Reliability study of the ductility of reinforced concrete beams based on the NBR 6118 (2014)

Ana Carolina da Silva Pacheco[1], Wellison José de Santana Gomes[2]

[1]*Dept. of Civil Engineering, Federal University of Santa Catarina*
João Pio Duarte da Silva S., 88037-000, Santa Catarina, Brazil
aanapacheco@hotmail.com
[2]*Dept. of Civil Engineering, Federal University of Santa Catarina*
João Pio Duarte da Silva S., 88037-000, Santa Catarina, Brazil
wellison.gomes@ufsc.br

Abstract. Semi-probabilistic methods are the procedures used in most structural design codes to try to ensure the safety of structures. However, there are several uncertainties related to the structural models, as well as uncertainties in the loads, in the geometric properties of the elements and in the mechanical properties of the materials employed. Therefore, it becomes necessary to apply probabilistic methods to verify safety, and structural reliability methods appear in this scenario as a way to calculate probabilities of failure taking into account the uncertainties involved. Several studies were developed about the reliability related to the strength of reinforced concrete beams. However, reliability analyzes focusing on the ductility of these structural elements are still rare in the literature. In the present paper it is intended to determine the level of reliability related to the ductility of reinforced concrete beams designed in accordance with the Brazilian code for the design of reinforced concrete structures (NBR 6118). For this, an analytical model which tries to adequately represent the non-linear behavior of the beams is implemented. To evaluate the structural reliability, a limit state function based on strains in the tensile rebar is presented. A code in MATLAB is developed, in which the Monte Carlo simulation is implemented and a beam is evaluated for a rage of concrete strength from 20 to 90 MPa. From this study, it is concluded that in some cases the reliability related to ductility can be considered low.

Keywords: beams, reinforced concrete, ductility, structural reliability, Monte Carlo simulation.

1 Introduction

One of the properties that affects the behavior of reinforced concrete elements is the ductility. It is defined as their capacity to withstand large plastic deformations without significant loss of strength, reaching failure only after considerable accumulation of plastic deformation energy. When a structural element has these characteristics, it is said it has ductile behavior. Otherwise, it is said to have brittle behavior, and it will fail in a brittle way [1].

In the design of reinforced concrete structures, it is necessary to ensure that there is adequate strength to prevent failure, as well as enough ductility so that, if it occurs, it is not brittle. In the context of a structural system, the ductility of the elements guarantees the ability to redistribute internal stresses as the cross sections undergo plasticization. This ductile behavior avoids sudden ruptures, increasing the security levels of the structural systems in relation to ultimate limit states [2, 3].

From the design point of view, several structural design codes require that the ductility of the elements is guaranteed, by imposing restrictions over the steel's minimum strain and the normalized neutral axis depth in the flexural design of cross sections. This approach is easy to consider in the design phase, as it is defined by known control parameters during the design process. However, several important mechanisms that interfere in the general behavior of reinforced concrete beams are not explicitly taken into account, such as: the damage evolution and, consequently, the cracking over time while the load acts during the use of the edification; and the contribution of the tensile concrete between cracks [4].

A comprehensive inelastic nonlinear analysis provides an accurate method for evaluating the adequacy of ductility of structural systems. However, such an analysis often is not practical in routine design, which usually relies heavily on meeting design code requirements. Some recommendations for modifying the code provisions have been proposed previously, but these studies were conducted primarily in a deterministic framework [5].

2 Ductility of reinforced concrete beams

In the design of reinforced concrete beams, it must be ensured that there is satisfactory safety. The safety is conditioned to the verification of limit states, which are situations in which the structural element performs inadequately for its purpose. Limit states can be classified into ultimate or service limit states. The ultimate limit state is associated with any form of ruin that paralyzes the use of the structure, while the service limit state corresponds to the condition in which the use of the structure becomes impaired, due to excessive deformation or cracking.

According to Araújo [6], the ultimate limit state, corresponding to the failure of a cross section, can occur due to concrete crushing or excessive deformation of the tensile rebar. The NBR 6118 [7] admits the ultimate limit state when the strain distribution along the height of a cross section falls into one of the domains illustrated in Fig. 1.

