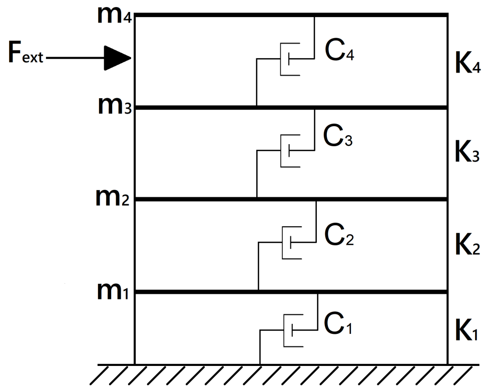


Figure 1. Shear Build Model

The Shear Building Model shown in Fig. 1 is a 4 degree of freedom model, and at this point does not have any nonlinear components. To derive its equations of motion, one can consider each floor of this model as a damped spring mass and solve it using Newton's second law.

Through the dynamic analysis of the Shear building, some characteristics of the model can be noticed, such as the structural damping [3] (as this is a simplified case the viscous damping is used, since it presents simple solutions that satisfy certain applications), besides the fact that the columns exert a restoring force analogous to the force of a spring [1] when its mass moves from its equilibrium position. For a first approach, the columns will be treated as linear springs following Hooke's law ($F = -kx$), thus obtaining eq. (1):

$$m\ddot{x} = \vec{F}(t) - kx - c\dot{x}. \tag{1}$$

The negative sign present in the spring force and damping is due to the fact that they act contrary to the displacement oriented in the positive direction adopted, being that the direction and analysis of horizontal movement. As the model presents one more degree of freedom, the slabs are influenced by the structural damping and the spring force of the adjacent blocks, so the equation for the first floor can be rewritten in the form of eq. (2):

$$m_1\ddot{x}_1 = -kx_1 - c\dot{x}_1 + k_1(x_2 - x_1) + c_1(\dot{x}_2 - \dot{x}_1). \tag{2}$$

Successively, the procedure is applied to the four masses taking into account their couplings, thus a system of 4 differential equations of second order is obtained, where the external force is concentrated on the last floor as shown in Fig. 1, so eq. (3) is obtained:

$$\begin{cases} m_1\ddot{x}_1 = -k_1x_1 - c_1\dot{x}_1 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1). \\ m_2\ddot{x}_2 = -k_2(x_2 - x_1) - c_2(\dot{x}_2 - \dot{x}_1) + k_3(x_3 - x_2) + c_3(\dot{x}_3 - \dot{x}_2). \\ m_3\ddot{x}_3 = -k_3(x_3 - x_2) - c_3(\dot{x}_3 - \dot{x}_2) + c_4(\dot{x}_4 - \dot{x}_3) + k_4(x_4 - x_3). \\ m_4\ddot{x}_4 = -k_4(x_4 - x_3) - c_4(\dot{x}_4 - \dot{x}_3) + F_w\cos(\omega t). \end{cases} \tag{3}$$

To numerically simulate this system of ODE's that govern the behavior of the Shear Building it is necessary to transform the second order equations into a system of first order equations. The transformation of the four second-order ODEs into eight first-order ODEs will be done according to eq. (4):

$$\begin{cases} \dot{x}_1 = v_1. \\ \dot{v}_1 = \ddot{x}_1 = \frac{-k_1x_1 - c_1v_1 + k_2(x_2 - x_1) + c_2(v2 - v1)}{m_1}. \\ \dot{x}_2 = v_2. \\ \dot{v}_2 = \ddot{x}_2 = \frac{-k_2(x_2 - x_1) - c_2(v_2 - v_1) + k_3(x_3 - x_2) + c_3(v_3 - v_2)}{m_2}. \\ \dot{x}_3 = v_3. \\ \dot{v}_3 = \ddot{x}_3 = \frac{-k_3(x_3 - x_2) - c_3(v_3 - v_2) + c_4(v_4 - v_3) + k_4(x_4 - x_3)}{m_3}. \\ \dot{x}_4 = v_4. \\ \dot{v}_4 = \ddot{x}_4 = \frac{-k_4(x_4 - x_3) - c_4(v_4 - v_3) + F_w\cos(\omega t)}{m_4}. \end{cases} \tag{4}$$

