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OPTIMIZATION OF A PRESTRESSED CONCRETE WIND TURBINE TOWER USING DIFFERENTIAL EVOLUTION

Jonatas M. F. C. Martins[1], A. Macário C. de Melo[1], Evandro Parente Junior[1]

[1]*Laboratório de Mecânica Computacional e Visualização (LMCV), Departamente de Engenharia Estrutural e Construção Civil, Universidade Federal do Ceará*
Campus do Pici, Bloco 728, 60440-900, Ceará, Brazil
jonatascacau@alu.ufc.br, macario@ufc.br, evandro@ufc.br

Abstract. Optimization procedures are being increasingly used in wind tower structure design in order to provide more efficiency. In the face of recent advances, towers are getting bigger in order to perform better by providing better winds for the turbines. Thus, prestressed concrete towers emerge as an excellent solution. This work aims to apply and evaluate the Differential Evolution algorithm in order to reduce costs and improve the performance of prestressed wind towers. The Finite Element Method is used for structural analysis. The algorithm is evaluated in terms of accuracy and computational efficiency.

Keywords: Wind Turbine Towers, Prestressed Concrete, Differential Evolution, Finite Elements Method.

1 Introduction

Interest in the use of clean alternative energy sources has grown considerably in recent years and that comes from an increasing concern with the longevity of life on the planet. Even with the high development of turbine technology, it is necessary to position them at higher heights so that they have better wind conditions. Thus, the challenge for the sector has been to devise structural solutions so that the towers are increasingly taller and at the lowest possible cost, in order to increase the competitiveness of wind energy compared to others. Hau [1] comments that the cost of the towers can represent up to 30% of the total costs in the implementation of wind energy. Therefore, performing a safe and efficient tower design is a very important aspect.

In this context, optimization techniques can be used to reduce the cost of the tower structure. In structural optimization, due to its ability to avoid local minima, it is common to use heuristic algorithms over mathematical programming methods. Regarding prestressed concrete towers, several works can be found that use these methods.

Bai et al. [2] use the Particle Swarm Optimization (PSO) algorithm associated with the Finite Element Method (FEM) to determine the lowest construction cost, obtaining good results. Al-Kaimakchi et al. [3] used Genetic Algorithms (GA) in the multi-objective optimization of steel, prestressed concrete and hybrid towers. The results indicate that prestressed and hybrid concrete towers have a lower cost than steel towers for heights greater than 80m, and that this difference increases with height reaching 30% at 150m. Melo [4] also uses GA associated with FEM to reduce material costs for towers with one and two segments, obtaining good results.

Another commonly used heuristic algorithm proposed to deal with a continuous design space is Differential Evolution (DE) [5]. This heuristic has been applied in the optimization of several mathematical and structural problems obtaining excellent results. Works that focus on comparing PSO and DE generally find that DE is superior both in terms of precision and speed of convergence [6, 7]. However, applications in prestressed towers have not yet been explored.

Therefore, this article aims to apply and evaluate DE in the optimization of prestressed wind towers. For this, the implementation of analysis made by Melo [4] in the Matlab software will be used, associated with optimization using the Biologically Inspired Optimization System (BIOS) software [8].

This article is organized as follows. In Section 2 the prestressed concrete towers are described in more detail. In Section 3 the focus is on the analysis method used. In Section 4 the optimization model is presented together with the DE. The results are presented in Section 5, and in Section 6 the main conclusions are discussed.

2 Prestressed Wind Tower

In a simplified way, the main components of a wind turbine are the tower, rotor, nacelle, and blades. The tubular towers today are the most common, due to the speed of execution and, mainly, for the stiffness that they provide. Generally, the wind turbines are built in packs and as commented before, the tower has a relevance in the final price of the wind turbine. Therefore, reducing costs becomes even more relevant in the face of large-scale production of towers.