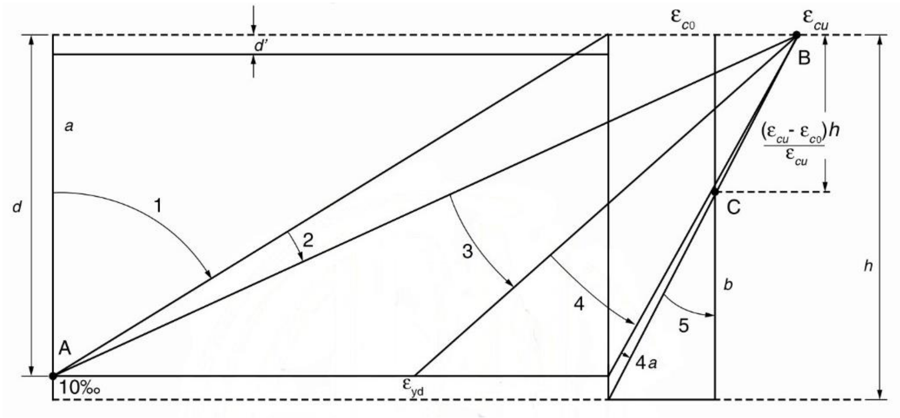


Figure 1. Design domains of a cross section

In simple bending, which is the predominant solicitation in beams, failure can occur in domains 2, 3 and 4. A beam is considered to have ductile behavior when the steel yields, that is, when the steel's strain at failure is greater than the steel's yield strain, which occurs in domains 2 and 3. In domain 4, however, due to the excess of reinforcement, the steel does not yield and the rupture occurs by concrete crushing, without prior notice. In the design of beams, this type of situation can be avoided with the use of compressive reinforcement [6]. NBR 6118 has as one of its objectives to guarantee the functionality of structural elements and prevent their fragile rupture. Thus, it requires a minimum tensile reinforcement ratio, in order to avoid brittle failures at the boundary between domains 1 and 2 [8]. In addition, NBR 6118 started to adopt, since its last version released in 2014, a limitation for the normalized neutral axis depth that leads to a failure distant from domain 4 and ensures greater ductility for the beams. In this way, part of domain 3 is eliminated.

3 Probabilistic procedure

3.1 The proposed limit state

In order to analyze the level of reliability related to the ductility of reinforced concrete beams designed in accordance to the NBR 6118, a limit state function based on the strain of the tensile rebar is used. This function was adapted from Baji *et al.* [5]. The limit state function proposed by these authors is based on the fact that brittle failure occurs when the steel's strain on the ultimate limit state is smaller than the steel's yield strain, which corresponds to domain 4 of design. However, the NBR 6118 indicates that part of domain 3 must be eliminated in order to ensure greater ductility, as presented in section 2. Thus, in this paper, brittle failure is deemed to occur when the steel's strain in the ultimate limit state (ε_s) is less than a certain limit strain ($\varepsilon_{s,lim}$). Then, the limit state function is given by:

$$g = \frac{\varepsilon_s}{\varepsilon_{s.\lim}} - 1. \tag{1}$$

In the proposed limit state function, the ultimate limit state is deemed to occur by concrete crushing. Thus, considering an elastic-perfectly-plastic behavior for the steel reinforcement and referring to Fig. 2, the strain at tensile rebar can be calculated as:

$$\varepsilon_s = \varepsilon_{cu}\left(\frac{1}{x/d} - 1\right), \tag{2}$$

where ε_{cu} is the ultimate strain of concrete, x is the neutral axis depth and d is the effective height of the beam's cross section.

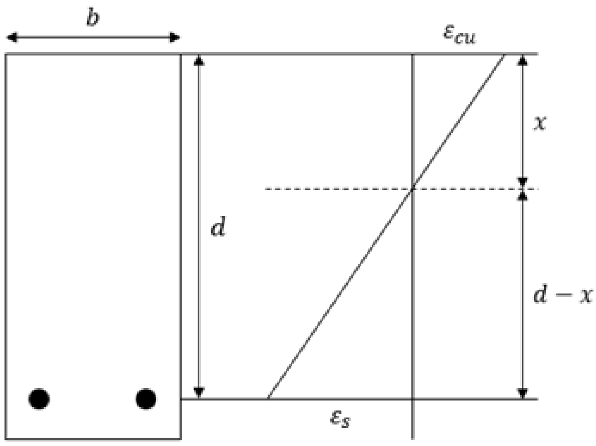


Figure 2. Strain diagram at the design state

By introducing a model error (θ) in predicting the neutral axis depth, the limit state function shown in eq. (1) can be rewritten as:

$$g = \frac{\varepsilon_{cu}}{\varepsilon_{s,\lim}}\left(\frac{1}{\theta(x/d)} - 1\right) - 1. \tag{3}$$