In the nonlinear model a duffing spring will be introduced in the third story of the structure of Fig. 2. Nonlinear springs have different values of k depending on the applied force [8], being necessary to replace the elastic constant k by two constants β and α, but not in a trivial way because the α parameter is interacting with the position term raised to the cube as shown in the equation of the duffing spring force in eq. (5).

$$F = -\beta x - \alpha x^3. \tag{5}$$

Therefore, after implementing the duffing spring force in the system and developing the equation, results the eq. (6).

$$\begin{cases} \dot{x}_1 = v_1. \\ \dot{v}_1 = \ddot{x}_1 = \frac{-k_1 x_1 - c_1 v_1 + k_2(x_2 - x_1) + c_2(v2 - v1)}{m_1}. \\ \dot{x}_2 = v_2. \\ \dot{v}_2 = \ddot{x}_2 = \frac{-k_2(x_2 - x_1) - c_2(v_2 - v_1) + \beta(x_3 - x_2) + \alpha({x_3}^3 - {x_2}^3) + c_3(v_3 - v_2)}{m_2}. \\ \dot{x}_3 = v_3. \\ \dot{v}_3 = \ddot{x}_3 = \frac{-\beta(x_3 - x_2) - \alpha({x_3}^3 - {x_2}^3) - c_3(v_3 - v_2) + c_4(v_4 - v_3) + k_4(x_4 - x_3)}{m_3}. \\ \dot{x}_4 = v_4. \\ \dot{v}_4 = \ddot{x}_4 = \frac{-k_4(x_4 - x_3) - c_4(v_4 - v_3) + F_w \cos(\omega t)}{m_4}. \end{cases} \tag{6}$$

3 Wind force

For the study of a model system with an external force, the wind will be used as excitation, acting as a normal force to the surface of the structure. In order to make the model more faithful to reality, specific standards [9] were used to calculate the wind force intensity in several types of structures and different regions, therefore, the wind force F_w acting on a structure or a structural component can be determined directly using the expression below:

$$F_w = c_s c_d \cdot c_f \cdot q_p(z_e) \cdot A_{ref}. \tag{7}$$

where:

$c_s c_d$ = Structural factor, which takes into account structure factors such as dimensions (width, height), geometry, and also dependent on turbulence factors all described in the standard.

c_f = Force coefficient for the structure or structural element, which considers the geometry where there is contact with the wind, such as rectangular or polygonal normal or tangential to the surface, all described in the standard.

$q_p(z_e)$ = Peak velocity pressure, which depends on other factors such as air density, average height of the structure, basic wind speed and other mathematical expressions all described in the standard.

A_{ref} = Wind contact area with the structure surface.

All calculations referring to the wind force were simplified using the most favorable conditions of the structure and environment to make the case simpler, thus not being necessary to use more elaborate methods as also described in the standard, because the objective of this work is the analysis of the structure with an external force acting and not an in-depth analysis of the behavior of the wind and its force acting on structures.

4 Results

The results were obtained by numerical integration of the systems of eqs. (4) and (6), using the fourth order Runge-Kutta method [10]. All analyses in time and frequency were obtained and plotted using the scientific libraries NumPy, SciPy and Matplolib. The results were divided into two topics. In the first, a wind force compatible with reality was implemented seeking to simulate and analyze possible conditions in both a linear spring and a duffing spring. After that, in the second topic, the amplitude of the external force is increased seeking to observe emerging nonlinear phenomena due to the implementation of the nonlinear spring.

4.1 Small amplitudes

For the analysis of small amplitudes of the system we use as structural parameters the values compatible with reality, taken from the book [11], to observe how the structure behaves under the influence of such values, being the wind speed 56 m/s, where the force under the structure is calculated through the equation described in section 3, the spring constant and mass for the first floor $K_1 = 31522823[N/m]$ $M_1 = 350254[kg]$, first floor $K_2 = 21015215[kg]$ $M_2 = 262691[kg]$, second floor $K_3 = 10507608[kg]$ $M_3 = 175127[kg]$, third floor $K_4 = 5253804[kg]$ $M_4 = 87564[kg]$ and for the duffing spring $\alpha = 1260912.96[N/m]$ e $\beta = 10507608[N/m]$, with the structural damping calculated using the system's mass and stiffness matrix based on 2 factors that vary for each structure [3] (the viscous damping ratios) being 0.00009M + 0.0001K.