As for the materials used, they can be steel, concrete or hybrid. The most adopted solution for many years were the metal towers formed by hollow cone truncated segments. However, the efficiency of this alternative decreases when the tower height increases. To circumvent this problem, prestressed concrete towers have been adopted. This solution has the following advantages: lower maintenance cost; greater flexibility in the construction process, using sliding forms or lifting segments; better dynamic response, which provides less vibration and material fatigue; and better transport possibilities, using prefabricated segments made in plants or on the wind farm's own yard. In addition, the lower environmental impact of the material can also be highlighted. In addition to being recycled, the concrete has a lower CO_2 content incorporated than a conventional steel tower.

In terms of prestressing, it can be of the bonded (internal) or unbonded (internal or external) type, or a combination of these, for example using bonded pre-tension on prefabricated parts and unbonded post-tension on the tower as a all. According to Melo [4], the choice of external prestressing has the following advantages: use of thinner concrete thicknesses, ease of installing tendons in different stages of construction, ease of inspection and tendon replacement, greater tolerance to fatigue under dynamic loads, lower friction losses and simpler demolition procedure.

3 Analysis Model

The Finite Element Method (FEM) is a numerical method, widely used to solve stress and strain analysis problems. Due to its versatility and ability to refine the results, this method is the most used in the structural analysis of wind towers [4]. Different analysis models can be used, from simple frame element to complex shell element models.

The more complex and refined the structural model, more time-consuming is the analysis process. In the case of optimization, mainly in the application of heuristic algorithms, the efficiency of the analysis model is very important in view of the need to evaluate several candidate solutions. In this sense, Barroso Filho [9] evaluated different models of analysis of reinforced concrete towers using the FEM, obtaining differences of less than 6% between the results of models with beam and shell elements. Melo [4] carried out a comparative study in the nonlinear analysis of towers with externally prestressed tubular section in which the model using lower cost plane frame elements provided results that differed around 10% from the results of a model that used quadratic Reissner-Mindlin curved shell elements.

The formulation used in this work for the analysis of the externally prestressed concrete tower is presented by Melo [4] and implemented in the MATLAB software, being an adaptation of the Alves model [10]. The tower is simulated by Euler-Bernoulli planar frame elements for large displacements and moderate rotations, and each prestressing tendon is considered as a single element, associated with one or more frame elements.

The following hypotheses were considered regarding the prestressing tendons: all tendons are straight, vertical and evenly distributed on a concentric circle to the tower; all tendons have the same properties; are anchored at the base and top of the tower; in all sections there is double symmetry of the tendons in relation to the parallel and transversal axes to the direction of the wind loads; on each of these axes there are two diametrically opposed tendons; and all tendons are prestressed simultaneously.

Therefore, the number of tendons will be a quarter multiple and disregarding the torsion effects, the tendons arranged symmetrically in relation to the axis parallel to the wind direction have identical behavior. Thus, the spatial configuration can be simplified to a flat configuration as illustrated in Fig. 1, where the tendon areas correspond to the sum of the corresponding tendon areas Ap. The simplification for the flat model was what made it possible to use the plane frame elements.

The prestressing is modeled as resistant elements. The latter allows the interaction between the tendons and the concrete to be carried out and, therefore, the tendon contributes to the vector of internal loads and to the stiffness matrix of the structure. Thus, the analysis becomes more complete and robust [10].

In the case of unbonded tendons, as is done in this work, the modeling requires a more complex formulation since there is no compatibility of deformations between the materials by section, only the displacements in the anchorage are compatible. Aves [10] presents a formulation, also used by Melo [4], that the tendon deformation is determined considering the displacements of the entire structure due to the lack of strain compatibility between the tendon and the concrete. Thus, the displacements of each tendon segment are determined from the displacements