The limit strain can be calculated by $\varepsilon_{s,\lim} = \varepsilon_{cu}[-1 + 1/(x/d)]$, replacing ε_{cu} and x/d by the limit values established in NBR 6118. Such values are indicated in Tab. 1.

Table 1. Limit values of ε_{cu} and x/d for different concrete strengths

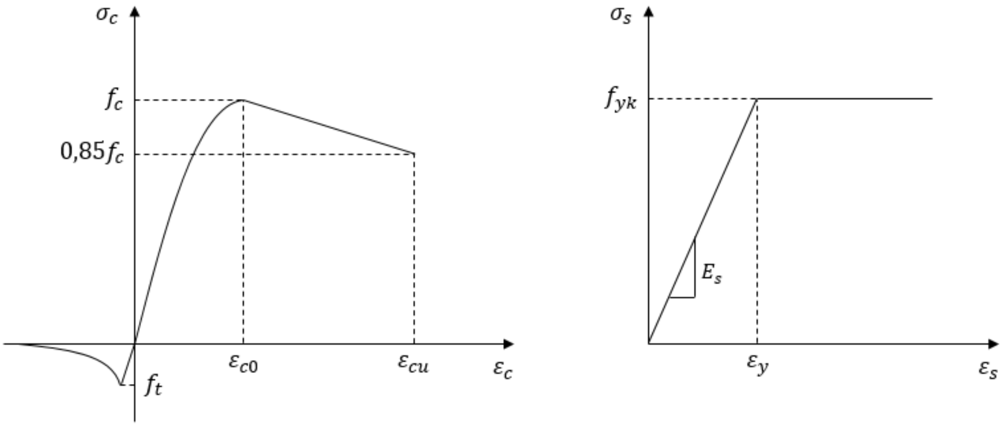
f_{ck} (MPa)	$\leq$ 50	> 50
ε_{cu}	0.0035	$0.0026 + 0.035\left(\frac{90 - f_{ck}}{100}\right)^4$
$(x/d)_{lim}$	0.45	0.35

Thus, the limit strains for different concrete strengths assume the values shown in Tab. 2.

Table 2. Limit strains for different concrete strengths

f_{ck} (MPa)	$\leq$ 50	60	70	80	90
$\varepsilon_{s,lim}$	0.00428	0.00536	0.00493	0.00484	0.00483

The neutral axis depth is determined using an iterative procedure, in which it is varied until the equilibrium of the cross section forces is attended. The stresses and strains of concrete and steel are obtained directly from the stress-strain curves of the materials, assuming an elastic-perfectly-plastic behavior for the steel and using the constitutive models of Hognestad [9] and Stramandinoli and La Rovere [10] for compressed and tensile concrete, respectively. In Fig. 3a and b, the considered stress–strain relationships for concrete and steel materials are shown.



(a) Concrete material

(b) Steel material

Figure 3. Assumptions for the material behavior

To represent the behavior of the cross section properly, it is divided into layers. Since the strain diagram is linear, it is possible to determine the strain in each layer by means of triangle similarity. Thus, knowing the strain diagram, it is possible to determine the stress on each layer through the stress-strain curve equation considered.

The model of Hognestad [9] for compressed concrete adopts the following equations:

$$\sigma_{c,i} = \begin{cases} f_c\left[2\left(\dfrac{\varepsilon_{c,i}}{\varepsilon_{c0}}\right)-\left(\dfrac{\varepsilon_{c,i}}{\varepsilon_{c0}}\right)^2\right], & for \quad \varepsilon_{c,i} \leq \varepsilon_{c0} \\ f_c\left[1-0.15\left(\dfrac{\varepsilon_{c,i}-\varepsilon_{c0}}{\varepsilon_{cu}-\varepsilon_{c0}}\right)\right], & for \quad \varepsilon_{c0} < \varepsilon_{c,i} < \varepsilon_{cu} \end{cases}, \tag{4}$$

where $\sigma_{c,i}$ is the stress in the layer i of the cross section, $\varepsilon_{c,i}$ is the strain in the layer i and ε_{c0} is the strain at the peak of the stress-strain curve.

