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Numerical formulation for advanced analysis of semi-rigid steel-concrete composite frames

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Abstract. The present work aims at the implementation and validation of a displacement-based two-dimensional numerical formulation including several sources of non-linearities in steel-concrete composite frames, such as second-order effects, plasticity and beam-to-column semi-rigid connections. The co-rotational-based approach is used to describe the finite element formulation, allowing large displacements and rotations in the numerical model. Rotational pseudo-springs are used at the ends of the finite element, where the gradual loss of stiffness is determined by combining the normal force and bending moment (NM) in the cross-section. The limiting of the uncracked, elastic and plastic regimes are defined in the NM diagram. In the cross-sectional analysis, the Strain Compatibility Method (SCM) is used to capture the axial strains in the section components. In this way, the constitutive models of the materials are described by continuous functions. The semi-rigid connections are also simulated by the rotational pseudo-springs at the finite elements ends, and the connection behavior is given by its moment-rotation relationship. A multi-linear model for beam-to-column connections is used. To validate the proposed numerical formulation, the results obtained are compared with numerical and experimental data available in the literature. Since the model proposed here starts with the concentrated simulation of nonlinear effects, an examination of the finite element mesh refinement is also carried out.

Keywords: Steel-concrete composite structures, semi-rigid frames, lumped plasticity, cross-sectional analysis, finite element method, non-linear analysis

1 Introduction

There are three basic pillars for the elaboration of a structural project: safety, time (preparation and execution) and economy. The optimization of the three variables is mainly related to the materials and analysis methods used. In civil construction, when it comes to the choice of materials, Lemes et al. [1]evidence that the use and concrete are the usual ones, in the way that they lead to a better physical and mechanical properties.

The main advantages of composite steel and concrete structures are related to increased strength and rigidity, protection from metallic elements (fire and corrosion) and cost-effectiveness. Maximiano [2] adds that this type of structural system also has advantages during execution, as it is possible to make the metal profiles support the concrete elements during the curing process, reducing costs with shoring and increasing the free space for movement of materials in the work.

Lemes [3], however, warns that the analysis of this type of mixed structural system presents challenges and in the search for its behavior optimization, and that advanced computational analysis stands out because it allows the development of numerical processes that make possible the solution of problems. problems that would be unfeasible once they extrapolate analytical analyzes and simplifications of design rules.

Weng and Yen [4] corroborate this reality by demonstrating experimentally that composite structures can approach or distance from the real structural behavior in the different forms of analysis existing in the different design standards. Faced with the constant need to understand the realistic way in which mixed steel and concrete structures behave, this research brings the non-linear analysis of structures (MRPR) approaching the analysis of mixed ported systems.

2 Methodology

The advanced analysis of structures is based on the process of discretization of the system by the Finite Element Method (FEM) and the physical nonlinearity of the lattice structures is commonly addressed through the Plastic Hinge Method (PHM) or through the Plastic Zone Method (PZM). In MRP, plasticity is considered to be concentrated only in the nodal points of the structural system model and therefore has the advantage of having low computational demand; the MZP considers the inelasticity along the entire length of the finite element (distributed plasticity), and thus presents a high computational demand. [3] [5]

The Refined Plastic Hinge Method (RPHM), which will be adopted in this work, is considered a simplified but efficient alternative to the MZP. With the refinement of this method, it becomes possible to consider and capture the transition from the elastic regime to the plastic regime gradually with the use of null-length springs located at the element nodes, as illustrated in Figure 1. Lemes et al. [1] explains that this process of system stiffness degradation starts when the elastic regime limit imposed by a plasticization start curve is reached by the combination of normal forces and bending moment, which means that the cross section has linear behavior. elastic until this limit is exceeded.

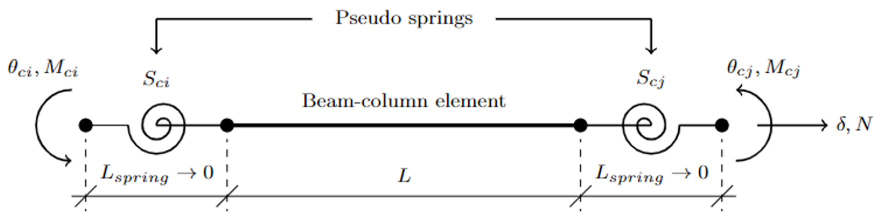


Figure 1. Hybrid finite element for semi-rigid connections simulation

It is important to highlight some considerations involving the FE formulation used in this paper:

- all elements are initially straight, prismatic, and the cross-section remains plane after deformation;
- the effects of global instability that may occur in three-dimensional problems (e.g., lateral and torsional buckling) are ignored considering a locking system out of plane;
- the effects of local instability are neglected, such as the buckling of the steel section plates, so the section can reach its full plastic rotation capacity;
- large displacements and rigid body rotations are allowed;
- the shear strain effects are ignored;
- yielding of the cross-section is governed by only normal stress; and
- the nonlinear behavior of the beam-to-column connections is defined exclusively by the flexure through the moment-rotation relationship.

With Figure 2 it is possible to better understand the element kinematics and the displacement notations (translations and rotations) used. For structural elements that have large displacements and/or large rotations, the global degrees of freedom contain the rigid motion and the deformational part; the co-rotational approach therefore aims to separate these parts.

Chang et al. [6] defines a local coordinate system (x', y') that moves continuously with the element, and is used to describe a deformational part of the motion whose rigid motion is described as displacements (u_{ig} and v_{ig}, and hard efforts $\alpha - \alpha_0$). He

The relation between global $(u_{ig}, v_{ig}, \theta_{ig}, u_{jg}, v_{jg}, \theta_{jg})$ and local $(\delta, \theta_i, \theta_j)$ degrees of freedom is obtained by a simple differentiation of the co-rotational displacements described in the function of global displacements and can be seen in Lemes [3]. In a matrix form, this relation is expressed by the following:

$$\Delta \mathbf{u}_l = \mathbf{B} \Delta \mathbf{u}_g \tag{1}$$

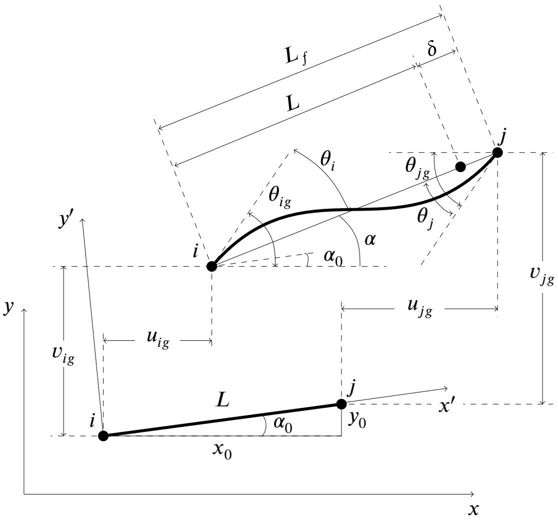


Figure 2. Displacements in global system coordinates

where $\Delta\mathbf{u}_l$ and $\Delta\mathbf{u}_g$ are the incremental displacements in local and global systems, respectively, and the transformation matrix **B** responsible for transforming the global displacements in local responses and vice-versa.

2.1 Element formulation

In the present work, the displacement-based formulation with concentrated plasticity in the nodal points is applied. In this case, the axial and flexural stiffness degradation occurs exclusively at the FE nodes. Then, the method is presented, introducing the material nonlinearity only. Some considerations and simplifications of this formulation can be seen in [3] [1].

In the structural system modelling, the hybrid beam-column finite element of length L, delimited by nodal points i and j (Figure 1), is used. This element has zero-length pseudo rotational springs at its ends, which are responsible for the plasticity simulation by means of the parameter S_c, discussed in Section 2.4. The finite element is referenced to the co-rotational system where the degrees of freedom are the rotations at nodes i and j, given by θ_i and θ_j , and the axial displacement in j, . The terms M_i, M_j and P represent the bending moments and the axial force in the respective degrees of freedom.

$$\begin{Bmatrix} \Delta N \\ \\ \Delta M_{pi} \\ \\ \Delta M_{pj} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 \\ \\ 0 & S_{ci} - \dfrac{S_{ci}^2\,(S_{cj} + k_{33})}{\beta} & \dfrac{S_{ci} k_{23} S_{cj}}{\beta} \\ \\ 0 & \dfrac{S_{cj} k_{32} S_{ci}}{\beta} & S_{cj} - \dfrac{S_{cj}^2\,(S_{ci} + k_{22})}{\beta} \end{bmatrix} \begin{Bmatrix} \Delta\delta \\ \\ \Delta\theta_{ci} \\ \\ \Delta\theta_{cj} \end{Bmatrix} \tag{2}$$

in which $\beta = (S_{pi} + k_{22})(S_{pj} + k_{33}) \;\; k_{32}k_{23}$

The terms $k_{11}, k_{22}, k_{23}, k_{32}$, and k_{33} are components of the beam-column stiffness matrix element, without the pseudo-springs, below, where the terms are further discussed in the following sections.