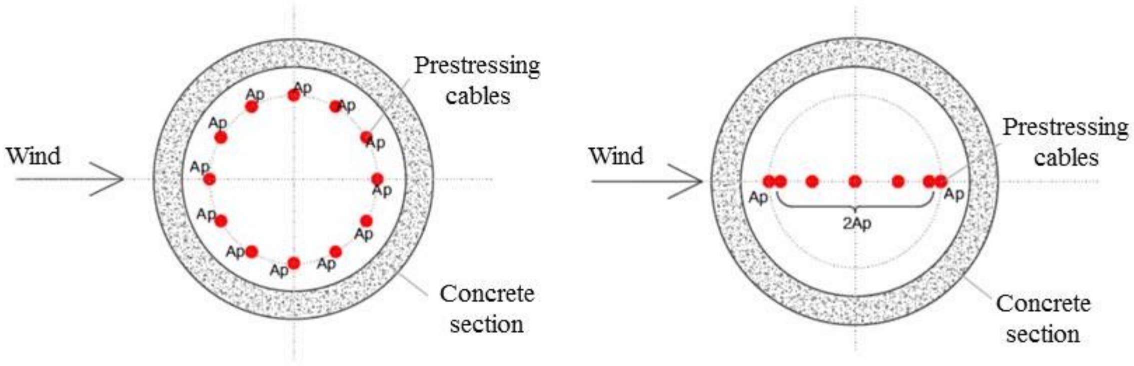

Figure 1. Hypothetical distribution of prestressing tendons in the cross section (Source: Adapted Melo [4])

presented by the plane frame elements associated with the tendon segment. Since friction is neglected, the strain along the tendon is uniform and therefore the tension in the tendon is constant.

For each tower design the analysis is run twice, once to check the Service Limit State (SLS) and once to check the Ultimate Limit State (ULS). In the structural analysis of the towers, the following steps are carried out in sequence: application of self-weight as external loading; application of prestressing; application of other external loads (turbine and wind forces). Due to the assumptions adopted, in the prestressing stage, it can be assumed that in the prestressing application stage, the tower tendons model can be replaced by a model with a single equivalent tendon, centered on the tower axis, with the same properties as the tendons of the original model.

In this application, there is no need to know the equilibrium path after the limit point, as the tower already violates the constraints imposed in the optimization. Therefore, the Load Control Method is used to solve the non-linear equilibrium equations. At each loading step, the equilibrium configuration of the structure for the external loading level is sought. When equilibrium is not reached, the structure loses stability and the analysis stops. This situation occurs when the structure has insufficient stiffness to resist the external loads.

4 Optimization Model

The optimization model of this work illustrated in Fig. 2 aims to minimize the costs of a prestressed concrete tower of a certain height H and for an outside diameter of the top D_{top}. The latter is fixed according to the nacelle adapter ring, which depends on the turbine used. To build the tower, it is necessary to define its dimensions and the number of prestressing tendons. Thus, the design variables of the model are:

$$\mathbf{x} = [D_1, D_2, ..., D_n, t_1, t_2, ..., t_n, t_{top}, N_c] \tag{1}$$

where n is the number of tower segments, D_1, D_2, …, D_n and t_1, t_2, …, t_n are the outer diameters and the base thicknesses of each segment, t_{top} is the thickness of the top of the tower, and N_c is the number of prestressing tendons. Due to the analysis model considered, the number of tendons is a discrete variable of a multiple of 4. The rest of the variables will be considered continuous. The objective function can then be calculated by:

$$f(\mathbf{x}) = V_c(\mathbf{x})C_c + M_{rs}(\mathbf{x})C_{rs} + M_{ps}(\mathbf{x})C_{ps} \tag{2}$$

where V_c is the volume of concrete, C_c the cost of concrete, S_r is the mass of reinforcement steel, C_r is the cost of reinforcement steel, S_p is the mass of prestressing steel, C_{sa} is the cost of prestressing steel and **x** the vector of the design variables.

Furthermore, the model has constraints regarding geometry, SLS and ULS. As for geometry, in order to avoid increasing diameters and thicknesses from the base to the top of the tower. A simplified way to reduce the risk of local buckling is to establish that the thickness/diameter ratios must respect a minimum value. Thus, we have the following normalized constraints:

$$D_i - D_{i-1} \leq 0,\ i = 2, 3, ..., n; \quad t_i - t_{i-1} \leq 0,\ i = 2, 3, ..., n; \quad \left(\frac{t}{D}\right)_{min} - \frac{t_i}{D_i} \leq 0,\ i = 1, 2, ..., n \tag{3}$$

