XLIII CILAMCE
21-25, November, 2022
FOZ DO IGUAÇU - PR - BRAZIL

A numerical model with an explicit representation of steel fibers for modeling SFRC beams subjected to torsion

Giuliano R. Balsamo, Luis A. G. Bitencourt Jr.

Department of Structural and Geotechnical Engineering, Polytechnic School at the University of São Paulo
Av. Prof. Almeida Prado, trav. do Biênio, 83, University of São Paulo - USP, 05508-070, São Paulo-SP, Brazil.
giuliano.balsamo@usp.br, luis.bitencourt@usp.br

Abstract. A numerical model with a discrete and explicit representation of steel fibers is used for modeling steel fiber reinforced concrete (SFRC) beams subjected to torsion. The numerical model is a combination of a fiber cloud, cement matrix, and the fiber-matrix interaction. It is well known that the addition of steel fibers to concrete increases the torsional and rotational strength, in addition to greater cracking control. In this context, this work aims to assess the capability of the numerical model to simulate experimental tests available in the literature of SFRC beams under torsion with steel fiber rates of 25 kg/m³ and 50 kg/m³. The results demonstrated that the numerical model proposed is appropriate to represent the failure process of beams under torsion and the numerical tool can be very useful in future studies in combination with analytical equations proposed by standard codes.

Keywords: SFRC; beams; torsion; numerical modeling.

1 Introduction

It is well known that the addition of steel fibers in the cement matrix improves the post-cracking behavior of the composite. The improve of toughness and ductility is result of “tension bridges” created by the presence of the fibers crossing the cracks, which avoid an abrupt crack propagation. Thus, the material can be used as secondary or primary reinforcement in structural elements, as it is suggested in some standards and recommendations. Among the models presented in the literature stands out is the designing model presented by *fib* Model Code 2010 [1]. This model adopts as the standard test for obtaining designing parameters, the 3-point bending test, according to EN 14651 [2]. From the force x CMOD curves (Linear measurement of the notch opening) obtained in these tests, residual strengths are evaluated and them used for the definition of the constituive model that describes the tensile behavior of the composite under tension, assuming a rigid-plastic or a linear elastic behavior. Such models are used to obtain the residual strengths in service (f_{Fts}) and ultimate (f_{Ftu}) conditions, calculated based on the values f_{R1} (equivalent CMOD of 0.5 mm) and f_{R3} (equivalent CMOD of 2.5 mm), as shown in eqs. (1) and (2), respectively.

$$f_{Fts} = 0,45 f_{R1}. \quad (1)$$

$$f_{Ftu} = f_{Fts} - \frac{w_u}{CMOD_3}\left(f_{Fts} - 0,5 f_{R3} + 0,2 f_{R1} \right) \geq 0. \quad (2)$$

Thus, SFRC structural members in pure torsion, without passive reinforcement, the following condition is required:

$$\sigma_1 \leq \frac{f_{Ftuk}}{\gamma_F}. \tag{3}$$

However, for SFRC structural members with passive reinforcement, the *fib* model Code 2010 [1] does not present a closed solution, instructing that the structural members must be tested in real scale. Thus, the objective of this work is to use a multiscale model proposed by Bitencourt Jr. [16] to simulate the SFRC beams in pure torsion in order to assess the capability of this approach to capture the failure process and to validate the numerical tool to be used in future research investigations in combination with experimental tests.

2 Methodology

To simulate the behavior of the SFRC structural members, a multiscale finite element model proposed by Bitencourt Jr. [3] was used, including the description of the crack propagation processes. Recently, Trindade et al. [4] used this approach to aid the design of beams reinforced with steel fibers and conventions rebars, obtaining promising results.

This numerical model allows the representation of the composite in 3 distinct phases: cement matrix, steel fibers, and fiber-matrix interaction. Initially, the geometry of the structural members is modeled using the preprocessing program GiD. A cloud of steel fiber is generated using a uniform isotropic random distribution and considering the wall effect of the mould. Then, coupling finite elements are inserted for simulating the interaction between the concrete elements and the truss elements. This coupling element for 3D simulations is a 5-noded tetrahedral elements and has the same node coordinates as the corresponding concrete element, and an additional internal node that corresponds to the node of the reinforcement inside the domain of the corresponding concrete element.

