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Global buckling of thin-walled laminated composite columns

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Abstract. The search for lighter and larger structural components makes the use of fiber-reinforced laminated components increasingly slender. However, the increase in slenderness makes the structure more flexible, which can cause stability problems and large displacements. As a result, buckling has a great influence on composite material column designs, so their failure can occur with stress lower than the strength of the material. In this way, the evaluation of the stability of these structures is of great importance because it allows for predicting the load capacity. However, one of the main objectives of the industry is to replace experimental tests with numerical simulations, since in tests of composite material structures, numerous and expensive tests are usually required. Therefore, this work aims to study of global buckling of laminated composite channel-section columns. Two approaches are employed in this context. The first approach consists of a three-dimensional beam finite element for stability analysis of thin-walled laminated composite. Regarding the second approach, it relies on the Rayleigh-Ritz framework assumes that the axial strain is neglected, and the column only buckles according to the minor axis of bending, evaluating the behavior of channel-section columns, with different layups when subjected to compressive loads. Both strategies are based on a fully coupled constitutive matrix, and the results obtained were compared with shell finite elements.

Keywords: Fiber reinforced composites, Thin-walled laminated columns, Stability, Global buckling.

1 Introduction

Fiber-reinforced composite materials have gained space in automotive, aeronautics, shipbuilding, and construction industries. This is because these materials present desired properties, such as high stiffness/weight and strength/weight ratios, thermal/acoustic insulation, and low thermal expansion. However, the behavior of composite structural members, especially columns, are much more complex than the behavior of their steel or aluminium counterparts. In-plane compressive loads, when high enough, can cause excessive deflections that may lead to failure. The load at which excessive out-of-plane deflections occur is called the buckling load [1]. The buckling strength of laminated structures depends on the material properties, composite layup, cross-section geometry, length, and boundary conditions. In terms of global buckling, this includes buckling modes where the half-wavelength has the same order of magnitude as the length of the compressed element and the cross-section remains undistorted.

The interaction between local and global buckling has already been evaluated experimentally for some time. Barbero and Tomblin [2] adapted an existing interaction equation aiming to consider the failure modes observed in the columns made of fiber-reinforced composite materials. D'Aguiar and Parente Jr. [3] presented a study on the behavior, performance, and failure of thin-walled composite channel section columns, showing the influence of the lamination scheme and the thickness on the local buckling of these columns. In addition, it was verified that the interaction between global and local buckling modes has a great influence on the post-critical behavior of the column and, consequently, on its load capacity.

The large elongation allowed by the fibers and resin allows the fiber-reinforced materials to remain linearly elastic for large deformations and deflections [4]. Therefore, the design specifications in structures of steel to avoid buckling problems can not be the same in material structures composites, since in steel structures buckling usually occurs in the plastic range with the influence of geometric and mechanical imperfections [5]. Therefore, this work seeks to present a Rayleigh-Ritz and a Finite Element approach to the evaluation of the global buckling load of laminated columns with channel cross-sections thin-walled composite columns with various layups and lengths. With this, it is possible to study the influence of the orientation of the plies on the critical load in the column with different lengths.

2 Stability of thin-walled laminated colums

Perfect or ideal columns are structural elements that are perfectly straight and subjected predominantly to axial compressive stresses. To determine the load by the which the structure loses the ability to resist increased loading in its configuration original and begins to show large deflections (buckling). It is important to note that composite columns have thin walls, buckling being an important design consideration [6].

In short columns, it is expected that local buckling occurs first, causing large deformations, following global buckling or material degradation [7]. However, for long columns of composite material it is observed that generally the global buckling (Euler) occurs before other instability modes. Therefore, in the next sections, two different methods used to obtain critical loads are presented.

2.1 Cross-sectional stiffness

The performance of the two approaches presented in this work to determine buckling loads depends on careful evaluation of the constitutive behavior of the laminated column. Defining such constitutive laws is not a straightforward task due to the geometry and layup of each segment of the cross-section. Several approaches are presented in the literature to obtain such a relation [8–11]. However, due to its implementation simplicity and the possibility to be applied for open and closed cross-sections with an arbitrary layup, and because of the good results that have been reported [12, 13], the theory proposed by Kollar and Pluzsik [14] is adopted in this work.

