XLIII CILAMCE
21–25, November, 2022
FOZ DO IGUAÇU – PR – BRAZIL

Optimization of 3d ground-structures with constraints of overlapping bars

José P. G. Carvalho[1], Afonso C. C. Lemonge [2], Beatriz de Souza Leite Pires de Lima[1], Patrícia Habib Hallak[2]

[1]*Civil Engineering Program - Federal University of Rio de Janeiro*
Av. Athos da Silveira Ramos, 149 Centro de Tecnologia - Bloco B, Sala 101, 21941-909, Rio de Janeiro/RJ, Brazil
jose.carvalho@coppe.ufrj.br, bia@coc.ufrj.br
[3]*Department of Applied and Computational Mechanics - Federal University of Juiz de Fora*
Rua José Lourenço Kelmer s/n, 36036-900, Juiz de Fora/MG, Brazil
afonso.lemonge@ufjf.br, patricia.hallak@ufjf.br

Abstract. The ground-structure is a manner of representing solid geometries in an optimization problem. Optimizing mass in ground-structures truss models means leaving the geometry with bars where the main stress chains occur, i.e., searching for its performance. In real-world applications, describing geometry as a ground-structure might be beneficial since the results are composed of bars and might be easier to manufacture. Also, depending on the length of the bar, it might represent multiple elements in a continuous traditional mesh, which will likely reduce the computational cost of the design's analysis. The solutions from traditional optimization processes with meshes consisting of polygonal elements and ground-structure approaches should be quite similar, as this last method has been largely used lately in literature. This work deals with topology optimization of 3d ground-structures truss models with constraints of non-overlapping bars and maximum material volume. The algorithm chosen is the Differential Evolution (DE) coupled with the Adaptive Penalty Method (APM) to handle the constraints.

Keywords: ground-structure, structural optimization, metaheuristic

1 Introduction

Topology optimization is a much desirable process to maximize the efficiency of structures. It consists of removing material from areas where it is not demanded and keeping it where the stresses occur. These structures are often solid geometries used in many engineering fields where maximum efficiency and weight reduction are needed, such as car parts, airplane parts, and others. The final optimized topologies must satisfy its requirements for performance and stability, leading to reliable results as a final product. Solving problems of this nature is not an easy task.

The ground-structure is traditionally found in the literature under the form of minimization *compliance* with constraints of material volume available. As important contributions in this area, the following papers can be related: Zegard and Paulino [1], Mela [2], Kanno [3], Zhang et al. [4] and others. However, many problems reflect very complex physical phenomena, and using deterministic algorithms is impractical due to the complexity of finding the derivatives of the objective function concerning design variables. In this sense, some examples of ground-structures have been recently expanded to analysis with natural frequencies, elastic critical load factor, stresses, and displacements, where an optimization process using metaheuristics is employed, as proposed in Carvalho et al. [5]. This paper deals with topology optimization of three-dimensional truss structure that minimizes the nodal displacements at the node where the load is applied, as a reference to *compliance*, with constraints of overlapping bars and available material volume.

This text is organized as follows: Section 2 describes the optimization problem on its mathematical form. Section 3 defines the proposed numerical example. Section 4 shows the results obtained for this problem and, finally, Section 5 shows the conclusions and future works.

2 The structural optimization problem

The formulation of the multi-objective structural optimization problem analyzed in this paper can be written as:

$$\begin{aligned} \min \quad & \delta_f(\mathbf{x}) \\ \text{s.t.} \quad & \text{non} - \text{overlapping bars and maximum volume constraints} \end{aligned} \tag{1}$$

where **x** is the vector containing the areas of the truss, and δ_f is the nodal displacement where the load is applied. In this paper, a three-dimensional ground-structure is optimized under constraints non-overlapping bars and maximum volume. The nodal displacements are obtained by solving the following equilibrium equation:

$$\underset{\sim}{u} = \underset{\approx}{K}^{-1}\underset{\sim}{F} \tag{2}$$

where $\underset{\sim}{u}$ is the displacements vector, $\underset{\approx}{K}$ is the stiffness matrix, and $\underset{\sim}{F}$ is the nodal vector force applied. The Finite Element Method (FEM) was used to discretize the ground-structure in bar elements and solve the problem.