2.1 Concrete modeling

The concretes model is discretized using 4-noded tetrahedral elements and its nonlinear behavior is described by a continuous damage model, proposed by Cervera [5], with independent damage variables to describe its tensile and compression behavior. The 4-noded tetrahedral element and its constitutive model is illustrated in Fig. 1a. To avoid numerical convergence problems, an implicit and explicit integration scheme (Impl-Ex) is employed to integrate the constitutive model. This Impl-Ex integration scheme can be better explained in Oliver [6] and Prazeres [7].

2.2 Reinforcement modeling

Steel fibers and rebars are represented by 2-noded truss elements. This element only allows deformations in their axial direction. Thus, for constitutive reasons, stiffness matrices and the internal forces vector are only dependent on the element axial direction. The material behavior is described by an elastoplastic model, as shown in Fig. 1b, where the model presents a branch of linear deformations until σ_y followed by a branch of permanent plastic deformations (H = 0). The branch of plastic deformations can have hardening (H>0), or softening behavior (H<0). A detailed description of the constitutive model can be found in Simo and Hugles [8] and Souza Neto [9].

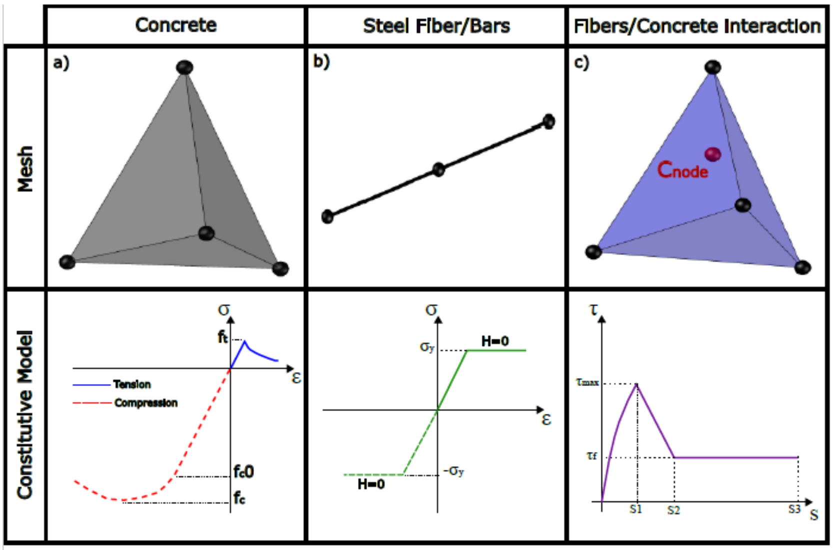

Figure 1. Finite elements and their respective constitutive models for each phase of the composite (adapted from Trindade et al. [15]).

2.3 Reinforcement/concrete interaction

The concrete and reinforcements (steel fibers and rebars) are initially discretized in finite elements in a totally independent way. Then, coupling finite elements (CFEs) proposed by Bitencourt Jr. [10] are inserted to describe the interaction between the non-matching finite element meshes. As stated, this element has 5 nodes, 4 nodes from the concrete element and 1 from the reinforcement (located inside the corresponding concrete element).

The CFEs define the compatibility of displacements and transfer forces between the non-matching meshes. Thus, can be defined as a relative displacement [[U]] as the difference between the displacement of the coupling node (C_{node}), shown in Fig. 1c, and the displacement of the material point (Xc), defined by the concrete finite element shape functions [N_i]. The interaction force arises from following equation:

$$F\left(\left[\left[U\right]\right]\right)=C\left[\left[U\right]\right]=C\left(D_{C_{node}}-\sum_{i}^{n}N_i\left(X_c\right)D_i\right). \tag{4}$$

Considering that the fibers deformation only occurs in its axial direction, the formulations start from the local coordinate systems of the element. Therefore, the coupling element internal force F_{CFE}^{int} and the stiffness matrix K_{CFE} are given by eq. (5) and eq. (6).

$$F_{CFE}^{\text{int}}=B_{CFE}^{T}CB_{CFE}D_{CFE}. \tag{5}$$

$$K_{CFE}=\frac{F_{CFE}^{\text{int}}}{\partial D_{CFE}}=B_{CFE}^{T}CB_{CFE}. \tag{6}$$

To find the global correspondent of F_{CFE}^{int}, it should consider the [R] orthogonal rotation matrix between the local and global coordinate systems. Such components compose the solid internal force and its stiffness matrix, presented by Bitencourt Jr. [10] and described by eq. (7) and eq. (8), where [A] is the finite element assembly operator.