Kollar and Pluzsik [14] proposed a general theory to analyze thin-walled laminated beams in which analytical expressions for flexibility matrix (and stiffness matrix) for open and closed sections are presented. In this theory, each segment of the cross-section consists of flat segments where each one may have different layers with arbitrary layups, and effects due to transverse shear deformation and warping are neglected; as a main result, the generalized strain vector ($\boldsymbol{\varepsilon}$) can be related to the generalized stress vector ($\boldsymbol{\sigma}$) through the symmetric flexibility matrix, $\mathbf{F}$:

$$\begin{Bmatrix} \varepsilon_x \\ \kappa_y \\ \kappa_z \\ \beta \end{Bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix} \begin{Bmatrix} N_x \\ M_y \\ M_z \\ T \end{Bmatrix} \Longrightarrow \boldsymbol{\varepsilon} = \mathbf{F}\boldsymbol{\sigma}, \tag{1}$$

where ε_x is the normal strain in x direction, κ_y and κ_z are respectively the beam curvatures about y and z axes, β is the rate of change of twist angle (θ_x) along x direction, N_x is the axial force, M_y and M_z are respectively the bending moments about y and z axes, and T is the torque (see Fig. 1). Observe that the stiffness matrix, $\mathbf{C}$, can be obtained by inverting $\mathbf{F}$, i.e., $\mathbf{C} = \mathbf{F}^{-1}$. Also, note a fully 4x4 coupled constitutive matrix can be obtained through this approach depends on the geometry and layup of the cross-section.

2.2 Finite Element Method

Regarding the first approach adopted in this work, the two-node three-dimensional beam finite element for geometrically nonlinear analysis of thin-walled laminated composite beams presented in [12] is used. This element uses the Total Lagrangian formulation to deal with large displacements and moderate rotations, and in which nonlinearity is only considered in the axial strain. Furthermore, the element formulation is based on Bernoulli-Euler-Navier bending hypothesis and Saint Venat's torsion hypothesis, and possess six degrees of freedom per node (three translations and three rotations).

This element can capture post-buckling behavior, but a linearized buckling analysis that consists in solving the generalized eigenproblem is employed for determining the buckling load of laminated composite columns [15]:

$$(\mathbf{K} + \lambda \mathbf{K}_g)\, \boldsymbol{\phi} = \mathbf{0}, \tag{2}$$

where λ are the eigenvalues associated with the buckling load factors, and $\boldsymbol{\phi}$ are the eigenvectors (or buckling

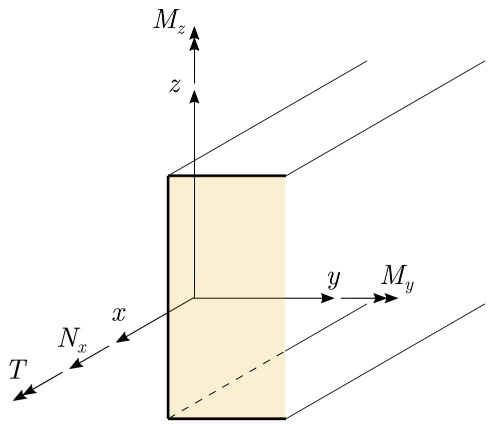


Figure 1. Channel-section beam: forces and global coordinate system whose origin coincides with the mechanical centroid of the cross-section. Colored region indicates that all forces are acting in the same cross-sectional plane.

modes). The two global matrices, $\mathbf{K}$ (global linear stiffness matrix) and $\mathbf{K}_g$ (global geometric stiffness matrix), are assembled through their corresponding element stiffness matrices which are, respectively, obtained by integration over the element length, L:

$$\mathbf{k} = \int_L \mathbf{B}^{\mathrm{T}}\mathbf{C}\mathbf{B}\,\mathrm{d}x \tag{3}$$

and

$$\mathbf{k}_g = \int_L N_x \mathbf{A}^{\mathrm{T}}\,\mathrm{d}x, \tag{4}$$

with $\mathbf{C}$ being the constitutive matrix defined as $\mathbf{C} = \mathbf{F}^{-1}$ (see Eq. (1)). The matrix $\mathbf{A}$ and the displacement-strain matrix $\mathbf{B}$ are obtained by evaluating the derivatives of standard finite element shape functions, which are in turn assembled through Lagrangian (linear) and Hermite (cubic) polynomials according to the nodal degrees of freedom of element. More details on how to assemble matrices $\mathbf{A}$ and $\mathbf{B}$ can be found in [12].

