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Parametric optimization of an off-road car suspension system aiming for a compromise between comfort and safety

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Abstract. This work addresses the the parametric optimization of an off-road car suspension system. The aim is to find a compromise between comfort and safety while respecting functional constraints such as maximum deflections for the suspension and the tires, as well as the tendency of oversteering on cornering. The vertical vehicle dynamic is used to evaluate both the comfort and safety criteria for different operational conditions: uniform rectilinear movement; braking; curves; and longitudinal acceleration. Tires were subjected to base excitation, obtained from the Power Spectral Density (PSD) of the road profile. The tendency to oversteer on cornering was measured by comparing the frontal and rear torsional stiffness. The Finite Element Method (FEM) was implemented to solve the transient equilibrium problem needed to evaluate the comfort and safety criteria as well as the functional constraints. The design variables considered in the parametric optimization were the springs and tires stiffness, and dampers damping. The optimization algorithm used was a modified version of the Particle Swarm Optimization (PSO). Results show that it is possible to obtain a feasible final configuration minimizing the objective function, thus improving both safety and comfort criteria.

Keywords: Parametric optimization; Suspension system; Oversteering; Safety; Comfort.

1 Introduction

This work considers an off road car designed to the SAE Brazil Baja competition with rules defined by [SAE]. Hence, for this purpose, the components that are applicable are vehicle, frame-work, engine, drive train, driver, steering system, wheel, suspension system, tires and road. According to [2], the suspension system is responsible to absorb vibrations transmitted from the ground and also to keep tires in contact with the ground to guarantee handling stability. This system consists of springs, dampers, tires, and guiding elements,[3].

Regarding road vehicles, there are many different metrics to define a proper design. Among these many criteria, vehicles must be designed to attend ride safety and comfort. Ride safety can be related to dynamic forces acting on the wheels while ride comfort is related to the acceleration perceived by the occupants [3]. Nonetheless, comfort conflicts with handling stability (safety), such that when comfort is improved, safety gets worse and vice-versa [4]. This compromise is specially hard to achieve in the design of off-road cars aimed to be used in country roads, where the vehicle is subjected to complex ground excitation patterns along with severe longitudinal and lateral forces. According to [4], optimization algorithms have been used to modify suspension parameters to improve performance.

Beyond the suspension system, structural stiffness also influences the overall dynamical behavior. Nonetheless, most works aiming to study suspensions use a simplified model for the rest of the car, as for example quarter car models, used by [4, 5] and [6], with 2 Degrees of Freedom (DOFs). To maximize comfort, all of them defined the vehicle body vertical acceleration as objective function. The same objective function was also used by [7] and [8] using half car models with 5 DOFs. Full car models were used by [9] and [10] with 7 DOF. According to [11], who also adopted the full car model with 7 DOF, it can properly represent the dynamic of the vehicle. These works show that the use of full car models makes it possible to evaluate angular movements of pitch and roll in addition to vertical body and wheel movements. However, if there is no intention to consider angular accelerations as comfort criteria, then one can use a quarter car model, with the advantage of being the simplest model [12].

Despite the advantages found in each one of the simplified methods, it is clear none of them takes into account the influence of the body of the car in the total stiffness as well the details such as the guiding elements and suspension assembly angles. Based on this fact, a method which can be used to analyze the total car stiffness, including the suspension system and structural elements is necessary to obtain accurate results. The most used method for this purpose is the Finite Element Method (FEM).

2 Proposed formulation

Assuming the car is modeled by a finite element model comprised of space frame elements, springs, dampers and lumped masses, one can represent the discrete dynamic equilibrium problem as

$$\mathbf{M}\ddot{\mathbf{U}}(t) + \mathbf{C}\dot{\mathbf{U}}(t) + \mathbf{K}\mathbf{U}(t) = \mathbf{F}(t), \tag{1}$$

where $\mathbf{M}$ is the global inertia matrix, $\mathbf{C}$ is the global damping matrix, $\mathbf{K}$ is the global stiffness matrix, $\mathbf{U}(t)$ is the global displacement vector and $\mathbf{F}(t)$ is the global vector of external forces. Time derivatives are denoted by a single (for velocities) and double (for acceleration) dots.

