XLIII CILAMCE
21-25, November, 2022
FOZ DO IGUAÇU - PR - BRAZIL

Modal analysis of a simply supported beam subjected to a moving mass

Baddyo K. S. P. Silva[1], Reyolando M. L. R. F. Brasil[2]

[1]*PEF-Polytechnical School, University of São Paulo*
Av. Prof. Luciano Gualberto, 380 - Butantã, São Paulo - SP, 05508-01
baddyo.silva@usp.br
[2]*CECS-Federal University of ABC*
Alameda da Universidade, SN, 09606-045, São Bernardo do Campo, São Paulo, Brazil
reyolando.brasil@ufabc.edu.br

Abstract. This study presents a computational analysis of the dynamic behavior of a simply supported beam subjected to a moving mass and corresponding load that travels along its entire length with different speeds. The first analysis aims to understand the behavior of the structure and to determine transversal response of the structure. Here, a discretized simply supported beam model was developed, based on the Finite Element Method, using numerical integration by Newmark's Method for the solution of ordinary differential equations and obtaining the displacements of the structure in the time domain, to evaluate its behavior due to moving masses and loads. The analysis is made in different velocities and damping rates aiming to find the maximum transverse response, and comparing with the static response to find the maximum amplification coefficients. This article aims to evaluate the frequencies and vibration modes associated with the maximum displacements found and its relation with the different velocities applied.

Keywords: Moving loads. Structural Dynamics. Newmark's method. Modal Analysis.

1 Introduction

The structures are submitted to loads that vary in function of time, causing dynamic effects. The development of trustful models that evaluate the responses of structures is essential to warrantee its security.

This paper presents a case study, in which a simply supported beam is subjected to a moving mass in longitudinal motion, and aims to find the transverse responses of the structure for different velocities using optimization methods to find the velocity that causes the maximum displacement of the structure, it aims to relate the frequencies associated with its maximum displacements.

2 Modelling

The studied system is shown in Figure 1. It consists of a simply supported beam, with length L, modulus of elasticity E, cross section area A, moment of inertia about the y-axis I and density ρ. Along the beam, the moving mass M travels with constant speed v(t).

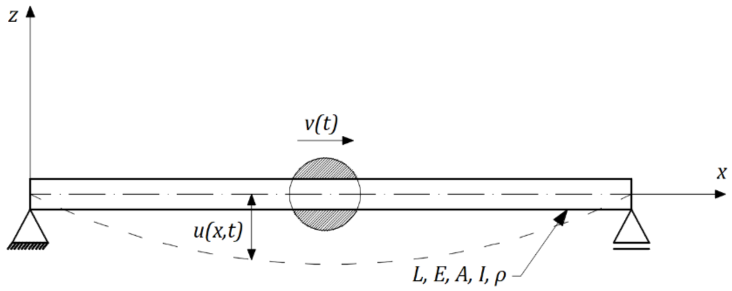


Figure 1. Simply-supported beam subjected to a moving mass system

The model is then discretized and analyzed using the finite element method where the real structure is represented as a model consisted of several elements, as shown in Figure 2, with several degrees of freedom. Each element has mass, stiffness and corresponding damping, leading to the generation of matrices of mass, stiffness and damping of each element.

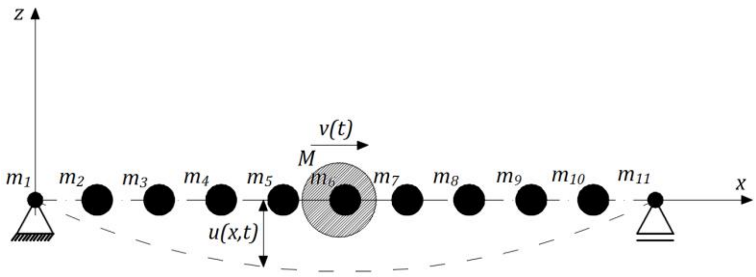


Figure 2. Discretized model

The local matrices are then converted to global matrices, and the equation of the system's motion is considered in the matrixial form as