When analyzing both the linear and the nonlinear system (Fig. 2) and comparing their temporal responses, an interesting phenomenon is observed where the responses remain virtually the same, as observed in Fig. 2 (a), (b), (c) e (d), with only minor variations in amplitude, a phenomenon that is present in all floors.

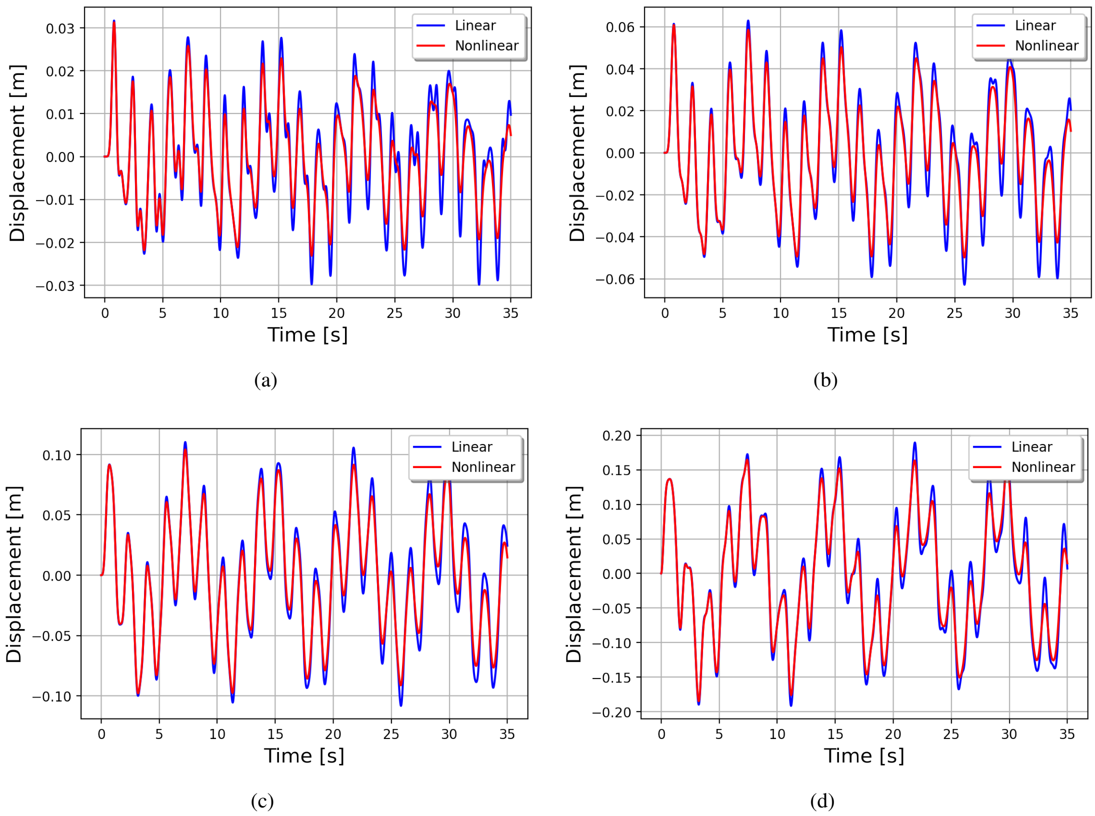


Figure 2. a) Displacement x Time (Ground), b) Displacement x Time (Fist floor) c) Displacement x Time (Second floor), d) Displacement x Time (Third floor)

Another very important tool for the analysis of nonlinear systems is the continuous wavelet transform (CWT)[12, 13] which returns the signal in the frequency domain, thus allowing us to observe important phenomena not visible in the time domain, for example, the phenomenon of frequency scattering [13], very present in nonlinear dynamics, being a great indicator of nonlinear systems. However, this phenomenon does not occur in CWT for the nonlinear case of Fig. 3, and the comparisons between the linear and nonlinear system, as found for the signal in the time domain, are virtually identical with slight differences.