In the case of the SLS, the fundamental frequency of the tower must be within a safe interval ($f_{inf} < f < f_{sup}$) so that resonance does not occur. Regarding the maximum lateral displacement at the top of the tower $\Delta_{top,ELS}$ in

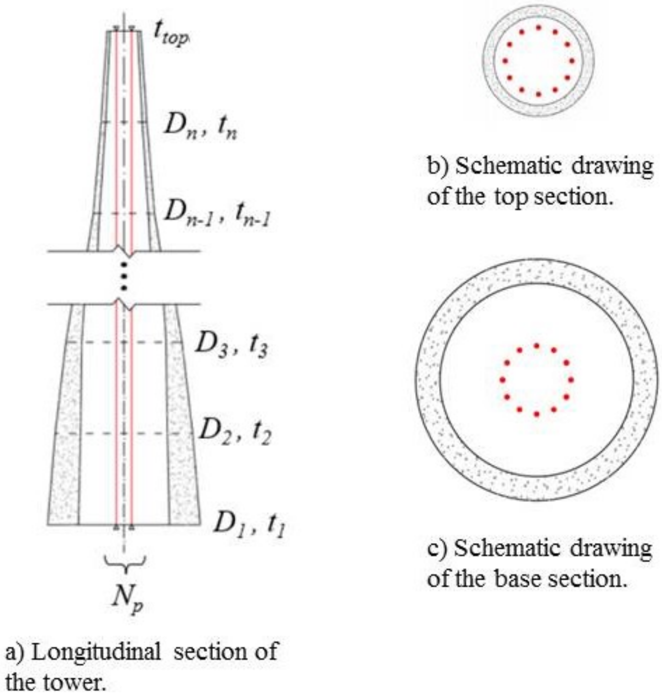

Figure 2. Design variables illustration (Source: Adapted Melo [4])

order to guarantee the limit state of excessive deformation must not exceed the established limit Δ_{lim}. Verification of the decompression limit state consists of ensuring that there is no tensile stress in the concrete. Thus, the stress in the concrete in the least compressed fiber on the windward face $\sigma_{c,max,ELS}$ should result in a value less than or equal to zero. Another check is regarding the limit state of excessive compression in which the stress in the concrete in the most compressed fiber on the leeward face $\sigma_{c,min,ELS}$ which must not exceed, in modulus, the established limit stress $\sigma_{c,lim,CE}$. These constraints are written as:

$$1-\frac{f}{finf} \leq 0; \quad \frac{f}{fsup}-1 \leq 0; \quad \frac{\Delta_{top,ELS}}{\Delta_{lim}}-1 \leq 0; \quad \frac{\sigma_{c,max,ELS}+fck}{fck}-1 \leq 0; \quad \frac{|\sigma_{c,min,ELS}|}{\sigma_{c,lim,CE}}-1 \leq 0. \quad (4)$$

For the ULS, the stress in the most compressed fiber of the concrete $\sigma_{c,min,ELU}$ must not exceed, in modulus, its Compressive strength $\sigma_{c,lim,ELU}$ and the tensile stress in the most tensioned prestressing tendon (closest to the windward face) $\sigma_{p,max,ELU}$ must not exceed the its tensile strength limits $\sigma_{p,lim,ELU}$. These are written as:

$$\frac{|\sigma_{c,min,ELU}+fck|}{\sigma_{c,lim,ELU}}-1 \leq 0; \quad \frac{\sigma_{p,max,ELU}}{\sigma_{p,lim,ELU}}-1 \leq 0. \quad (5)$$

In order to solve the optimization problem, this work proposes the use of Differential Evolution (DE). This algorithm was initially proposed by Price [5] and is characterized by simplicity in its handling, as it has the population size N_p, the scale factor F and the crossover probability C_r as a control parameter. The algorithm is known as an efficiency and robustness algorithm in the search for a global optimal solution in a continuous domain [6].