The constraints are normalized to work according to the adaptive penalty method adopted and written as:

- Overlapping bars constraint:

$$n_{c_j}(\mathbf{x}) - 1 < 0, \quad 1 \leq j \leq m_c, \tag{3}$$

  where **x** is the vector containing the cross-sectional areas of the bars, n_{c_j} is the number of bars that cross the $j-th$ bar and m_c is the number of bars.
- Volume constraint:

$$\frac{v(\mathbf{x})}{\bar{v}} - 1 \leq 0; \tag{4}$$

  where **x** is the vector containing the cross-sectional areas of the bars, v is the volume of the structure and $\bar{v}$ is the maximum allowable volume.

The optimization algorithm used in this paper is the Differential Evolution (DE) proposed by Storn and Price [6]. Furthermore, the Adaptative Penalty Method was chosen to handle the constraints [7]. The structural analysis simulator is performed in C language compiled to MATLAB ecosystem, where the main optimization and constraint algorithms are carried out. Regarding computational time consumed, the machine employed for the optimization has a processor, Ryzen 7 3700X, and 16Gb of RAM. The evolution of the population was performed with parallel computation support of MATLAB, where each new individual was evaluated into one of the 8-cores available, making the process, theoretically, eight times faster.

3 Numerical experiment - cantilever beam

This section describes the numerical experiment as follows: it consists of a cantilever beam with a down-force applied at the free extremity, as depicted in Fig. 1. It has 832 bars, and symmetry was considered for the cross-sectional areas of the bars (design variables) to improve the performance of the optimized structure, reducing the search space to the size of 453 variables. As mentioned, overlapping bars are not allowed, the material density was equal to 7800 $\frac{kg}{m^3}$, elasticity modulus was equal to 210 GPa, and the maximum allowable volume was set to 0.3m³. The mesh has 10m x 4m x 4m, and the load P was set equal to 1000 kN. The lower and upper bounds for the cross-sectional areas are 0 cm^2 and 20 cm^2, being the *cutoff* cross-sectional area (where bars with a cross-sectional area equal to or less than this value are removed from the structure) variable (6 cm^2, 12 cm^2, 16 cm^2 and 24 cm^2) to investigate different topologies for these different values.

Since it is a very complex problem, the experiment was evaluated into 20 independent runs, with a population size of 100 individuals throughout 2500 generations each. A reduced order model (ROM) (Sanders et al. [8]) was used when the algorithm adopted co-linear bars.

4 Results

Figs. 2-4 shows the different topologies obtained for different values of *cutoff* cros-sectional area. The objective function and constraints values are shown in Table 1.

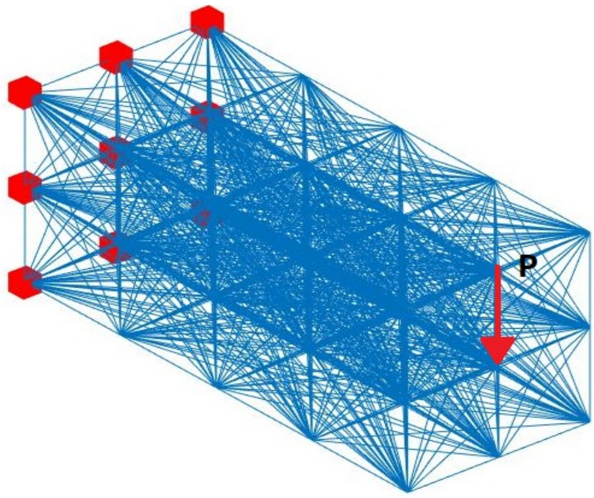

Figure 1. Ground-structure of the cantilever beam with 832 bar.

