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Implementation and Validation of a 6-Degree-of-Freedom Nonlinear Model for an Aircraft

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Abstract. Aircraft dynamic modeling at SAE Brazil Aerodesign competition has become a fundamental part of the project to compute the payload and performance characteristics of the aircraft. With each year, the airplanes become more refined and the analysis for the project, more complex. The need to evaluate in more detail the behavior of the aircraft arises to ensure flight handling quality and verify that the mission requirements have been fulfilled, even before building a prototype, specially since the beginning of the COVID-19 pandemic. Thus, in alternative of the 3 degrees of freedom linear models described in the literature, it is proposed the use of a more complete model, with 6 degrees of freedom, to describe with more fidelity the flight dynamics of the aircraft. The implementation, written in Python, computes the movement of the airplane with twelve state variables in each instant of the analysis, which is done by numerical simulation with either Euler's or Runge Kutta's method. It was observed that the developed software can be used to estimate handling quality parameters with the aeronautical regulations and, as a result, refine the project towards a more competitive aircraft capable of ranking the team higher at the national competition.

Keywords: UAV, dynamic model, 6 DOF, flight simulator, aerodesign.

1 Introduction

The advent of the COVID-19 pandemic, the virtualization of the SAE Brazil Competitions combined with the in-person activities restriction on the Universities Campuses, forced the teams to develop more robust software oriented analyses for the project elaboration aiming for more reliable unmanned aerial vehicles(UAV), given the inability to perform field tests. One of the analyses is related to flight mechanics and quality, which is regulated by military standards such as MIL-F-8785C [1], and others such as Part 23 of Administration [2] for light aircraft, and is summarized in characterizing and qualifying the aircraft behavior under different flight conditions. The situation described above aligned with the need for a more detailed dynamic model due to the peculiarities and differences of the aircraft built for the competition in comparison to conventional ones, was conceived as the problem whose solution developed by the Delta do Piauí Aerodesign team, was the implementation in python of a non-linear dynamic model with 6 degrees of freedom to assist in the design of flight mechanics. In this work, it is presented the modeling of coefficients for characterization of aerodynamic and propulsive forces and moments, and then the details of the model in state space is presented and finally, the model is validated using the Cessna 182 aircraft in comparison with the literature by Roskam [3].

2 Aircraft Modeling

Since building and testing an aircraft is somewhat costly, its mathematical modeling has been explored extensively in the literature ranging from Hanke and Nordwall [4] decades ago to Wang and Ma [5] in recent years. According to Stevens, Lewis and Johnson [6], the core of a dynamic model are the rigid body equations of motion, which can be established by applying Newton's second law of motion considering the earth as an inertial referential and under the flat earth assumption. The equations present in Stevens, Lewis and Johnson [6] are used as the basis of the model:

$$ {}^{b}\dot{\mathbf{V}}^{frd}_{b/e} = \sum \mathbf{F}^{frd}/m - \boldsymbol{\omega}^{frd}_{b/e} \times \mathbf{V}^{frd}_{b/e}. \tag{1} $$

$$^{b}\dot{\boldsymbol{\omega}}_{b/e}^{frd} = \left(I^{frd}\right)^{-1}\left[\sum \mathbf{M}^{frd} - \boldsymbol{\omega}_{b/e}^{frd} \times \left(I^{frd}\boldsymbol{\omega}_{b/e}^{frd}\right)\right]. \tag{2}$$

Where the subscript b/e means that the property relates the aircraft(body) to the earth; the superscript frd indicates that the vector is written in the $forward - right - down$ coordinate system fixed to the aircraft, while the superscript b indicates that the derivative is taken in the fixed referential of the aircraft. Then $\boldsymbol{V}$ and $\boldsymbol{\omega}$ indicate the linear and angular velocities respectively, while $^{b}\dot{\boldsymbol{V}}$ and $^{b}\dot{\boldsymbol{\omega}}$ indicate the acceleration vectors, m is the vehicle mass and I the inertia tensor. the other variables, $\sum \mathbf{F}^{frd}$ and $\sum \mathbf{M}^{frd}$ represents, the sum of the gravitational and aeropropulsive forces and moments, respectively, which are related to the free body diagram of Fig. 1 and eqs. 3 and 4:

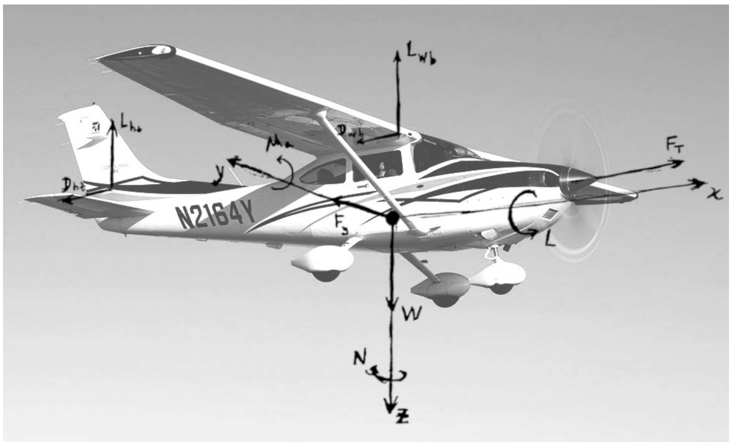


Figure 1. **Aircraft Free Body Diagram**

Source: adapted from Estaff [7]

$$\sum \mathbf{F} = \mathbf{L}_{wb} + \mathbf{L}_t + \mathbf{D}_{wb} + \mathbf{D}_t + \mathbf{F}_T + \mathbf{W} + \mathbf{F}_Y. \tag{3}$$

$$\sum \mathbf{M} = \mathbf{M}_a + \mathbf{N} + \mathbf{L} + \mathbf{M}_T. \tag{4}$$

In which $\mathbf{L}_{wb}$ and $\mathbf{D}_{wb}$ are the lift and drag of the wing-fuselage assembly, $\mathbf{L}_t$ and $\mathbf{D}_t$ of the tail; $\mathbf{F}_y$ being the lateral aerodynamic force and $\mathbf{F}_T$ the thrust; $\mathbf{W}$ is the weight of the aircraft; $\mathbf{M}_a$, $\mathbf{L}$, and $\mathbf{N}$ are the pitch, roll, and yaw moments, respectively, with $\mathbf{M}_T$ being the thrust pitching moment.

The velocities are computed by evaluations of the relations between the coordinate systems through the transformations defined in the eqs. 5 e 6 which are related to the rotation matrices E_u and E_n:

$$\boldsymbol{\omega} = E_u\dot{\boldsymbol{\Phi}}. \tag{5}$$

$$\mathbf{V} = E_N\dot{\mathbf{r}}. \tag{6}$$

Where r symbolizes the position vector of the vehicle center of mass with respect to the earth's fixed referential and $\dot{\boldsymbol{\Phi}}$ is the Euler angles rates vector defined by:

$$\dot{\boldsymbol{\Phi}} = [\dot{\phi} \quad \dot{\theta} \quad \dot{\psi}]^T. \tag{7}$$

with $\phi \in [-\pi, \pi]$, $\theta \in [-\pi/2, \pi/2]$ e $\psi \in [-\pi, \pi]$ being the bank(roll), pitching(attitude) and yaw angles.

Finally, to describe the state of the aircraft over time, Stevens, Lewis and Johnson [6] organizes a state vector composed of velocities and positions, whose derivative is related to the eqs. 1, 2, 5, 6 as follows:

$$\dot{\mathbf{X}} = [\dot{\mathbf{V}} \quad \dot{\boldsymbol{\omega}} \quad \dot{\boldsymbol{\Phi}} \quad \dot{\mathbf{r}}]^T = \begin{bmatrix} \sum \mathbf{F}/m - \boldsymbol{\omega} \times \mathbf{V} \\ I^{-1}\left[\sum \mathbf{M} - \boldsymbol{\omega} \times (I\boldsymbol{\omega})\right] \\ E_u^{-1}\boldsymbol{\omega} \\ E_N^{-1}\mathbf{V} \end{bmatrix}. \tag{8}$$

The aerodynamic model used for characterization of the coefficients follows the approach of Roskam [3], Finck [8] and Roskam [9], which characterizes them as functions of the state variables and deflections of the controls(δ):

$$|\mathbf{F}(\mathbf{X}, \boldsymbol{\delta})| = \overline{q} S C_f(\mathbf{X}, \boldsymbol{\delta}). \tag{9}$$