$$F^{\text{int}}=A_C F_C^{\text{int}}+A_{CFE}F_{CFE}^{\text{int}}+A_F F_F^{\text{int}}. \tag{7}$$

$$K=A_C K_C+A_{CFE}K_{CFE}+A_F K_F. \tag{8}$$

3 Multiscale model

The numerical model is assessed through the simulations of SFRC beams tested by Facconi et al. [12]. In this work, the authors divide the specimens into 3 categories: PC, SFRC25 and SFRC50 with the same geometry. The PC category includes pure concrete beams, the SFRC25 includes SFRC beams with a fiber density of 25 kg/m³ and, the category SFRC50 includes SFRC beams with fibers density of 50 kg/m³ dispersed in the cementitious matrix. For geometry, the beams have a total length of 2400 mm and constant rectangular cross section of 300 x 300 mm.

To induce twisting to the specimens, Facconi et al. [12] applied a vertical P/2 load on side arms at beams head with the same cross section. All specimens are composed of 4 longitudinal reinforce bars with Ø18 mm diameter and with Ø10 mm diameter cross spaced every 50 mm to avoid deformations during the load application and to avoid cracks formation. Should be noted that the beams head boundary conditions allow the free rotation of the cross section. At central region with 1200 mm length is used as a monitoring region for the control of the cross section relative rotation and the cracks opening control.

Table 1 presents the finite element meshes used. Table 2 presents the concrete parameters used in the constitutive model proposed by Cervera [5]. As can be seen, the same finite element mesh was employed in the SFRC25 and SFRC50 simulations, and to consider the distinct number of fibers, a geometric proportionality (cross section) was adopted.

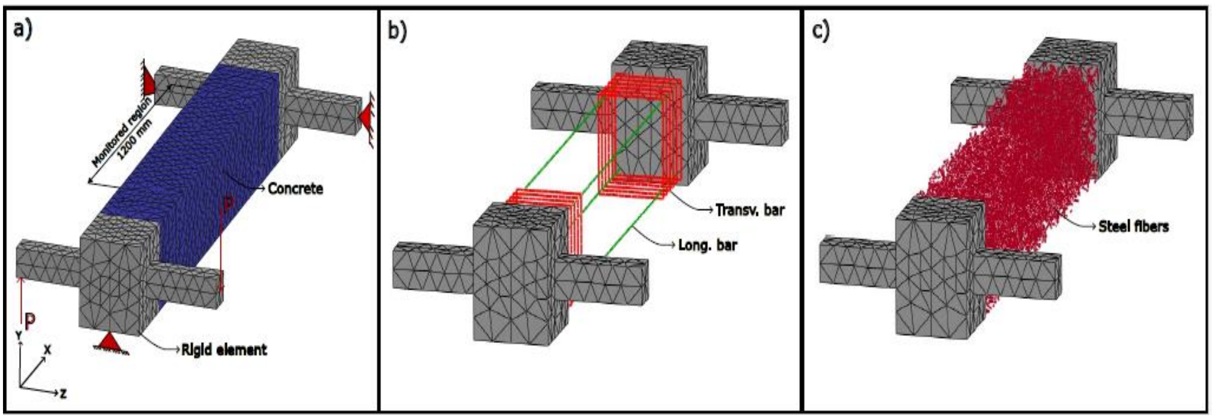

Figure 2. Numerical models constructed: a) loading and boundary; b) representation of longitudinal and transverse reinforcements; and c) cloud of steel fibers.

Table 1. Finite element meshes.

Mesh	Finite element model		
	PC	SFRC25	SFRC50
2-noded truss	880	67.930	67.930
4-noded tetrahedral	17.746	10.582	10.582
5-noded tetrahedral	884	90.284	90.284
Total	19.510	168.796	168.796

Table 2. Mechanical properties of concrete.

Concrete parameters	Finite element model			Unit
	PC	SFRC25	SFRC50	
fc0	25,00	25,00	25,00	MPa
ft	1,76	1,92	1,68	MPa
Ec	31.000	33.000	28.300	MPa
v	0,20	0,20	0,20	-
Gf	0,10	0,25	0,25	N/m
Compressive parameters	A = 1,0 e B = 0,89			

Table 3. Mechanical properties of fiber-matrix interaction (CFEs).