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{u}}\} + [\boldsymbol{C}]\{\dot{\boldsymbol{u}}\} + [\boldsymbol{K}]\{\boldsymbol{u}\} = p(t) \tag{1}$$

where $[M]$ is the mass matrix, $[C]$ the damping matrix and $[K]$ the stiffness matrix of the system. Mazzilli et al. [1] report that it is a sufficient condition for the damping to be of the proportional type such that the damping matrix [C] is a linear combination of the mass and stiffness matrices, expressed by

$$[C] = \sum_b a_b [M]([M]^{-1}[K])^b \tag{2}$$

where the particular case of the Rayleigh damping can be considered

$$[C] = a_0[M] + a_1[K] \tag{3}$$

where the factors a_0 and a_1 are obtained imposing damping rates ξ arbitrarily adopted for two chosen modes, finding the solution for the system

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \tag{4}$$

To solve the problem of moving loads, the moving mass M is considered changing position between the nodes, changing the mass matrix $[M]$ at each time step, depending on the constant speed $v(t)$, solving the nonlinear problem in a linearized form.

In order to obtain system responses, it is necessary to integrate the equations of motion, so the Newmark method is used for direct integration, Toledo [2] reports that displacements and velocities are developed in Taylor series, with the rest being calculated approximately according to free parameters that are fixed later. Brasil and Silva [3] report that, given the vectors of displacements, velocities and accelerations, their values are determined in an instant $t + \Delta t$.

$$u_{t+\Delta t} = u_t + \Delta u \tag{5}$$

$$\dot{u}_{t+\Delta t} = b_0 \Delta u - b_2 \dot{u}_t - b_3 \ddot{u}_t \tag{6}$$

$$\ddot{u}_{t+\Delta t} = b_1 \Delta u - b_4 \dot{u}_t - b_5 \ddot{u}_t \tag{7}$$

The coefficients $b_0, b_1, b_2, b_3, b_4, b_5$ are chosen in order to approximate the variation of the vectors, obtaining a system of algebraic equations that allows to find the increments of displacements in the step,

$$\hat{K} \Delta u = \hat{p}_{t+\Delta t} \tag{8}$$

with equivalent stiffness

$$\hat{K} = b_1 M + b_0 C + K \tag{9}$$

and equivalent step load

$$\hat{p}_{t+\Delta t} = p_{t+\Delta t} + M(b_2 \dot{u}_t + b_3 \ddot{u}_t) + C(b_4 \dot{u}_t + b_5 \ddot{u}_t) - K u_t \tag{10}$$

from which the displacement velocities and accelerations of the next step are determined.

The concept of optimization is then applied to find the maximum displacements in the midspan considering different constant speeds of displacement of the mass over the bar element to obtain the solution of the problem. Brasil and Silva [4] present the concept of the golden ratio optimization, from which, in an iterative way, the solution of the problem can be found, this method uses the golden ratio, defined by Euclid as

$$\varphi = \frac{1+\sqrt{5}}{2} \tag{11}$$

to find the minimum of a function, an interval is defined so that the minimum value is sought, where two points chosen considering the golden ratio are compared. The functions are applied to these points, and the minimum between them is verified, after that, one of the points becomes the new limit of the interval, excluding the left or right domain, and the analysis is repeated, the interval is then reduced by 61.8% at each iteration and is repeated until the minimum of the function is found.

In an analytical study, for a cantilever beam with the free left end, the exact cyclic frequency can be expressed as

$$\omega_i = i^2 \pi^2 \sqrt{\frac{EI}{l^4 \mu}} \tag{12}$$

Rao [5] points the mode shape equation as,

$$\Phi_i(x) = \sin \frac{i\pi x}{L} \tag{13}$$

Applying them, it is possible to find the vibration modes of the system as presented in Figure 3.

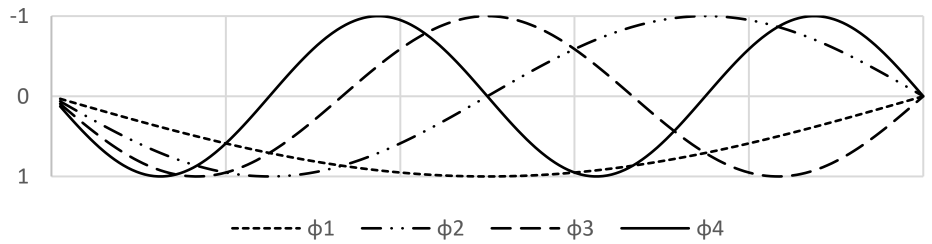


Figure 3. Shapes of the first four modes of vibration for simply supported beams

Fryba [6] points as a way to find the critical speeds the equation

$$v_{cr(i)} = \frac{i\pi}{L} \sqrt{\frac{EI}{\mu}} \tag{14}$$

From which we relate with the speed applied to evaluate the excited modes.