Where $\overline{q}$ is the dynamic pressure and S the lifting surface platform area. While for the propulsive forces and moments the hypothesis of Roskam [9] for variable pitch aircraft, such as the Cessna 182, was used, that the power is constant for small velocity perturbations and the thrust modulus then can be described by:

$$|\mathbf{F}_T| = \frac{P}{|\mathbf{V}|}. \tag{10}$$

Finally, eq. 8 can be rewritten as a nonlinear ordinary differential equation:

$$\dot{\mathbf{X}} = f(\mathbf{X}, \boldsymbol{\delta}). \tag{11}$$

Which can be solved by the numerical methods such as Euler's and fourth order Runger Kutta, seen in Kaw, Kalu, and Nguyen [10] .

3 Methodology

To validate the model, the responses of a unit pulse commands, described in Fig. 2, obtained by the model were compared to those predicted by the transfer functions described in Roskam [9].

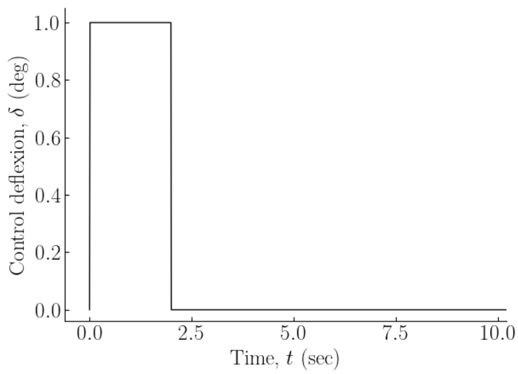


Figure 2. **Pulse Representation for Controls**

Source: Authors

3.1 Inputs

The geometry, inertia, and derivative information used as input are seen in the following Tables 1 and 2, as well as the flight conditions in Table 3 in which the aircraft was trimmed as to be used as starting point for the control responses.

Table 1. **Cessna 182 Geometric and Inertial Properties**

	Value in SI
S	16.16 m^2
$\overline{c}$	1.49 m
b	11 m
m	1202 kg
I_{xx}	1285.32 kg·m^2
I_{yy}	1824.93 kg·m^2
I_{zz}	2666.90 kg·m^2
I_{xz}	0 kg·m^2

Source: Adapted From Roskam [9]

Table 2. **Cessna 182 Derivatives**

	Value in rad^-1		Value in rad^-1
C_{D_0}	0.0270	C_{m_0}	0.0400
C_{D_α}	0.1210	C_{m_α}	-0.6130
C_{L_0}	0.3070	$C_{m_{\dot{\alpha}}}$	-7.2700
C_{L_α}	4.4100	C_{m_q}	-12.4000
$C_{L_{\dot{\alpha}}}$	1.7000	$C_{L_{\delta_e}}$	0.4300
C_{L_q}	3.9000	$C_{m_{\delta_e}}$	-1.1220
C_{l_β}	-0.0923	C_{n_p}	-0.0278
C_{l_p}	-0.4840	C_{n_r}	-0.0937
C_{l_r}	0.0798	$C_{l_{\delta_a}}$	0.2290
C_{y_β}	-0.3930	$C_{l_{\delta_r}}$	0.0147
C_{y_p}	-0.0750	$C_{n_{\delta_a}}$	-0.0216
C_{y_r}	0.2140	$C_{n_{\delta_r}}$	-0.0645
C_{n_β}	0.0587	$C_{y_{\delta_r}}$	0.1870

Source: Adapted from Roskam [9]

Table 3. **Flight Conditions**

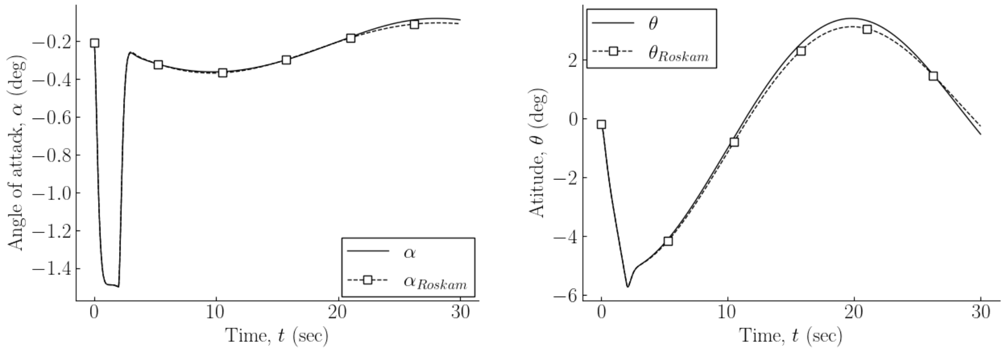
	Value in SI
Altitude, h	1524m m^2
TAS	67.09 m/s