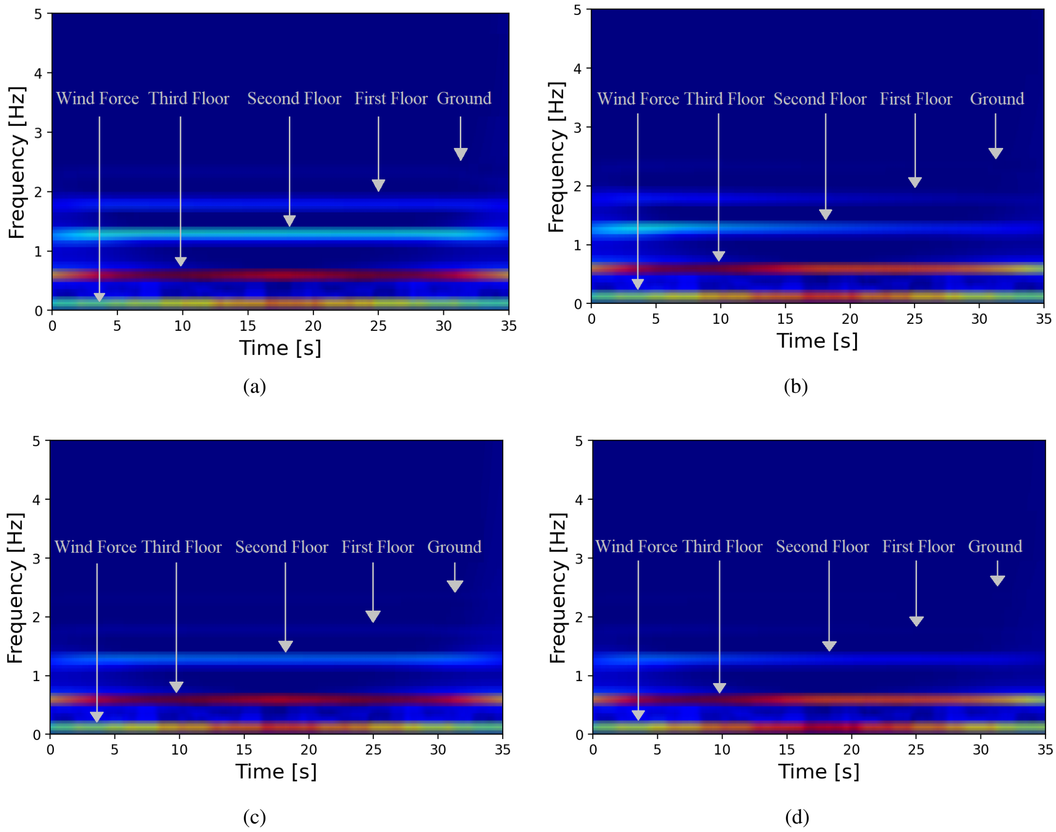


Figure 3. a) CWT linear (Ground) b) CWT nonlinear (Ground) c) CWT linear (Fist floor) d) CWT nonlinear (Fist floor)

4.2 High amplitudes

In this topic, only the amplitude of the external force was increased to an unrealistic value of 400 m/s, so that the nonlinear effects presented by the Duffing spring could be observed when it is in the nonlinear regime, as can be seen in Fig. 4, which from a certain displacement exhibits a nonlinear restoring force.