$$\begin{aligned} k_{11} &= \frac{E_s A}{L} & k_{22} &= \frac{E_s(3I_{efci} + I_{efcj})}{L} \\ k_{23} = k_{32} &= \frac{E_s(I_{efci} + I_{efcj})}{L} & k_{33} &= \frac{E_s(I_{efci} + 3I_{efcj})}{L} \end{aligned} \tag{3}$$

2.2 Concentrated plasticity approach

The co-rotational-FE model is adopted in the structural systems used, which brings a finite element beam-column, defined by nodes i and j, as shown in Fig. 2. So the finite element equilibrium, in incremental form, is given by:

$$\Delta \mathbf{f}_c = \mathbf{K}_c \Delta \mathbf{u}_c \tag{4}$$

The inelastic flexure terms of the matrix $\mathbf{K}_l$, are obtained by a similar approach proposed by Ziemian and McGuire [7]. In order to avoid any numerical integration in calculating element stiffness matrices during the analysis, the flexure terms are calculated considering the moment-curvature relationship $(M \times \Phi)$ tangent varying linearly along the finite element length to the likely situation of a linear moment gradient [7]. Thus:

$$EI\left(x\right) = \left[\left(1 - \frac{x}{L}\right) EI_{T,i} + \frac{x}{L} EI_{T,j}\right] \tag{5}$$

where $EI_{t,i}$ and $EI_{t,j}$ are the tangent flexural stiffness, obtained as described in Section **??**, in the nodal points i and j, respectively.

The reduced stiffness matrix (only the first fraction of flexure terms - $k_{cl(2,2)}$, $k_{cl(2,3)}$, $k_{cl(3,2)}$, $k_{cl(3,3)}$ - see Appendix A), $\mathbf{k}^*$, is defined using the second derivative of Hermite interpolation functions used in Eq. (**??**) [8], described in **N**, that is:

$$\mathbf{k}^* = \int_0^L \mathbf{N}^T EI_T\left(x\right) \mathbf{N} dx \tag{6}$$

in which:

$$\mathbf{N} = \left[\frac{2}{L}\left(2 - \frac{3x}{L}\right) \qquad \frac{2}{L}\left(1 - \frac{3x}{L}\right)\right] \tag{7}$$

2.3 Stiffness of pseudo-springs

The parameter S_c, in the Figure 1. is given per unit of flexural stiffness in radians, being defined within 3 domains. To determine its value, it is necessary to analyze what situation the structure is in. When in an elastic regime (limited by the plasticization start moment), S_c is taken as infinite, adopting a high value (numerical infinity). When the internal forces reach the moment of total plasticization, the complete degradation of the flexural rigidity is considered, that is, there is the formation of a plastic hinge. Thus, S_c is taken as null. Numerically, a very small value is adopted to avoid singularity problems. Finally. when in elastoplastic regime (an intermediate request in relation to the values mentioned below), it is considered that the loss of stiffness happens gradually through Equation 8, so that L is the finite element length; M_{pr} and M_{er} are, respectively, the ultimate strength and early plasticization moments, defined according to the procedure described in Section 3; E_a is the modulus of elasticity of steel; and Ihom is the moment of inertia of the homogenized section, obtained as described in 2.4

$$S_c = \frac{E_a I_{hom}}{L}\left(\frac{M_{pr} - M}{M - M_{er}}\right) \tag{8}$$

Being the moment of inertia of the homogenized section, I_{hom}, determined as follows form:

$$I_{hom} = \left[E_a + \frac{E_b}{E_a} I_b + \frac{E_c}{E_a} I_{efc}\right] \tag{9}$$

2.4 Effective moment of inertia of the cross section

When the element is in an uncracked state, its cross section is intact and, consequently, its moment of inertia is also intact. However, with the amplification of the load throughout the load history of the structure, the cracking of the concrete component of the mixed cross section of the element can occur, which causes its area to decrease, also modifying the value of the moment of inertia.