The tension-stiffening model of Stramandinoli and La Rovere [10] allows to obtain the tensile stress of the concrete through the following equations:

$$\sigma_{c,i} = \begin{cases} E_c \cdot \varepsilon_{c,i}, & for \quad \varepsilon_{c,i} \leq \dfrac{f_t}{E_c} \\ f_t \cdot \exp\left(-\iota \cdot \varepsilon_{c,i} \cdot \dfrac{E_c}{f_t}\right), & for \quad \dfrac{f_t}{E_c} < \varepsilon_{c,i} < \varepsilon_y, \\ 0, & for \quad \varepsilon_{c,i} \geq \varepsilon_y \end{cases} \tag{5}$$

where E_c is the concrete's elasticity modulus, f_t is the maximum tensile capacity of the concrete, ε_y is the steel's yield strain and $\iota = 0.017 + 0.255(\eta \cdot \rho_e) - 0.106(\eta \cdot \rho_e)^2 + 0.016(\eta \cdot \rho_e)^3$, being η the relation between the steel's and the concrete's elasticity modulus and ρ_e the effective reinforcement ratio.

3.2 Structural reliability analysis

As is well-known, the probability of failure P_f and the reliability index β are used as a measure of structural safety. They are given as follows:

$$P_f = \Pr(g < 0), \tag{6}$$

$$\beta = -\phi^{-1}(P_f), \tag{7}$$

where ϕ^{-1} is the inverse standard normal distribution function.

There are several methods that can be used to calculate the probability of failure. The one used in this paper is the Monte Carlo simulation. It is implemented in MATLAB and 10 thousand simulations are used to evaluate de probability of failure.

3.3 Random variables

A probabilistic analysis requires the establishment of a set of random variables that allows evaluating the expected variability in a real structure. However, the number of variables that affect the actual behavior of a model can hardly be measured. From a research point of view, this number should be limited. A reasonable criterion in the selection of these variables is to consider those that are known to be significant in relation to the limit state studied. In this paper, the random variables indicated in Tab. 3 were considered for the reliability analysis.

For the maximum tensile capacity of the concrete (f_t), it was used the mean values indicated on NBR 6118 and the statistical model indicated on JCSS [11]. The concrete's elasticity modulus equation used is the one indicated by the *fib* Model Code [12].

Table 3. Statistical models for the random variables

Variable	Distribution	Unit	Mean (μ)	Standard deviation (σ)	Reference
b	Normal	mm	b	$4 + 0.006b \leq 10$	JCSS [11]
d	Normal	mm	d	15	Israel *et al.* [13]
f_c	Normal	MPa	$f_{ck} + 1.65\sigma$	4	NBR 12655 [14]
θ	Weibull	-	1.04	0.27μ	Baji *et al.* [5]
E_s	Lognormal	GPa	E_s	0.033μ	Mirza *et al.* [15]
A_s	Normal	cm²	A_s	0.015μ	Stucchi and Santos [16]
f_y	Normal	MPa	$1.22f_y$	0.04μ	Santiago [17]
ε_{cu}	Lognormal	-	0.0037	0.21μ	Baji *et al.* [5]

3.4 Description of the beams under study

To investigate the reliability related to the ductility of beams designed based on the criteria of NBR 6118, a cross section with a base of 20 centimeters and a height of 40 centimeters is analyzed. It is evaluated for a concrete strength range of 20 to 90 MPa and CA-50 steel (f_{yk} = 500 MPa). For the design of the cross section, the case of simple reinforcement is considered. In order to analyze the less favorable situation in terms of ductility, the largest tensile reinforcements allowed by the code are adopted. Simple reinforcement is used for $x/d \leq (x/d)_{lim}$, where $(x/d)_{lim}$ is equal to 0.45 for $f_{ck} \leq 50$ MPa and 0.35 for $f_{ck} > 50$ MPa. Therefore, it is possible to state that the largest tensile reinforcement occurs when x/d is equal to $(x/d)_{lim}$ itself. When considering $x/d = (x/d)_{lim}$ and following the design procedures indicated by the NBR 6118, the tensile reinforcements indicated in Table 4 are obtained.