DE starts with the random generation of the initial population of N_p members within a defined search domain, each solution being a m-dimensional vector of design variables. The next step is mutation, which simulates an evolving population controlled by the scale factor F. In the original formulation of DE, this operator is given by Price [5]:

$$v_{j,i} = x_{j,r0} + F\left(x_{j,r1} - x_{j,r2}\right) \quad (6)$$

where $x_{j,r0}$, $x_{j,r1}$ and $x_{j,r2}$ are candidate solutions, $r0$, $r1$ and $r2$ are natural numbers chosen at random from the range $[1, N_p]$ and $v_{j,i}$ is the mutated vector. This formulation is known as Rand/1 because the base vector $x_{j,r0}$ is chosen at random and only one difference vector is considered. Among the most diverse proposals made since its initial development, it is worth highlighting the different ways of carrying out the mutation and two of the most used are Best/1 and Current-to-best/1, respectively:

$$v_{j,i} = x_{j,best} + F\left(x_{j,r1} - x_{j,r2}\right) \quad and \quad v_{j,i} = x_{j,i} + F\left(x_{j,best} - x_{j,i}\right) + F\left(x_{j,r1} - x_{j,r2}\right) \quad (7)$$

where $x_{j,best}$ is the best design found during the optimization process.

A binomial crossover is then performed to increase the diversity of the population, controlled by the crossover probability C_r. Furthermore, to ensure that the resulting vector does not duplicate the current individual, the test parameter with randomly chosen dimensional index j_{rand} is taken from the mutant. After evaluating the final resulting population, selection is employed: if the mutant i-th design is worse than the individual i-th in the previous generation, the latter will not be replaced [5]. Thus, the old i-th individual x_i is compared with the current u_i, and the replacement only takes place if u_i shows an improvement over the previous one. These operators are described below:

$$u_{j,i} = \begin{cases} v_{j,i} & if \ (r_{c,j} \leq C_r \quad or \quad j = j_{rand}) \\ x_{j,i} & otherwise \end{cases} \qquad x_{j+i,i} = \begin{cases} u_{j,i} & if \quad f(u_{j,i}) \leq f(x_{j,i}) \\ x_{j,i} & otherwise \end{cases} \tag{8}$$

where $r_{c,j}$ is a random number uniformly distributed between 0 and 1, and $f(u_i)$ and $f(x_i)$ are the objective function values for the current and elderly populations, respectively. It is worth mentioning that for a constrained problem, constraints must be considered here, as a feasible design should not be replaced by an infeasible one. For each generation or iteration, the objective function is evaluated N_p times, trying to find the best design of the population until a stopping criterion is reached, usually related to a maximum number of iterations It_{max} or a maximum number of consecutive iterations without considerable improvement It_{stall}.

Regarding the treatment of constraints, a possible strategy instead of just excluding the unfeasible point, a procedure known as death penalty, is the application of a penalty so that the individual deviates from the optimal value, without being totally discarded. Among the several alternatives, the adaptive penalty presented by Lemonge and Barbosa [11] will be adopted. The penalty coefficients are proportional to the degree of violation of the constraints.

Finally, as it is a stochastic process, some artifice needs to be adopted in order to guarantee that the individuals are within the domain of the variable. Ribeiro et al. [7] perform a procedure in which the variable that violates the imposed limits assumes its own limit value and its speed is reduced by half in the opposite direction. These commented optimization strategies are implemented in the BIOS software from the Laboratory of Computational Mechanics and Visualization (LMCV) and more details can be seen in Barroso et al. [8].

5 Numerical Example

This example was initially proposed by Melo [4] and consists in the cost optimization of a prestressed concrete tower with 100 m and only one segment. Tab. 1 summarizes the characteristics adopted for the turbine considered in the design of the wind tower.

Table 1. Turbine features

Turbine Power	5 MW
Rotor Rotation Speed	11.2 rpm (0.187 Hz)
Operation Frequency Interval	0.250 Hz to 0.485 Hz
Rotor Diameter	128 m
Mass of the rotor-nacelle assembly	480076 kg
Tower height	100 m
IEC class	IIB

Regarding the constitutive model of the materials involved in the tower, the same considerations made by Melo [4] will be adopted for concrete, passive and active reinforcement. The author is based on the work of Gama [12] which presents the necessary considerations regarding the stress-strain diagrams in accordance with the Brazilian standard NBR 6118:2014 and which will be used in the SLS and ULS verifications.