There is no specific equation to simulate the cracking of concrete from the rotational stiffness of the pseudo-springs. Thus, the way to consider this effect in the formulation of the element is to insert the effective moment of inertia within the expression of Sp and in terms referring to the bending of the stiffness Matrix 2.

The value of the effective moment of inertia of the concrete, I_{efc}, depends on the relationship between the acting moment and the resistant moment of the section, and can be calculated through Equation 4, proposed by Branson and Metz [9] , I_{efc1}, so that if $M \leq M_{cr}$: $I_{efc1} = I_c$, if $M > M_{cr}$, the I_{efc1} is given by the Equation 10

$$I_{efc} = \left(\frac{M_{cr}}{M}\right)^3 I_c + \left[1 - \left(\frac{M_{cr}}{M}\right)^3\right] I_{cr}, \quad I_{efc1} \leq I_c \tag{10}$$

where M_{cr} and M are, respectively, the cracking moment and the bending moment acting on the section, I_c is the moment of inertia of the intact concrete section (initial slope of the moment-curvature relationship for zero normal stress) and I_{cr} is the moment of inertia of the cracked section (assessed in the non-linear analysis of the section at the load limit point of the moment curvature relationship).

3 Numerical analysis

In this section we study one of the composite frames tested and presented by Bui and Kim [10] and Bui et. al [11]. Such structures are frames with the same dimensions, as illustrated in Figure xx, however with different cross sections of the columns: a tubular section (P1) and another rectangular (P2), as can also be seen in the figure below.