3 Discussion and Results

For the study, two models were analyzed using Mathworks Matlab® to find the solutions, consisting of a simply supported concrete beam, with two different lengths $L = 100.00m$ and $L = 50.00m$, modulus of elasticity $E = 3.0x10^{10}$, cross section area $A = 45.00m^2$, moment of inertia about y-axis $I = 45.00\ m^4$ and density $\rho = 2500\ kg/m^3$. The moving mass $M = 22000\ kg$ moves at a constant speed. We analyzed the structure's response to different speeds between $v(t) = 1{,}00\ m/s$ and $v(t) = 200{,}00\ m/s$.

The model is then discretized to perform computational analysis using the finite element method in a model with 100 elements and 101 nodes, each node with two degrees of freedom, relative to vertical displacement and rotation, and axial efforts can be disregarded.

Stiffness matrices and mass matrices are then generated for each moving mass position.

Using the Newmark Method, the responses of the motion equation for each position of the mobile load are then calculated, as a function of the displacement speed, for different constant speeds between 1,00 m/s and 100,00 m/s, applying the golden ratio method of optimization to find the velocity that causes the maximum deflections for different ratios of damping between 0.005 and 0.1, and its respective maximum transversal deflection, the results are shown in Tables 1 and 2.

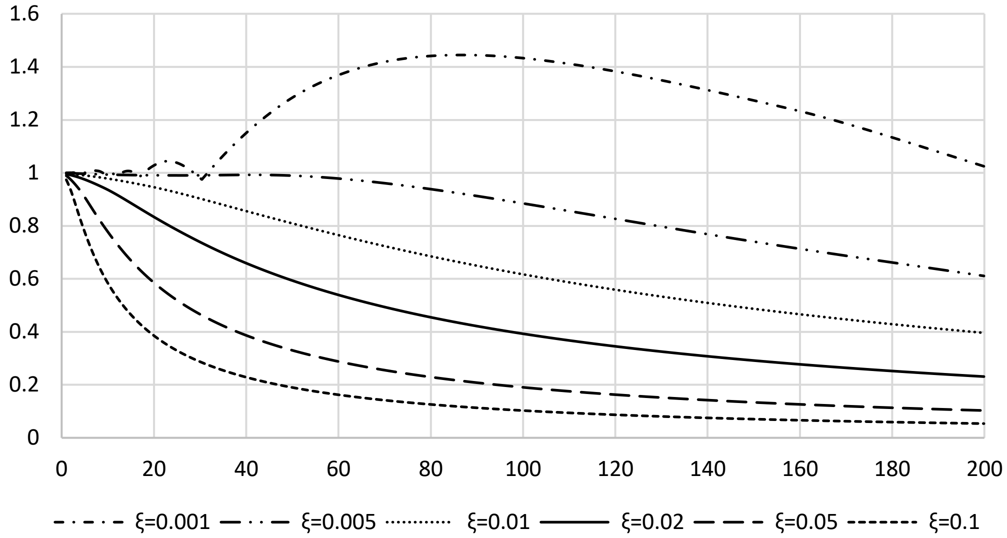
Table 1. Speeds of maximum transversal deflections for span of 50 meters

Damping rate ξ	Critical speed (m/s)	Maximum transversal deflections (mm)
0.001	86.6873	0.6174
0.005	39.7326	0.4249
0.01	1.0001	0.4243
0.02	1.0001	0.4241
0.05	1.0001	0.4225
0.10	1.0001	0.4173