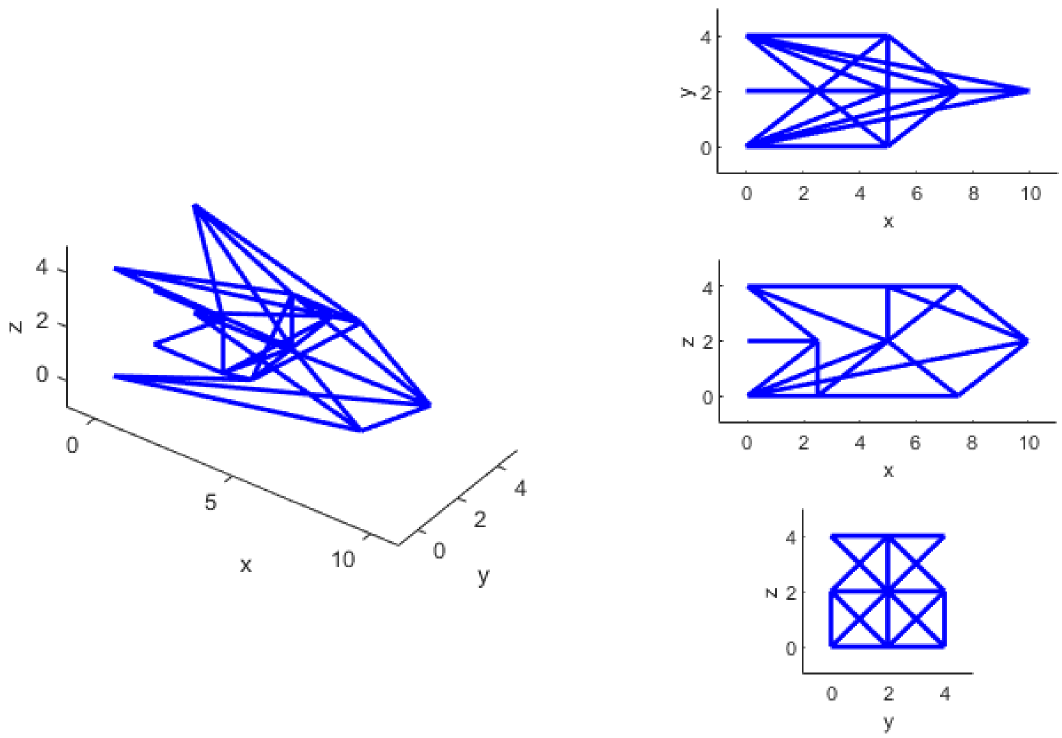

Figure 2. The best result found when *cutoff* cross-sectional area = 6 cm^2 (number of bars = 33).

Table 1. Results for different *cutoff* cross-sectional areas.

Cutoff area (cm^2)	6	12	18	24
Objective function (m)	0.041	0.060	0.085	0.055
Material volume (m^3)	0.30	0.30	0.16	0.25
Number of bars	33	47	4	6
Time (s)	1067	1055	1050	1061