2.3 Rayleigh-Ritz

The second approach used in this work for determining the buckling load of laminated composite columns relies on the Rayleigh-Ritz (RR) method. For this, it is assumed that the axial strain is neglected, and the column only buckles according to the minor axis of bending (i.e., axis z, see Fig. 2). As a result, Eq. (1) can be rewritten as:

$$\begin{Bmatrix} \varepsilon_x = 0 \\ \kappa_y = 0 \\ \kappa_z \\ \beta \end{Bmatrix} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix} \begin{Bmatrix} N_x \\ M_y \\ M_z \\ T \end{Bmatrix}. \tag{5}$$

Equation (5) can be solved for the axial force N_x and the bending moment M_y in terms of the remaining force components by extracting the first and second linear equations from the matrix:

$$\begin{Bmatrix} N_x \\ M_y \end{Bmatrix} = -\mathbf{Q}^{-1}\mathbf{R} \begin{Bmatrix} M_z \\ T \end{Bmatrix}, \tag{6}$$

with

$$\mathbf{R} = \begin{bmatrix} F_{13} & F_{14} \\ F_{23} & F_{24} \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}. \tag{7}$$

Substituting Eq. (6) into the third and fourth linear equation in Eq. (5), and recalling that $\mathbf{F}$ is symmetric (and consequently $\mathbf{R}$) leads to the following condensed/reduced relation:

$$\begin{Bmatrix} \kappa_z \\ \beta \end{Bmatrix} = \left(\begin{bmatrix} F_{33} & F_{34} \\ F_{43} & F_{44} \end{bmatrix} - \mathbf{R}^{\mathrm{T}} \mathbf{Q}^{-1} \mathbf{R} \right) \begin{Bmatrix} M_z \\ T \end{Bmatrix} \quad \Rightarrow \quad \boldsymbol{\varepsilon}_r = \mathbf{F}_r \, \boldsymbol{\sigma}_r, \tag{8}$$

where $\mathbf{F}_r$ is the reduced flexibility matrix, whose inversion yields the reduced stiffness matrix ($\mathbf{C}_r = \mathbf{F}_r^{-1}$):

$$\begin{Bmatrix} M_z \\ T \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{Bmatrix} \kappa_z \\ \beta \end{Bmatrix} \quad \Rightarrow \quad \boldsymbol{\sigma}_r = \mathbf{C}_r \, \boldsymbol{\varepsilon}_r \tag{9}$$

In this equation, C_{11} corresponds to the bending stiffness ($\overline{EI}_z$), C_{22} corresponds to the torsional stiffness ($\overline{GJ}$), and $C_{12} = C_{21}$ corresponds to the bending-torsion coupling stiffness ($\overline{S}$).