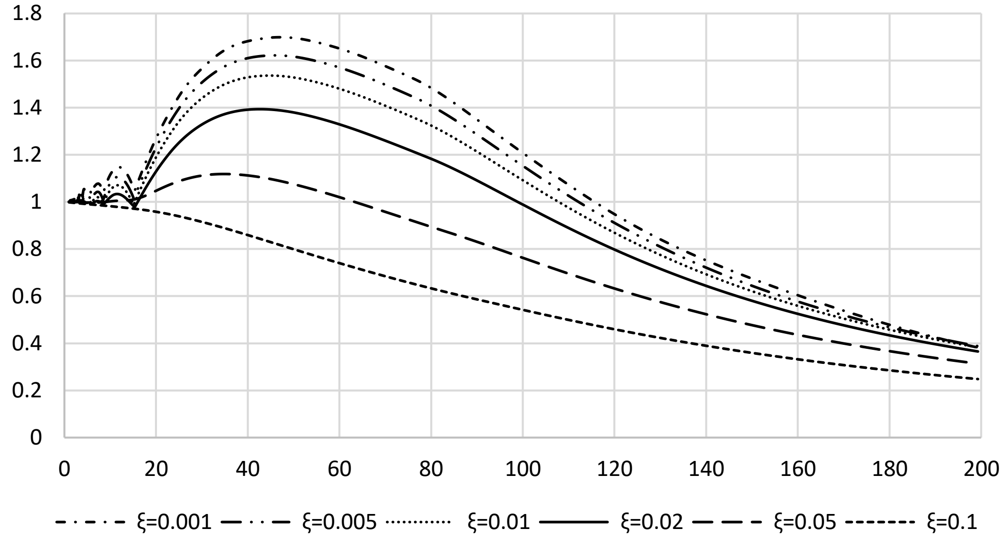
Table 2. Speeds and maximum transversal deflections for span of 100 meters

Damping rate ξ	Critical speed (m/s)	Maximum transversal deflections (mm)
0.001	46.5226	5.8026
0.005	45.6288	5.5399
0.01	44.6379	5.2482
0.02	42.5543	4.7629
0.05	34.7484	3.8258
0.10	1.0001	3.3943

The dynamic amplification coefficients obtained comparing the static deflection and the dynamic deflection are shown in Figure 3.



a)



b)

Figure 3. Relation between the displacement speed of the mobile mass and the dynamic amplification coefficient for simply supported structure for different damping rates for a) 50 meters span and b) 100 meters span

The critical speeds can be seen in Table 3.

Table 3. Critical Speeds for spans of 50 meters and 100 meters

i	50 m span (m/s)	100 m span (m/s)
1	217.66	108.83
2	435.31	217.66
3	652.97	326.48
4	870.62	435.31

The relation between the speeds of maximum deflection at midspan and the critical speeds for the first mode can be seen in the Table 4.

Table 4. relation between the speeds of maximum deflection at midspan and the critical speeds

Damping rate ξ	Vmax/Vcr in 50 m span (m/s)	Vmax/Vcr in 100 m span (m/s)
0.001	0.43	0.40
0.005	0.42	0.18
0.01	0.41	-
0.02	0.39	-
0.05	0.32	-
0.10	-	-

4 Conclusions

It is possible to observe that due to the movement of the mass, the structure responds with different amplification, depending on the adopted parameters. Considering the subcritical damping, for small speeds the deflection is practically equal to the static deflection, as the speed grows, the midspan deflection approaches its higher value, and then, applying higher speeds the deflection tends to lower values, approximating the values to zero.

The coefficients of dynamic amplification reach values higher than 1.7 for lower damping rates. It is possible to see that in speeds around 40% of the critical speed for the first modes occurs the maximum displacements of the beams for both studied models.

Acknowledgements. The authors acknowledge support by CNPq and FAPESP, both Brazilian research funding agencies.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] C. E. N. Mazzilli et al. Lições em Mecânica das Estruturas: Dinâmica. Blucher, 2016.
[2] R. C. P. L. Toledo, Um Estudo Sobre Métodos de Integração Direta para a Análise Dinâmica Não-Linear de Estruturas. MSc Thesis, Federal University of Rio de Janeiro, 1983.
[3] R. M. L. R. F. Brasil and M. A. Silva. Introdução à Dinâmica das Estruturas para a Engenharia Civil. Blucher, 2015.
[4] R. M. L. R. F. Brasil and M. A. Silva. Otimização de projetos de Engenharia. Blucher, 2019.
[5] S. S. Rao. Vibration of Continuous Systems. John Wiley & Sons, 2007.
[6] L. Fryba. Vibrations of Solids and Structures under Moving Loads. Thomas Telford, 1999.