The elements can be divided in two groups, structural and suspension elements. The stiffness ($\mathbf{K}$) and inertia matrices ($\mathbf{M}$) of the first one, composed by bar and space frame elements, are defined according to [13].

Proportional damping is used to model damping in space frame elements in this work. According to [14], the proportional damping matrix is

$$\mathbf{C}_e = \alpha\mathbf{M}_e + \beta\mathbf{K}_e \tag{2}$$

where α and β are defined for given damping ratio (ζ) related to two natural frequencies of the structure. The procedure to obtain α and β is detailed by [14].

The second group (suspension elements) is composed by springs, dampers and tires. The spring element can be defined as similar to bar elements, with axial stiffness given by K_s. For the damper element the axial stiffness is equal 0 and the axial damping is defined by C_d.

Tire elements were modeled as space frames with effective axial stiffness defined by K_{ti} and torsional, as well bending stiffness in y and z direction defined by large values ($K = 1\times10^{10}$), to insure just axial displacements and to avoid lateral rigid body modes.

Road profiles $z_{ri}(t)$ are imposed at the vertical DOF of each tire and are the main source of loading in this work. As tires work as compressive springs, the effective stiffness in each tire is

$$K_{Ti}(t) = \begin{cases} K_{ti} & if \quad z_{wi}(t) \leq z_{ri}(t) \\ 0 & if \quad z_{wi}(t) > z_{ri}(t) \end{cases} \quad i = 1..n_w, \tag{3}$$

where z_w is the vertical displacement of wheel i and n_w is the number of wheels. So, the effective force due to ground excitation is

$$F_{Ti}(t) = K_{Ti}(t) z_{ri}(t) \; i = 1..n_w. \tag{4}$$

The dynamic equilibrium equations were defined considering generic base excitation $z_{ri}(t)$ and other load sources $\mathbf{F}_g$. According to [3], the road profile can be approximated by the superposition of N sine waves in the form

$$z_r(s) = \sum_{n=1}^{N} (2\Phi(\Omega_n)\Delta\Omega)^{\frac{1}{2}} \sin(\Omega_n(s+\tilde{L}) - \psi_n), \tag{5}$$

where s is the longitudinal displacement of the vehicle, $\tilde{L}$ is 0 for front wheels and L (wheelbase) for rear wheels, Ω_n is the wave number and ψ_n represents the phase number of sine wave n, usually defined by a random number between 0 and 2π.

The Newmark-β method is the numerical method used to solve the second-order differential equation (1) [15]. The equilibrium equation is evaluated for different operational conditions, considering a country road profile and a constant velocity v_s [m/s] with ground profiles $z_r(t)$ and self weight $\mathbf{F}_w = -G[\mathbf{M}]_z$, since gravity acts along the negative vertical, z, direction. Each operational condition also considers additional loads $\mathbf{F}_g$:

- Uniform rectilinear movement: reference condition, $\mathbf{F}_g = \mathbf{0}$,
- Braking: Related to a braking condition when there is a negative longitudinal acceleration (forward inertial load) $\mathbf{F}_g = \mu_x G[\mathbf{M}]_x$, where μ_x is the maximum longitudinal friction coefficient and $G = 9.8065\ m/s^2$. It is assumed that the velocity does not change during the simulation;
- Curves: Related to a curve to the right side, $\mathbf{F}_g = \mu_y G[\mathbf{M}]_y$, where μ_y is the maximum lateral friction coefficient;
- Acceleration: Related to positive longitudinal acceleration condition, $\mathbf{F}_g = -a_{max}[\mathbf{M}]_x$, where a_{max} is the the maximum expected longitudinal acceleration. It is assumed that the velocity does not change during the simulation.

The operator $[\mathbf{A}]_k$ is defined as

$$[\mathbf{A}]_k = \begin{cases} A_{ii} & if \quad i \; isa \; k \\ 0 & if \quad \neg \; i \; isa \; k \end{cases} \quad i = 1..ngl, \;\; k = x, y, z, \theta_x, \theta_y, \theta_z \tag{6}$$

where $\mathbf{A}$ is a square matrix and ngl is the number of DOFs in the finite element mesh. The operation $i \; isa \; k$ is true if DOF i corresponds to a global direction k. Thus, $[\mathbf{A}]_x$ returns a vector with the diagonal values of $\mathbf{A}$ in the global x DOFs and 0 in the other DOFs, for example.