Table 4. Tensile reinforcements for different concrete strengths

f_{ck} (MPa)	20	30	40	50	60	70	80	90
A_s (cm²)	6.28	10.05	12.57	16.08	14.07	16.08	16.08	16.08
Tensile reinforcement	2φ20mm	5φ16mm	4φ20mm	2φ32mm	7φ16mm	2φ32mm	2φ32mm	2φ32mm

4 Results and discussions

Using eq. (3), reliability indices for non-ductile failure can be calculated. In Fig. 4 the resulting reliability indices for a wide range of concrete compressive strengths are shown. It is important to emphasize that the reliability indices are referred to the probability that the failure is brittle, given that the failure had already occurred. The results suggest a downward trend of the reliability index with the increase of f_{ck} for group I concretes ($f_{ck} \leq$ 50 MPa). For group II concrete (f_{ck} > 50 MPa), the reliability index tends to increase with the increase of f_{ck}. This is related to the influence of the normalized neutral axis depth (*x/d*) and the parameters of the rectangular stress block of the compressed concrete, used in the design of the tensile reinforcement. For group I concretes, the parameters of the rectangular stress block of the compressed concrete are constant, therefore, with increasing f_{ck}, the tensile reinforcement increases and this generates a reduction in the reliability index. For group II concretes, the limit for the normalized neutral axis depth is smaller, which generates an increase in the reliability index. Furthermore, for group II, the parameters of the rectangular stress block of the compressed concrete are calculated with the f_{ck}, and decrease as it increases. Thus, even with the increase of f_{ck}, the dimensioned tensile reinforcement does not increase considerably, which makes the reliability index increase, as the configuration of the beam moves away from the over-reinforced condition.

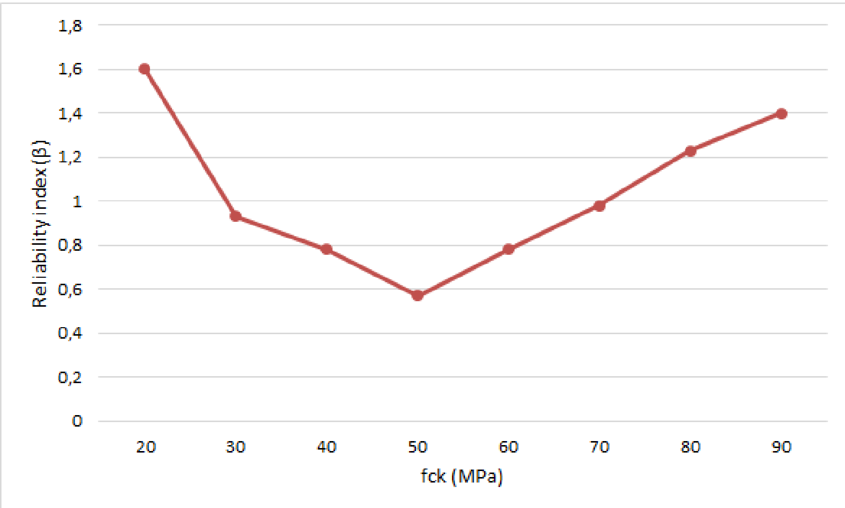


Figure 4. Reliability indices for different concrete strengths

To establish a conclusion about the level of safety achieved by the beams studied here, it is necessary to set the target value with which the reliability indices can be compared to. However, little information is found in the literature about appropriate values for reliability indices for ductility. Ito and Sumikama [18] proposed a target reliability index equal to 2.3, and Baji *et al.* [5] considered this same value to evaluate the reliability related to ductility for various codes. In the present paper, all reliability indices obtained are below 2, and some of them are very low. This highlights the need for more research on adequate reliability levels related to the ductility of reinforced concrete elements.