CFE - Fibers/concrete interaction		Unit
τmax	15,67	MPa
τf	3,33	MPa
S1	0,0001	mm
S2	0,0001	mm
S3	5,69	mm
α	0,70	-

In relation to the coupling elements, fiber-concrete interaction parameters were defined based on the fiber pullout studies performed by Mineiro et al. [14]. Thus, the parameters obtained for a uniform stress distribution model were applied along the fiber. Table 3 summarizes mechanical parameters adopted to describe the fiber-concrete interaction. A study of convergency was also performed to better understand the influence of the Impl-Ex scheme., considering 1000, 4000 and 8000 load steps. It is observed that 4000 load steps is enough to obtain the equilibrium curve.

4 Discussion of results

As results of the analyses, the relative rotations of the cross-sectional section were obtained, in the monitoring region, and its corresponding torque. In the Fig. 3, the results obtained for all numerical models are shown. As can be seen, numerical model - PC obtained torque results and rotations consistent with those presented by Facconi et al. [12]. In Tab. 4 can be seen that the maximum torque variation obtained by the model is 3,4% in relation to specimen tested in the literature. However, when the analysis is performed in relation to the torque corresponding to the twist presented, it is noted that the error decreases to 3,3%.

The numerical model SFRC25 presented a good representation of the post-cracking material behavior. It can be seen from Fig.3 that the curve obtained for the post-cracked regime decreases like the specimen tested by Facconi et al. [12]. Regarding the maximum torque obtained, it is observed that the error was 13,3%, but when compared with the corresponding twist presented in the literature, there is a decrease in the error to 7,2%, as shown in Tab. 4. For the numerical model SFRC50, a good representation of the behavior of the material in the post-cracked regime is obtained, as shown in Fig. 3. As is shown in Tab. 4, to the maximum torque obtained the error was 6,2%, but when compared with the corresponding twist presented in the literature, there is a decrease in the error to 5,2%.

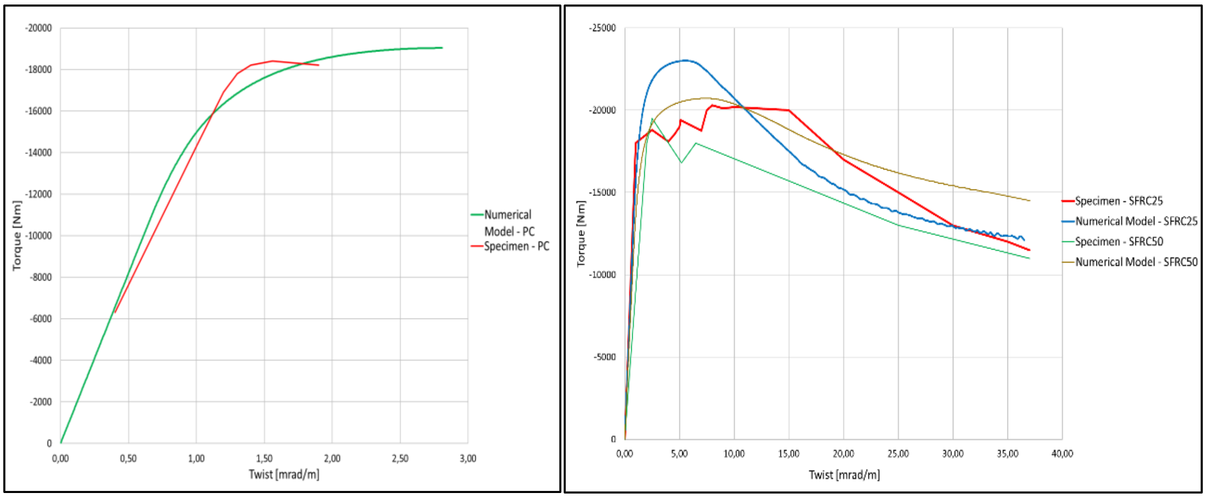

Figure 3. Twist vs Torque curves.

Table 4. Summary of the responses obtained.