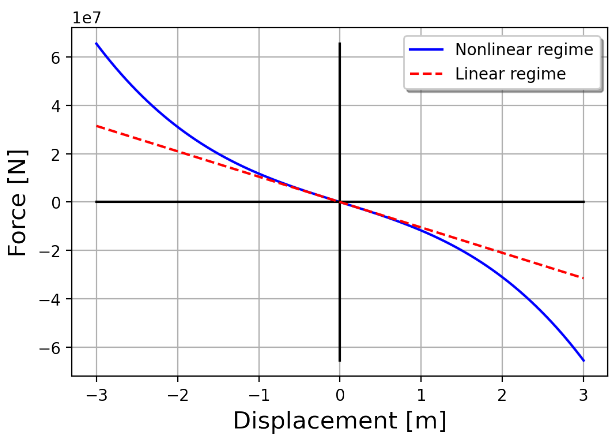
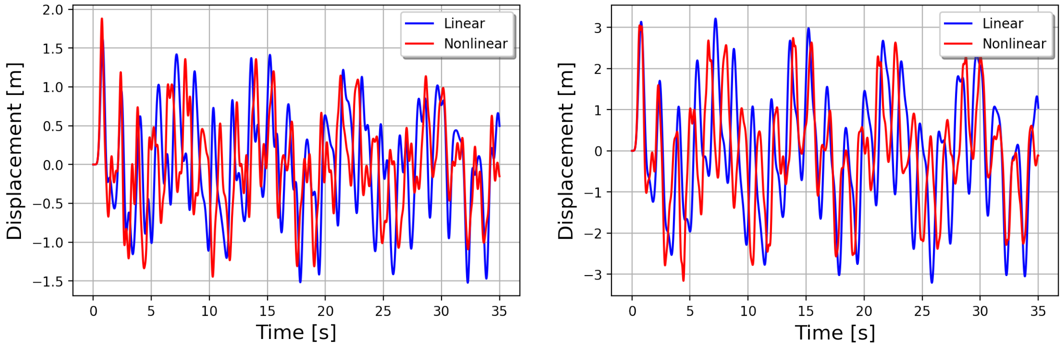


Figure 4. spring restoring force

In this topic it will only be shown the temporal response of the ground and first floor which are the most affected by nonlinearity. Unlike the first topic, this time one can observe some divergences in the temporal and frequency domain response of the linear system with the nonlinear one, as can be seen in Fig. (5).



(a) (b)

Figure 5. a) Displacement x Time (Ground), b) Displacement x Time (Fist floor)

This divergence presented in time corresponds to a scattering in the frequencies presented in the CWT. Figs. 6 (a) e (c)) again shows the same natural frequencies and the external force, but comparing with Figs. 6 (b) e (d), it is notable the scattering of frequencies that occurs in the region of 2 to 4 [Hz], this phenomenon as already said an indication of the nonlinearity of the system, thus proving as stated that for large amplitudes (high wind speed) it becomes more apparent the nonlinear behavior of the system.