The turbine loads were considered at the top of the tower as concentrated static loads and their values are presented in LaNier [13] for the respective wind models, presented in Tab. 2.

Regarding the wind loading on the tower, these were considered as static and concentrated forces applied to the frame nodes. The forces were calculated according to Melo [4]. The self-weight of each concrete frame element was calculated by multiplying the volume by the specific weight of 25 kN/m3 applied as a nodal force at

Table 2. Actions of the 5 MW turbine (Source: LaNier [13])

Wind Model	Thrust Force (kN)	Moment (kNm)	Axial Compression Force (kN)
EWM50 (ULS)	578.32	28568.27	4709.55
EOG50 (SLS)	1064.72	19337.25	4709.55

the base of the element. The prestressing force is considered by applying an effective tension of 1100 MPa to each tendon. Following the combinations adopted by LaNier [13], for the SLS and for the ULS we have:

$$ELS = 1.0D + 1.0TWL + 1.0W \quad and \quad ELU = 1.2D + 1.35TWL + 1.6W \tag{9}$$

where D is the Dead loads (permanent actions), TWL the Turbine Wind Loads (turbine actions), and W the Wind loads directly on the tower.

The evaluated tower has a hollow truncated cone shape. The parameters related to the DE operation, the limits of the design variables and the values of the parameters adopted in the constraints and objective function are all summarized in Tab. 3. The D_{min} is the value adopted for the diameter at the top of the tower, since this parameter is directly linked to the ring used, depending on the adopted turbine. Furthermore, for the constraints, a tolerance of 10^{-5} was adopted. It is worth noting that due to the analysis process along with the data transfer process between the software, the processing time was very high and only 4 optimizations were performed.

Table 3. Parameters considered in the optimizations

Differentiation	Best/1	t_{min}	0.2 m	Δ_{lim}	2H/400 = 0.5 m
F	0.85	t_{max}	0.8 m	$\sigma_{c,lim,CE}$	0.6fck=30 MPa
C_r	0.8	$N_{c,min}$	8	$\sigma_{c,lim,ELU}$	0.85fck/1.4 = 30.36 MPa
Generation	15	$N_{c,max}$	64	$\sigma_{p,lim,ELU}$	$fptk$/1.15 = 1617.39 MPa
Population	20	$(t/D)_{min}$	0.05	C_c	408.89 R$/m^3
D_{min}	3.6 m	f_{inf}	1.1P = 0.205 Hz	C_{sp}	9.3000 R$/kg
D_{max}	12 m	f_{sup}	2.6P = 0.485 Hz	C_{sa}	9.9751 R$/kg

Tab. 4 presents the results obtained using the DE and the results obtained by Melo [4].In this table, Sr is the success rate. Initially, it is worth noting that here the design variables were considered as continuous the reference solves a discrete problem. The number of tendons is limited to be a multiple of four, as discussed previously. Thus, this variable was rounded to the nearest multiple of a quarter. Furthermore, the cost values (fobj) are divided by the value of 1194169.68 calculated for the tower taken as a reference proposed by LaNier [13].

Table 4. Example results

Ref. Melo [4]				Present (Continuous Solution)				Present (Discrete Solution)			
Dbase	9.6	tbase	0.5	Dbase	9.6899	tbase	0.5089	Dbase	9.6	tbase	0.5
Dtop	3.6	ttop	0.3	Dtop	3.6	ttop	0.2388	Dtop	3.6	ttop	0.25
Nc	24	fobj	0.6653	Nc	16	fobj	0.5612	Nc	20	fobj	0.6021
Sr	80%	c6	-0.068	Sr	50%	c6	-0.036	Sr	-	c6	-0.051
c1	-0.625	c7	-0.499	c1	-0.628	c7	-0.713	c1	-0.625	c7	-0.705
c2	-0.400	c8	0.000	c2	-0.531	c8	-0.005	c2	-0.500	c8	-0.016
c3	-0.042	c9	-0.225	c3	-0.050	c9	-0.493	c3	-0.042	c9	-0.422
c4	-0.667	c10	-0.030	c4	-0.327	c10	-0.231	c4	-0.389	c10	-0.307
c5	-1.206	c11	-0.309	c5	-1.280	c11	-0.0125	c5	-1.244	c11	-0.165