Specimen	Numerical Model			Reference			Error	
	T,máx [Nm]	φ [mrad/m]	T,c [Nm]	φ [mrad/m]	T,máx [Nm]	φ [mrad/m]	T,máx [%]	T,c [%]
PC	19.032	2,80	17.790	1,56	18.400	1,56	3,4	3,3
SFRC25	23.000	5,57	21.770	8,39	20.300	8,39	13,3	7,2
SFRC50	20.717	7,40	18.490	2,06	19.500	2,06	6,2	5,2

5 Conclusions

To simulate the behavior of steel fiber reinforced concrete beams in pure torsion, a multiscale finite element model proposed by Bitencourt Jr. [3] was employed. The numerical analyses were performed considering as the reference the experimental tests performed by Facconi et al. [12]. The results obtained demonstrated that the numerical model was able to represent the structural member behavior under torsion, including the post-cracking regime. The curves in terms of rotation and torque values obtained present an error of 3.3% - 13.3% when compared to experimental results. Based on these results, the numerical model will be used as an aid tool in the design of SFRC beams under torsion in combination with experimental tests.

Acknowledgements. The authors would like to acknowledge the financial support of the National Council for Scientific and Technological Development - CNPq (Proc. n. 310401/2019-4 and 406205/2021-3) and São Paulo Research Foundation (Proc. n. 2019/24487-2).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors or has the permission of the owners to be included here.

References

[1] *fib* Model Code for Concrete Structures 2010. International Federation for Concrete Structures. Ernst & Soh. Lausanne, 2013.
[2] EN 14651. Test Method for Metallic Fiber Concrete - Measuring the Flexural Tensile Strength (Limit of Proportionality (LOP), Residual), 2007.
[3] Bitencourt Jr. LAG, Manzoli OL, Bittencourt TN, Vecchio FJ. Numerical modeling of steel fiber reinforced concrete with a discrete and explicit representation of steel fibers. Int J Solids Struct 2019;159:171–90.
[4] Trindade YT, Bitencourt LAG, Manzoli OL. Design of SFRC members aided by a multiscale model: Part II – Predicting the behavior of RC-SFRC beams. Compos Struct 2020, v. 241, p. 112079.
[5] Cervera M, Oliver J, Manzoli O. A rate-dependent isotropic damage model for the seismic analysis of concrete dams. Earthquake Eng Struct Dyn 1996;25(9):987–1010.
[6] Oliver J, Huespe A, Cante J. An implicit/explicit integration scheme to increase computability of non-linear material and contact/friction problems. Comput Methods Appl Mech Eng 2008;197:1865–89.
[7] Prazeres PGC, Bitencourt Jr. LAG, Bittencourt TN, Manzoli OL. A modified implicit-explicit integration scheme: an application to elastoplasticity problems. J Braz Soc Mech Sci Eng 2015:1–11.
[8] Simo J, Hugles TJR. Computational Inelasticity. Springer; 1998.
[9] de Souza Neto E, Perić D, Owen DRJ. Computational methods for plasticity: theory and applications. Wiley; 2008.
[10] Bitencourt Jr. LAG, Manzoli OL, Prazeres PGC, Rodrigues EA, Bittencourt TN. A coupling technique for non-matching finite element meshes. Comput Methods Appl Mech Eng 2015;290:19–44.
[11] Cunha V. Steel fibre reinforced self-compacting concrete – from micromechanics to composite behaviour [Ph.D. Thesis], Department of Civil Engineering, University of Minho, Portugal; 2010.
[12] Facconi L, Minelli F, Plizzari G, Ceresa P (2019) Experimental study on steel fiber reinforced concrete beams under pure torsion. In: DerkowskiWet al (eds) Proceedings of the fib symposium 2019 held in Kraków, Poland.
[13] EN 1992-1-1: Eurocode 2 (2005) Design of concrete structures. Part 1–1: general rules and rules for buildings.
[14] Mineiro MLR., Monte R., Manzoli, OL, Bitencourt Jr. LAG. An integrated experimental and multiscale numerical methodology for modeling pullout of hooked-end steel fiber from cementitious matrix Construction and Building Materials 344 (2022) 128215.
[15] Trindade YT, Bitencourt LAG, Monte R., Figueiredo AD, Manzoli OL. Design of SFRC members aided by a multiscale model: Part I – Predicting the post-cracking parameters. Compos Struct 2020, v. 241, p. 112078, 2020.
[16] Bitencourt Jr. LAG. Numerical modeling of failure processes in steel fiber reinforced cementitious materials. 2015. Doctoral Thesis, Polythecnical School at the University of São Paulo, São Paulo, 2015 (https://doi.org/10.11606/T.3.2014.tde-16112015-150922).