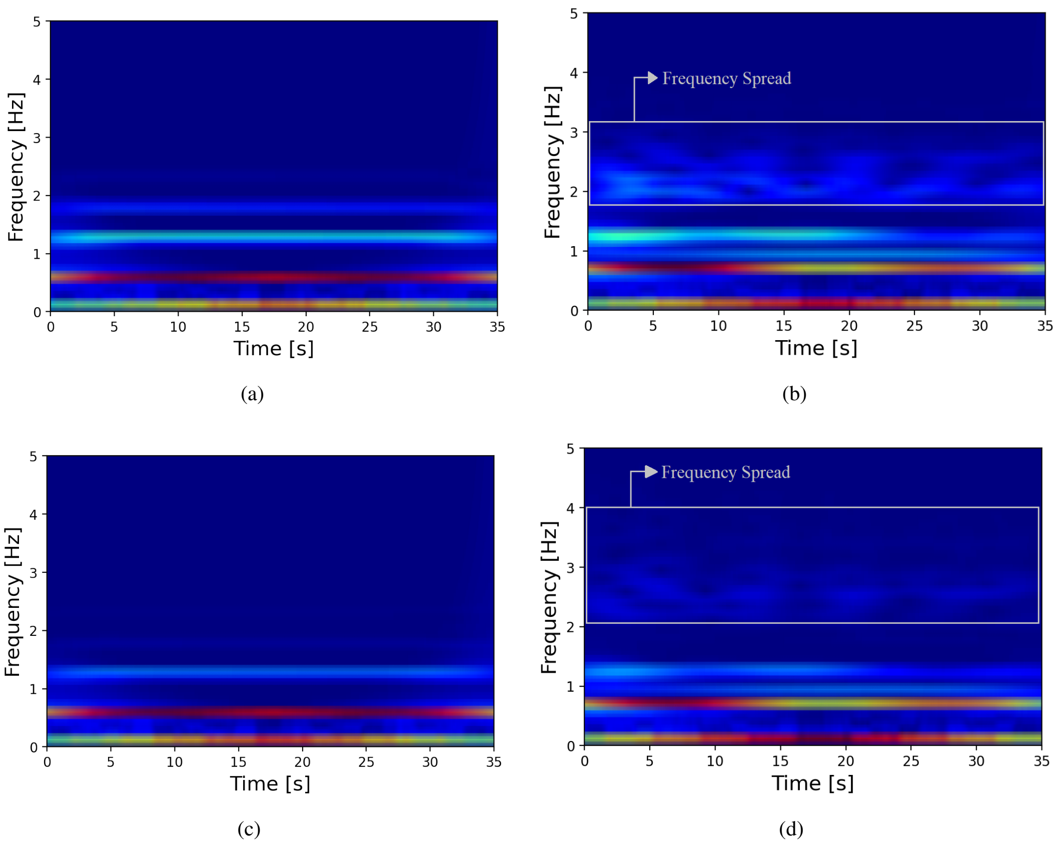


Figure 6. a) CWT linear (Ground) b) CWT nonlinear (Ground) c) CWT linear (Fist floor) d) CWT nonlinear (Fist floor)

5 Conclusion

As observed, the phenomena described in the section where observe the indifference between the linear and nonlinear model 4.1 can be explained by the fact that it is under a relatively small wind force in relation to its mass and stiffness, which will cause small displacements as can be seen in Fig. 2. When analyzing Fig. 4 4 notice a range between -1 and 1 of the x-axis where the duffing spring behaves almost like a linear spring, thus being able to explain the fact that the linear regime is so similar to the nonlinear one, because until a certain displacement the nonlinear effects are not apparent and as it is a realistic model of both stiffness and wind speed it is necessary that there is no large displacement of the structure, being designed to meet these requirements, because it would be catastrophic a structure of such magnitude oscillating with large amplitudes. Therefore, it ends up being necessary to increase the wind speed to make the nonlinearities of the system more noticeable, as described in section 4.2, where the wind speed is significantly increased, which consequently leads to a notorious nonlinear phenomenon due to the external force, as seen in Fig. 6. Thus, in real structures under the influence of a relatively small wind force (low displacement amplitude), the linear and nonlinear system dynamics are practically indifferent.

Acknowledgements. The authors acknowledge support by CNPq (Brazilian research funding agency).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] Taylor, John R. Classical mechanics. Bookman Publisher, 2013.

[2] Hayek, Sabih I. "Mechanical vibration and damping." Digital Encyclopedia of Applied Physics (2003).

[3] Soriano, Humberto Lima. Introduction to the dynamics of structures. Rio de Janeiro: Elsevier, 2014.

[4] Moghaddam, H., and R. Karami Mohammadi. "Ductility reduction factor of MDOF shear-building structures." Journal of earthquake engineering 5.03 (2001): 425-440.

[5] Yuan, Ping, Zhifeng Wu, and Xingrui Ma. "Estimated mass and stiffness matrices of shear building from modal test data." Earthquake engineering & structural dynamics 27.5 (1998): 415-421.

[6] Moon, Francis C. Chaotic and fractal dynamics: introduction for applied scientists and engineers. John Wiley & Sons, 2008.

[7] Alhan, Cenk, and Melih Sürmeli. "Shear building representations of seismically isolated buildings." Bulletin of Earthquake Engineering 9.5 (2011): 1643-1671.

[8] Aranha, Norberto, et al. "Hooke's law and nonlinear springs, a case study." Brazilian Journal of Physics Teaching 38 (2016).

[9] Standard, British. "Eurocode 1: Actions on structures—." (2006).

[10] Wang, Yong, Kasey J. Hill, and Erin C. Foley. "Computer programming with Python for industrial and systems engineers: Perspectives from an instructor and students." Computer Applications in Engineering Education 25.5 (2017): 800-811.

[11] Chopra, Anil K. Dynamics of structures. Pearson Education India, 2007.

[12] Weeks, Michael. Digital Signal Processing Using MATLAB & Wavelets added for testing purpose. Jones & Bartlett Publishers, 2010.

[13] Abreu,F.L and Marcus Varanis and Pedro Augusto Beck and Clivaldo Oliveira and Balthazar, Jos Manoel, Remarks on nonlinear dynamics of a suspension bridge model,2021.