5 Conclusions

In general, a lack of uniformity in the reliability indices obtained for different concrete strengths was observed. It was found that the probability of the occurrence of brittle failure can reach almost 30%, and the worst levels of reliability obtained are for beams designed with concrete strengths equal to 40 and 50 MPa. The results showed that the ductility decreases by increasing the concrete strength for group I concretes, and increases by increasing the concrete strength for group II concretes. This occurs, mainly, due to the influence of the normalized neutral axis depth limit and the parameters of the rectangular tension block of the compressed concrete considered in the design of the tensile rebar in each case.

This paper showed that, although the strength-related limit state has been extensively studied, the reliability requirements to achieve minimum ductility have received little attention. It has been found that the uncertainty associated with the ductility limit state is high, and in part this is because the ductility depends on multiple random variables and there is a relatively high degree of uncertainty associated with each one.

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References

[1] M. Machado. Estudo experimental da ductilidade de vigas em concreto armado reforçadas à flexão utilizando compósitos com tecido de fibras de carbono. Master thesis, Pontifícia Universidade Católica do Rio de Janeiro, 2005.
[2] I. Kara and A. Ashour, "Moment redistribution in continuous FRP reinforced concrete beams". *Construction and Building Materials*, vol. 49, pp. 939-948, 2013.
[3] M. Ko, S. Kim and J. Kim, "Experimental study on the plastic rotation capacity of reinforced high strength concrete beams". *Materials and Structures*, vol. 34, n. 5, pp. 302-311, 2001.
[4] C. Nogueira and I. Rodrigues, "Novo modelo de dimensionamento de vigas em concreto armado à flexão simples considerando o fator de ductilidade". *Revista IBRACON de Estruturas e Materiais*, vol. 13, n. 1, pp. 120-141, 2020.
[5] H. Baji, R. Ronagh and R. Melchers, "Reliability of ductility requirements in concrete design codes". *Structural Safety*, vol. 62, pp. 76-87, 2016.
[6] J. M. Araújo. *Curso de concreto armado*. Dunas, 2014.
[7] Associação brasileira de normas térmicas (ABNT). *NBR 6118: projeto de estruturas de concreto – procedimento*. Rio de Janeiro, 2014.
[8] R. Silva and C. Araújo, "Estudo das taxas mínimas de armadura de tração em vigas de concreto armado segundo a ABNT NBR 6118:2014". *Essentia*, vol. 16, pp. 38-54, 2015.
[9] E. Hognestad. A study of combined bending and axial load in reinforced concrete members. University of Illinois at Urbana-Champaign, 1951.
[10] R. Stramandinoli and H. La Rovere, "An efficient tension-stiffening model for nonlinear analysis of reinforced concrete members". *Engineering Structures*, vol. 30, n. 7, pp. 2069-2080, 2008.
[11] JCSS. *Probabilistic model code*. Technical University of Denmark, 2002.
[12] CEB-FIP. *fib Model Code 2010*. Draft Model Code. Ernst & Sohn, 2010.
[13] M. Israel, B. Ellingwood and R. Corotis, "Reliability-based code formulations for reinforced concrete buildings". *Journal of Structural Engineering*, vol. 113, n. 10, 1987.
[14] Associação brasileira de normas térmicas (ABNT). *NBR 12655: concreto de cimento Portland – preparo, controle, recebimento e aceitação – procedimento*. Rio de Janeiro, 2015.
[15] S. Mirza and J. Macgregor, "Variability of mechanical properties of reinforcing bars". *Journal of the Structural Division*, vol. 105, n. 5, pp. 921-937, 1979.
[16] D. Santos, F. Stucchi and A. Beck, "Confiabilidade de vigas projetadas de acordo com as normas brasileiras". *Revista IBRACON de Estruturas e Materiais*, vol. 7, n. 5, pp. 723-746, 2014.
[17] W. Santiago. Calibração baseada em confiabilidade dos coeficientes parciais de segurança das principais normas brasileiras de projeto estrutural. Phd Thesis, Universidade de São Paulo, 2019.
[18] K. Ito and A. Sumikama, "Probabilistic study of reduction coefficient for balanced steel ratio in the ACI code". *ACI Structural Journal*, vol. 82, n. 5, pp. 701-709, 1985.