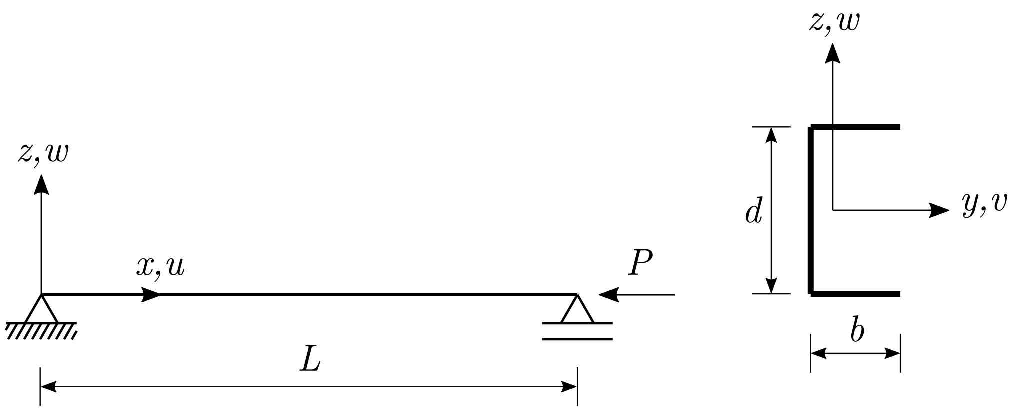


Figure 2. Column geometry and boundary conditions.

In this study, only columns with both ends pinned are considered, as shown in Fig. 2. In line with the Rayleigh-Ritz framework, a generalized buckling formula can be derived based on the energy approach. Therefore, the total potential energy function of the system can be expressed as:

$$\Pi = \int_L \frac{1}{2} \boldsymbol{\varepsilon}_r^{\mathrm{T}} \boldsymbol{\sigma}_r \, \mathrm{d}x - Pu = \frac{1}{2} \int_L \left(\overline{EI}_z \kappa_z^2 + 2\overline{S} \kappa_z \beta + \overline{GJ} \beta^2 \right) \mathrm{d}x - \frac{P}{2} \int_L \left(\frac{\mathrm{d}v}{\mathrm{d}x} \right)^2 \mathrm{d}x, \tag{10}$$

where the first integral corresponds to the strain energy which integrand is obtained through Eqs. (8) and (9), and the second integral refers to the work done by the applied force P. Once again, observe the fact that buckling may only take place according to the minor axis of bending; as such, only the lateral displacement v in the plane yz is considered.

For the boundary conditions shown in Fig. 2, the following displacement fields are assumed based on the isotropic Euler first buckling mode:

$$v(x) = a \sin\left(\frac{\pi x}{L} \right) \quad \text{and} \quad \theta_x(x) = b \left[1 - \cos\left(\frac{\pi x}{2L} \right) \right], \tag{11}$$

where $u(x)$ and $v(x)$ are, respectively, the axial and lateral displacements, and a and b are the degrees of freedom. Observe that the twist angle θ_x is fixed at one end and free at the other end. Therefore, the terms κ_z and β can respectively be expressed as:

$$\kappa_z = \frac{\mathrm{d}^2 v}{\mathrm{d}x^2} \quad \text{and} \quad \beta = \frac{\mathrm{d}\theta_x}{\mathrm{d}x}. \tag{12}$$

Solving the expression obtained by minimizing Eq. (10) concerning to a and b for the load P leads to the following expression for buckling load:

$$P_{cr} = \frac{9\pi^2 \overline{EI}_z \overline{GJ} - 64 \overline{S}^2}{9 \overline{GJ} L^2}. \tag{13}$$

It is interesting to emphasize the fact that the corresponding Euler formula ($P_{cr} = \pi^2 EI_z \,/\, L^2$) for isotropic columns can be obtained from the expression above by neglecting the coupling term $\overline{S}$.

3 Numerical examples

Finite element models of thin-walled columns of channel sections were developed using the ABAQUS [16] to assess the accuracy of the approaches proposed in this work. These models are discretized using quadratic shell elements based on the Reissner-Mindlin Theory with eight nodes and reduced integration (S8R). The columns considered in this work were previously studied by Debski et al. [17] with a focus on local buckling. The web measures $80\,\mathrm{mm}$ and is divided into 8 parts, while the flanges measure $40\,\mathrm{mm}$ and are divided into 4 parts. Columns with lengths from $1\,\mathrm{m}$ to $3.5\,\mathrm{m}$ are considered, discretized using square elements of $0.1\,\mathrm{m}$ size. Thus, as the length increases, the total number of elements also increases.

Boundary conditions were applied to a point at the centroid of the cross-section, where this point had rigid body constraints with only translational degrees of freedom associated with the rigid body, called PIN in the ABAQUS software [16]. The boundary conditions at the bottom of the constrained column were U1, U2, U3 and UR3, while the boundary conditions at the top of the constrained column were U1, U2 and UR3. These boundary conditions are consistent with a simply supported column for both local and global buckling modes. A unit load was applied to the top point. A typical model is shown in Fig. 3.