Source: Adapted From Roskam [9]

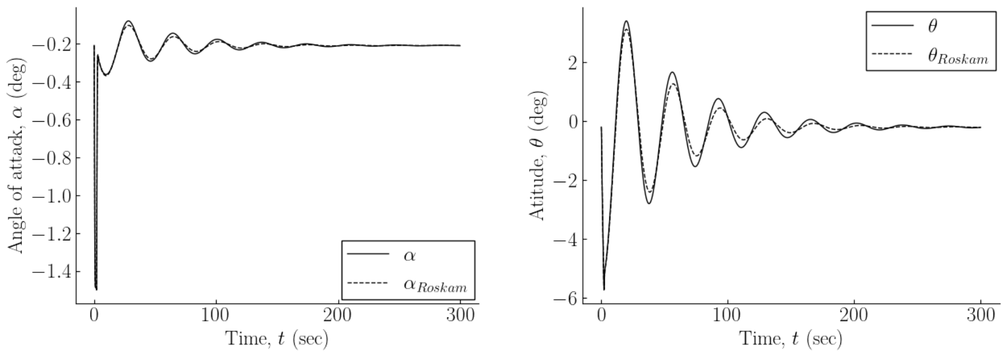
Finally, in relation to the powerplant, information from Cessna [11] was adopted, which is a power of 175.24 kW.

4 Results and Discussion

Numerical simulations were performed with the Runge Kutta method at times of 30s and 5 min for pulse disturbances, with a time step of 0.01s, in order to evaluate the difference between the model and the literature, and the results begin with elevator responses shown in Fig. 3:



(a) Angle of Attack, 30s (b) Attitude, 30s

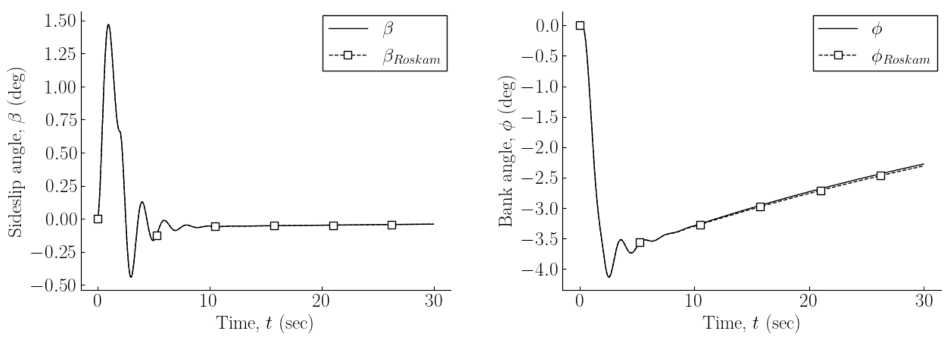


(c) Angle of Attack, 5min (d) Attitude, 5min

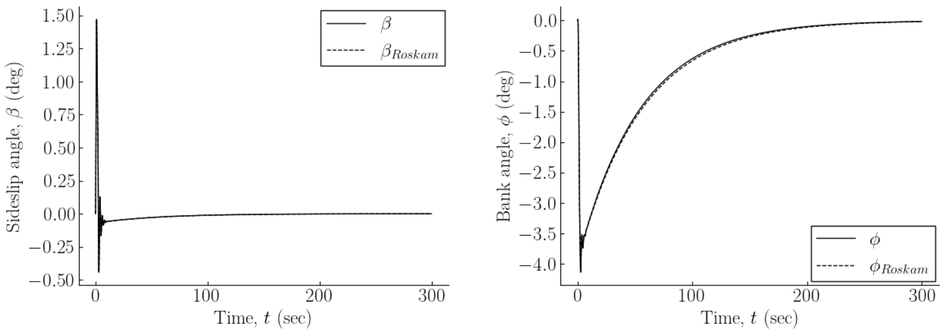
Figure 3. **Dynamic responses to a 1° elevator pulse for 2s**

Source: Authors

In Fig. 3, one can see that the responses of the model and the literature are quite similar in a smaller time analysis range, while in the larger one the difference in terms of damping is noticeable, since with the angle of attack the damping of the model was lower and in the case of the aircraft attitude the opposite occurred. The rudder results are show next in Fig. 4.