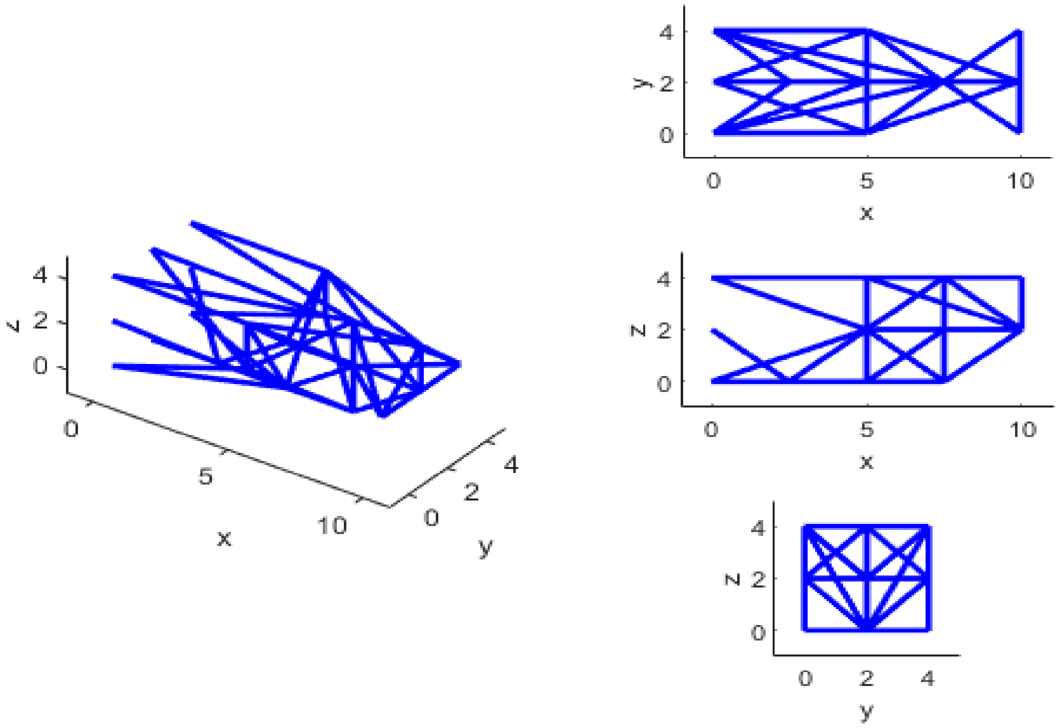

Figure 3. Topology obtained when *cutoff* cross-sectional area = 12 cm^2 (number of bars = 47).

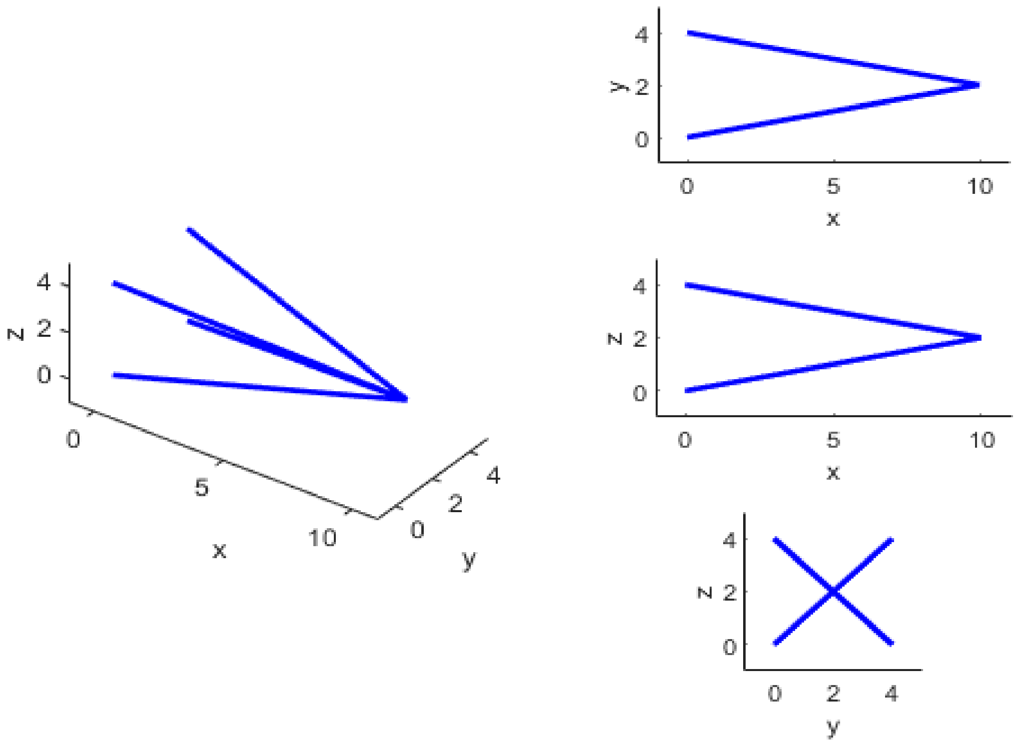

Figure 4. Topology obtained when *cutoff* cross-sectional area = 18 cm^2 (number of bars = 4).