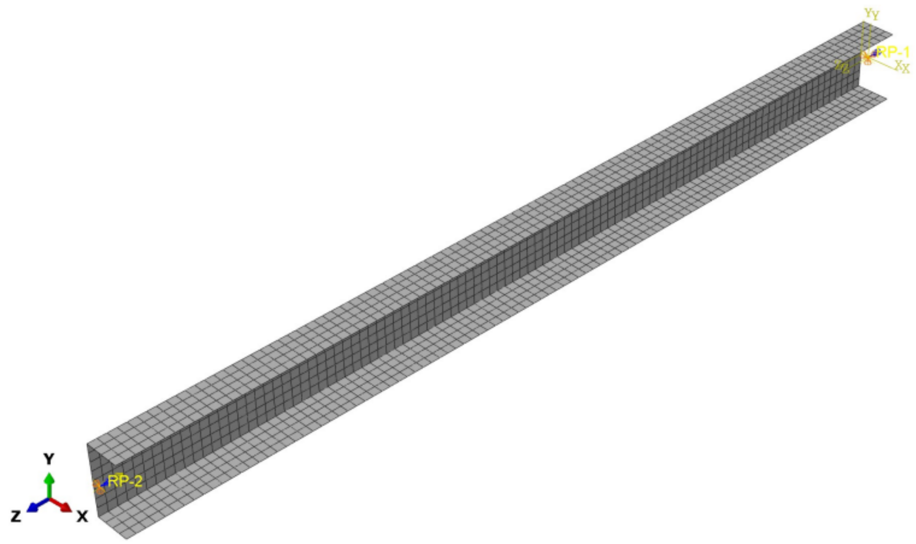


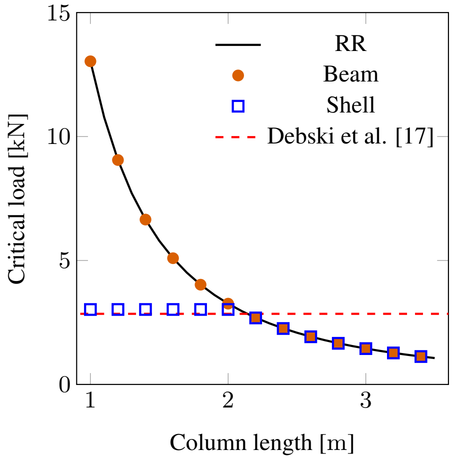
Figure 3. Column model adopts in ABAQUS and its discretization

Taking as a reference the laminated columns of the thin-walled channel section from the experimental tests carried out by Debski et al. [17], Hexcel's HexPly M12 carbon-epoxy material was adopted for the finite element models. The material properties are given by $E_1 = 130\,710\,\mathrm{MPa}$, $E_2 = 6360\,\mathrm{MPa}$, $G_{12} = 4180\,\mathrm{MPa}$ and $\nu_{12} = 0.32$. Four different composite layups with 8 plies in each were considered, each ply with 0.131 mm and total thickness $h = 1.048$ mm, they are: L1 $[0/-45/45/90]_s$, L2 $[0/90/0/90]_s$, L3 $[45/-45/0/90]_s$ and L4 $[45/-45/90/0]_s$.

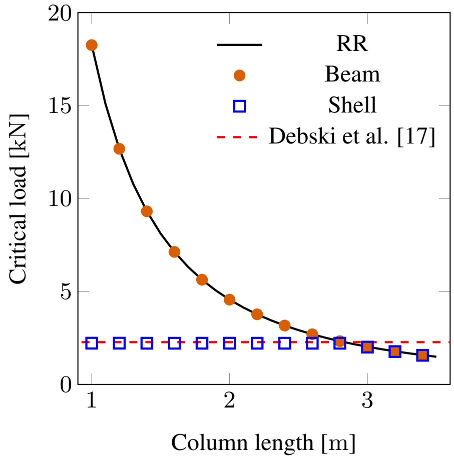
The buckling loads of the columns with different lengths and the four layups were evaluated using the Rayleigh-Ritz method, beam finite element models (with 15 elements) and shell finite element models. The obtained results are shown in Fig. 4. In addition, the experimental local buckling obtained by Debski et al. [17] for layups L1, L2, and L3 are also shown in this figure. The results of the shell model for short columns are in very good agreement with the available experimental results, showing the correct consideration of the geometry, layup, and boundary conditions.