(a) Beta, 30s (b) Bank Angle, 30s

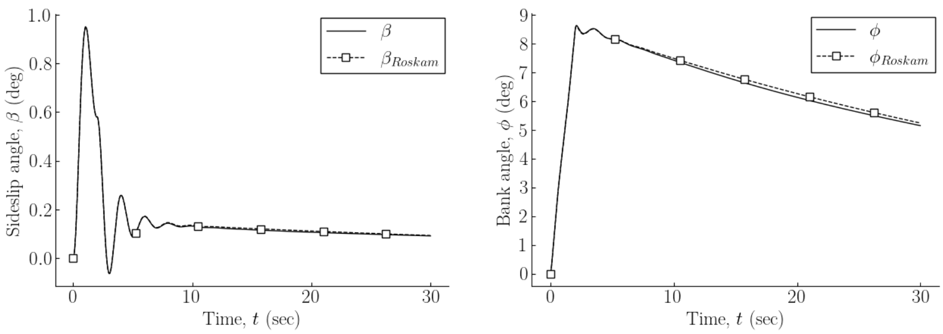


(c) Beta, 5min (d) Bank Angle, 5min

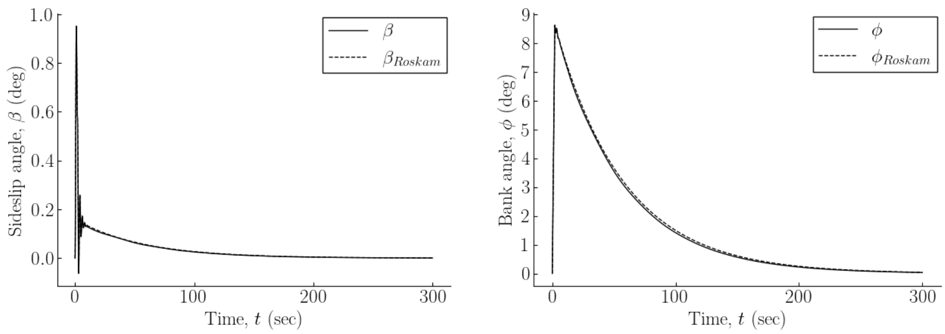
Figure 4. **Dynamic responses to a 1° rudder pulse for 2s**

Source: Authors

In Fig. 4, one can see almost perfect agreement between the literature and model results as opposed to what was observed earlier in the longitudinal responses, and such is the case for the aileron response seen in Fig. 5.



(a) Beta, 30s (b) Bank, 30s



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(c) Beta, 5min (d) Bank Angle, 5min

Figure 5. **Dynamic responses to a 1° aileron pulse for 2s**

Source: Authors

Finally, it can be seen that the results of the simulations performed using the 6 degrees of freedom nonlinear model are mostly in agreement with the results of the linear 3 degree model in the literature, and the differences may be due to the absence of more faithful propulsion methods and information, which is why the difference was more in the longitudinal variables while the lateral and directional variables were more similar.

5 Conclusions

In the present work, we compared the results obtained from a 6-degree-of-freedom nonlinear model subjected to unit pulses from the controls, with the transfer function responses presented in Roskam [9], for the Cessna 182 aircraft. Despite the inaccessibility of more accurate data and the confection of a more sensitive propulsion model, the predictions of the implemented model presented good conformity with the expected results.

Thus, even though further testing is needed, the program developed can be used by the team to guide flight mechanics design decisions for the SAE Brazil Aerodesign competition, provided that the necessary geometry and derivative inputs can be obtained, the analysis conditions are well established and the limitations of the model known, as well as the knowledge to take the results with a grain of salt.

It is proposed for future work, the verification of the aircraft behavior in more elaborate flight conditions, such as symmetric maneuvers, the implementation of a more altitude sensitive propulsive model and studies on more efficient ways of solving the equations of motion.

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