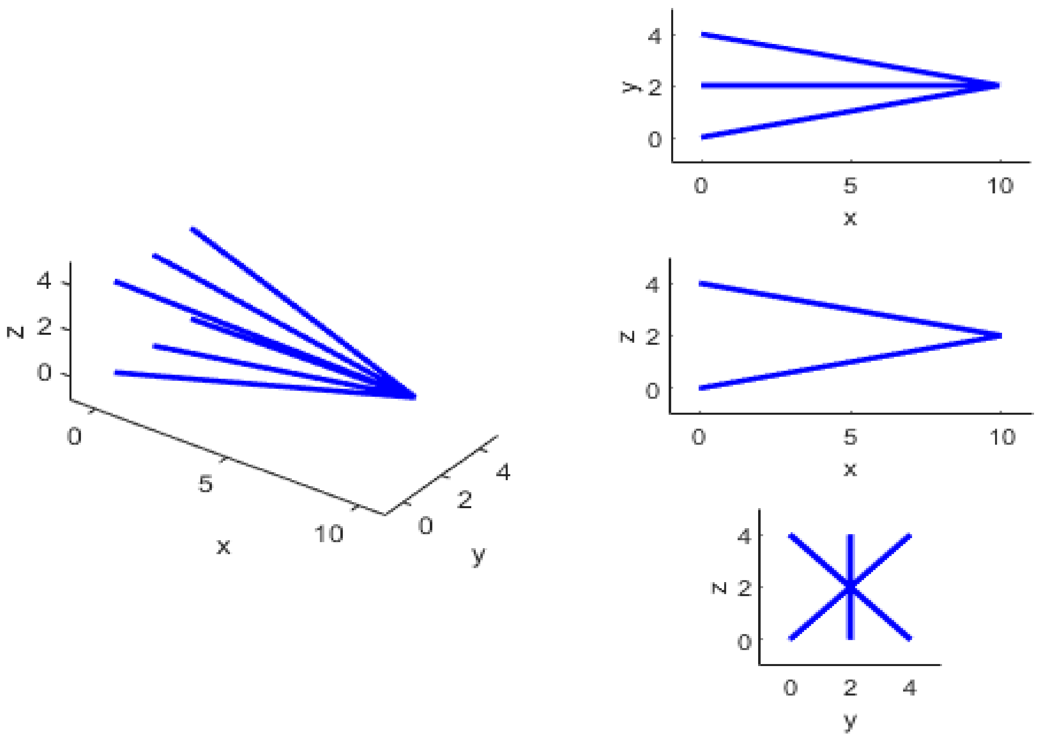

Figure 5. Topology obtained when *cutoff* cross-sectional area = 24 cm^2 (number of bars = 6).

5 Conclusions and future works

The results showed interesting results and intuitive geometries. The obtained topologies showed that once the *cutoff* cross-sectional area increases (*cutoff* higher than 50% of the upper bound), the final number of bars is reduced. One might expect that reducing the overall number of bars would imply fewer bars with higher values of areas resulting in higher objective function values and lower volume, which could be observed for *cutoff* = 16 cm^2. However, the result for *cutoff* = 24 cm^2 showed a topology with extra 2 bars than the previous case that led to an increase in volume and reduction of objective function value, adding more stiffness to the structure. For both cases, the topology was visually according to what was expected due to the physical problem. Hence, one might conclude that this methodology worked for this kind of problem. Since it is a very complex problem to solve involving many design variables, it is expected that an increase in the size of the initial population as well as the number of generations, the results can be more consistent and follow a better pattern (increase/decrease in volume and objective function). Future works are intended to run this problem with a frame model and insert stresses, natural frequencies, and critical load factors as new parameters to the optimization problem.

Acknowledgements. The authors wish to thank the CNPq (Grants 309042/2020-8 and grant 308105/2021-4), FAPEMIG (Grant TEC PPM-00174-18), and CAPES (finance code 001) for their support.

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