Evaluating the results, it is initially noticed that the optimal value obtained had an improvement of 15.64%. It can be seen that the optimal design obtained in this work has smaller diameters, thicknesses and fewer tendons than the reference. Regarding the constraints, they are named from 1 to 11 as presented. It can be seen that the ones that are most critical in the solution obtained were c3, c6, c8 and c11. In the reference they were the same changing only c11 by c10.

In order to improve the comparison, the solution obtained was evaluated by approximating the variables to the intervals considered by Melo [4]. Thus, the closest possible discrete solution to the obtained solution was considered. To respect the proposedconstraints, the number of tendons increased from 16 to 20. The viable solution obtained provided an improvement of 9.50% in relation to the cost of the reference.

6 Conclusions

The Differential Evolution was evaluated in the optimization of prestressed concrete wind towers and obtained excellent results. Despite comparing it with a solution obtained in the application of a Genetic Algorithm that treated the discrete variables, the results are promising, managing to improve the solutions achieved and keeping the individuals feasible. Using the values obtained with the continuous individuals and treating them as discrete so that they respect the domain proposed by the reference, a solution was obtained that also managed to provide a lower cost. Finally, it is also worth mentioning that the solution used only 300 individuals to reach the optimum and the reference solution needed 3200, showing the efficiency of DE. This reduction on computational cost without impair the efficiency make it very attractive to be used in surrogate based optimizations.

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References

[1] E. Hau. Wind turbines, 2013.

[2] H. Bai, J.-M. Cherfils, Y. Aoues, and D. Lemosse. Optimization of a tall wind turbine tower, 2017.

[3] A. Al-Kaimakchi, S. Jung, M. Rambo-Raddenberry, and G. Amirinia. Optimization of alternative towers for wind turbines in low wind resource regions, 2017.

[4] M. A. Melo. Oatimização de torres protendidas para turbinas eólicas. Master's thesis, Universidade Federal do Ceará, 2020.

[5] K. V. Price, R. M. Storn, and J. A. Lampinen. *Differential Evolution: A pratical approach to Global Optimization*. Springer, 2005.

[6] M. Iwan, R. Akmeliawati, T. Faisal, and H. M. Al-Assadi. Performance comparison of differential evolution and particle swarm optimization in constrained optimization. *Procedia Engineering*, vol. 41, pp. 1323–1328, 2012.

[7] L. G. Ribeiro, M. A. Maia, E. Parente Jr.., and A. M. C. D. Melo. Surrogate based optimization of functionally graded plates using radial basis functions. *Composite Structures*, vol. 252, 2020.

[8] E. S. Barroso, L. G. Ribeiro, M. A. Maia, da I. B. Rocha, E. Parente, and de A. M. Melo. Bios: An object-oriented framework for surrogate-based optimization using bio-inspired algorithms. *Structural and Multidisciplinary Optimization*, vol. 65, n. 7, 2022.

[9] N. F. Barroso Filho. Um estudo comparativo de procedimentos de análise estrutural para torres eólicas de concreto armado. Master's thesis, Universidade Federal do Ceará, 2018.

[10] J. C. Alves. Análsie de vigas com protensão não-aderente submetidas a cargas de curta e longa duração via método dos elementos finitos. Master's thesis, Universidade Federal do Ceará, 2020.

[11] A. C. C. Lemonge and H. J. C. Barbosa. An adaptive penalty scheme for genetic algorithms in structural optimization. *International Journal for Numerical Methods in Engineering*, 2003.

[12] P. V. C. N. d. Gama. Contribuições ao dimensionamento de torres eólicas de concreto. Master's thesis, Escola Politécnica da Universidade de São Paulo, 2015.

[13] M. LaNier. Lwst phase i project conceptual design study: Evaluation of design and construction approaches for economical hybrid steel/concrete wind turbine towers; june 28, 2002 – july 31, 2004, 2005.