It is important to note that the critical loads calculated by the Rayleigh-Ritz and beam element approaches present an almost perfect agreement for the four different layups considered in this work. Furthermore, these results are also in almost perfect agreement with the shell FE model for long columns, where the global mode is dominant. This occurs because the global mode adopted by the Rayleigh-Ritz method is very close to the one obtained by shell model, as depicted in Fig. 5.

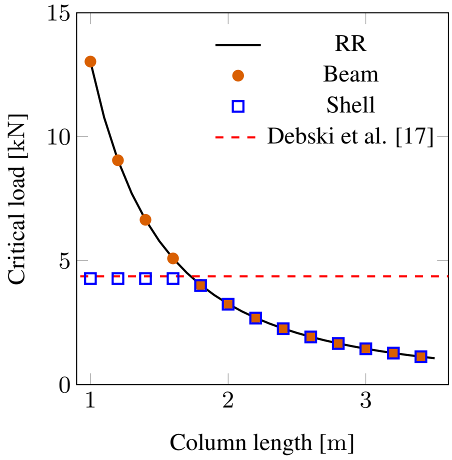
The results also show that there is a strong influence of the composite layup on the column strength, with layup L2 presenting the highest global buckling loads, but the smaller local buckling load. On the other hand, the other three layups present almost identical global buckling loads, but different local buckling loads. The transition length when the critical mode changes from local to global is also strongly dependent on the composite layup. On the other hand, the buckling modes for the length of $3.0\,\mathrm{m}$ presented in Fig. 5 reveal the similarity between global buckling modes for different layups.



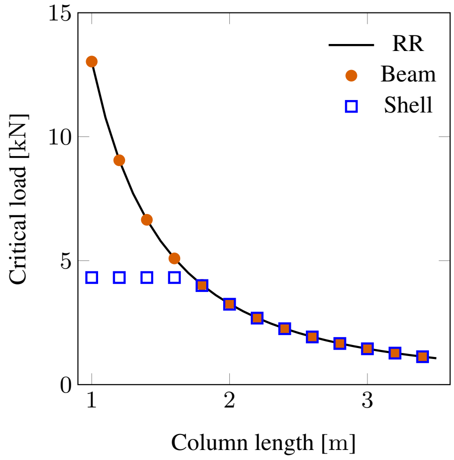
(a) Layup L1



(b) Layup L2



(c) Layup L3



(d) Layup L4

Figure 4. Critical load versus column length.

(a) Layup L1

(b) Layup L2

(c) Layup L3

(d) Layup L4

Figure 5. Buckling modes ($L = 3\,\mathrm{m}$).

4 Conclusions

In this work, Rayleigh-Ritz and Finite Element approaches were presented for the evaluation of the global buckling loads of laminated columns with thin-walled channel cross-sections. The results show the strong influence of composite layup and column length over the buckling load of laminated columns. On the other hand, the global buckling mode was practically the same for all layups considered in this work.

The Rayleigh-Ritz approach contributes to a better understanding of the instability mechanisms and the behavior of columns of fiber-reinforced composites. Furthermore, this approach leads to excellent results despite its simplicity. The accuracy of this approach is related to the approximate buckling modes adopted in this work.

The beam element model is an alternative that considers all couplings and helps to confirm the results obtained by the Rayleigh-Ritz method. Finally, the shell element model, adopted to validate the simplified approaches presented in this work, was able to capture the transition from local to global buckling leading to accurate results for both cases. Therefore, these finite element models can be used for more complex stability studies including the post-buckling behavior and the assessment of imperfection sensitivity.

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