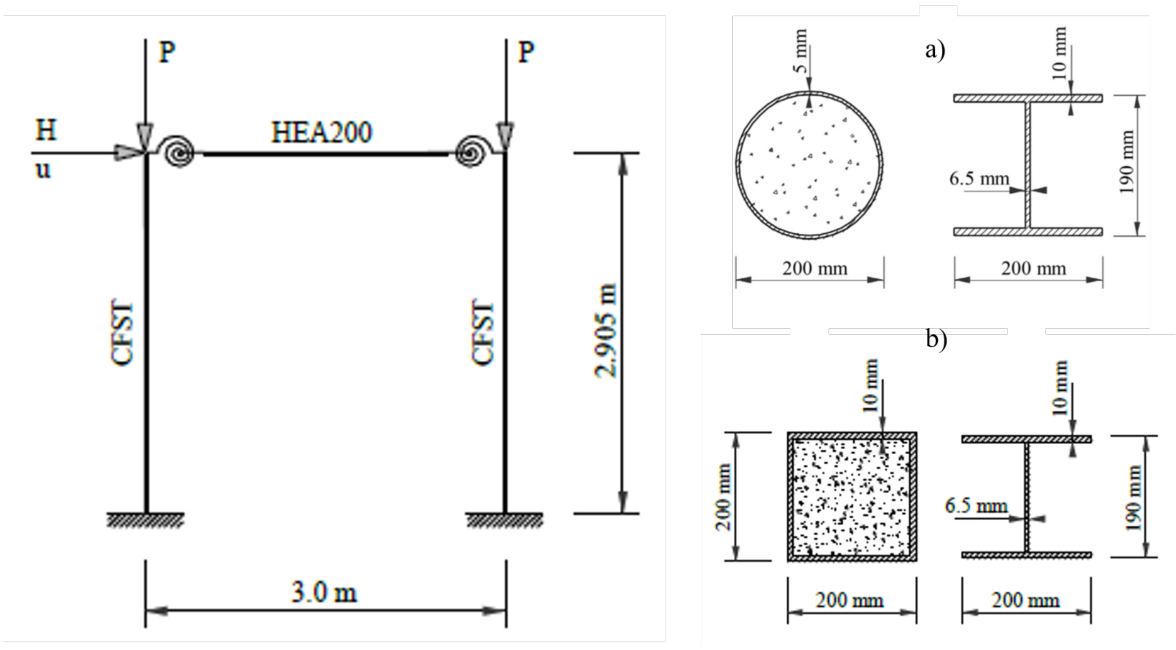


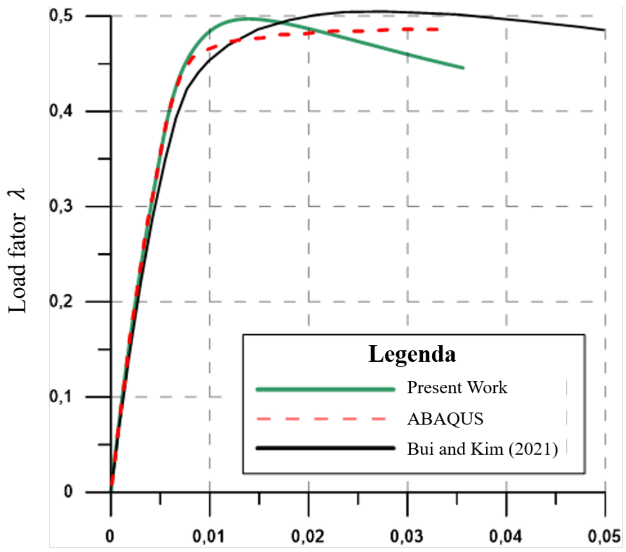
Figure 3. The portico and the two cross sections

Both structures have applied loads P vertically at the top of the column, and a horizontal load H at the top of the column on the left. The authors establish the values for P=28000 Kn and H = 35 kN. The results obtained for frames P1 and P2 are obtained through an increasing load factor λ applied to these curved loads, in order to obtain the load x displacement curve and analysis using the Branson and Metz Equation (1963). It is interesting to say that a low sensitivity to the influence of the mesh was noticed, demonstrating that less refined meshes generate already convergent results.

As for the properties of the materials used, the authors cite that for steel, E_s = 200 GPa, υ = 0.3, and the yield strength (f_y) and ultimate strength (f_u) are 250 MPa and 400 MPa, respectively. The concrete compressive strength (f_c) the concrete colors is 38 MPa.

In the graphs below, illustrated in the figure 4, it is possible to verify the results obtained by the formulation proposed in this work, with P1 on the right and P2 on the left. In the same graphs it is also possible to visualize the results obtained by [10] and Bui et. al [11], being one of the curves obtained through its formulation and the second curve obtained through the ABAQUS software.

In both structures the behavior obtained is very similar between the three curves. In P1 the initial stiffness is practically the same in the three results and the critical loads obtained are very close. In the case of P2, there is a



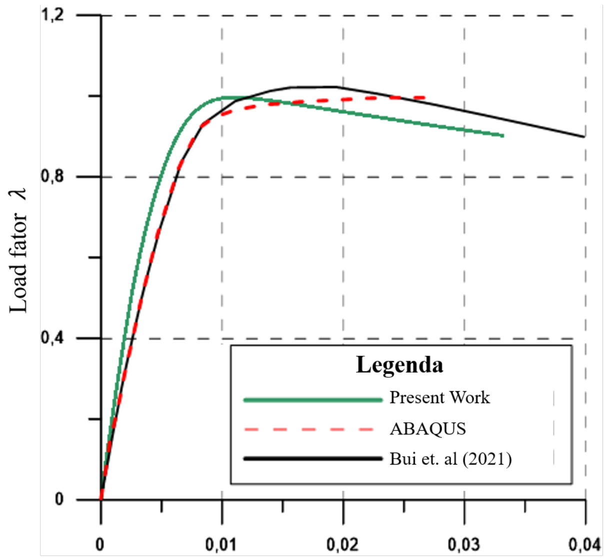


Figure 4. Equilibrium of P1 (left) and P2 (right)

small difference in the initial stiffness as well as in the critical loads obtained.

4 Conclusions

This article presents a consideration between nonlinear cross-section for steel-concrete analysis considering rigid and semi-rigid connections to beam and columns. (The formulation governing the beam-column element is constructed as linearity effects are not column effects P - δ P - Δ), and material nonlinearities. and uses o concentration the analysis of displacements at nodal points for an analysis of steel-concrete beams Method of Rotula Plastica Refindo to capture a transition from the elastic regime to the plastic regime gradually with the use of null-length springs located at the nodes of the element.

The example of a frame section is consistent, as it was simulated considering the results presented in the literature [10] [11]. Some differences considered can be considered small, but can be justified by things. yield strength at the flange and web of the steel section; and not linear process stop criterion.

A test on the finite element mesh refinement was also performed. It was verified in the presented results that the formulation presents low sensitivity to mesh refinement, highlighting that less refined meshes already present satisfactory results in both methodologies. A sensitivity analysis to the input data was also performed verifying the sensitivity of the formulation presents. In general, it is concluded that the formulation is efficient. Given this, it is likely that the presentation will be considered satisfactory results and can be considered in structural construction.

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