Hence, the external force $\mathbf{F}(t)$ is

$$\mathbf{F}(t) = \mathbf{F}_T(t) + \mathbf{F}_w + \mathbf{F}_g. \tag{7}$$

According to [16], comfort is defined as the Root Mean Square (RMS) of the perceived vertical acceleration $(\ddot{z}(t))$. According to [3], the RMS value of $\ddot{z}_i(t)$, at a given position i is defined as

$$C_i = \left(\frac{1}{t_f - t_0} \int_{t_0}^{t_f} \ddot{z}_i(t)^2 dt \right)^{1/2}, \tag{8}$$

where t_0 and t_f are the initial and final integration time and i are body connection to the suspension system. Hence, the objective function of the comfort criterion can be written as

$$\epsilon_c = \sum_{i}^{n_b} \frac{C_i}{C_{0i}}, \tag{9}$$

where C_{0i} is the value of C_i for the initial configuration, $n_b = 4$ is the total of body connections with suspension system and i corresponds to the global DOFs of front left, rear left, front right and rear right.

According to [3], the criterion to maximize safety is to minimize the dynamic wheel load variation, which can be obtained by the equation

$$S_i = \left(\frac{1}{t_f - t_0} \int_{t_0}^{t_f} \left[\frac{F_{T_i}^S - F_{T_i}^D(t)}{F_{T_i}^S} \right]^2 dt \right)^{\frac{1}{2}}, \tag{10}$$

where

$$F_{T_i}^S = K_{t_i} z_{w_i 0}, \tag{11}$$

is the static load, $z_{w_i 0}$ is the initial displacement of the wheel i,

$$F_{T_i}^D(t) = K_{t_i}(z_{w_i}(t) - z_{r_i}(t)), \tag{12}$$

is the dynamic load, K_{t_i} is the axial stiffness of the tire i and $z_{w_i}(t)$ and $z_{r_i}(t)$ are the wheel and road vertical displacements at i.

Normalizing by the initial suspension configuration and adding the contribution of each wheel load, the safety criterion can be calculated according to

$$\epsilon_s = \sum_{i=1}^{n_w} \frac{S_i}{S_{0i}}, \tag{13}$$

where i is the global degree of freedom correspondent to the vertical movement of each wheel, n_w is the number of wheels and S_{0i} is the value of S_i for the initial suspension configuration.

With the definitions of the comfort and safety criteria, it is possible to define the objective function as a linear combination

$$\epsilon(\boldsymbol{x}) = \alpha_c \sum_{j=1}^{o} \beta_j \epsilon_{cj}(\boldsymbol{x}) + \alpha_s \sum_{j=1}^{o} \beta_j \epsilon_{sj}(\boldsymbol{x}), \tag{14}$$

where α_c and α_s are the weight factor for each criteria (comfort; safety), β_j is the weight factor for each operational condition and ϵ_{cj} and ϵ_{sj} are the values of ϵ_s and ϵ_s for each one of the o operational conditions. Design variables considered in this work are suspension (k_{sf} and k_{sr}), tires stiffness (k_{tf} and $k_t r$), and suspension damping (c_f and c_r), as represented by

$$\mathbf{x} = \left\{ \begin{array}{cccccc} k_{sf} & k_{sr} & c_f & c_r & k_{tf} & k_{tr} \end{array} \right\}^T. \tag{15}$$

The constraint function related to the maximum deflection in the suspension is given by

$$g_1 = \frac{ds_{max}}{\bar{ds}} - 1, \tag{16}$$

where $\bar{ds}$ is the maximum value that ds_{max} can assume for an acceptable setup and

$$ds_{max} = \max(|ds_i(t)|)\ \forall i = 1, 2, ..., n_s, \tag{17}$$

where n_s is the number of the suspension set (spring plus damper elements), $||ds_i(t)||$ is the norm of $ds_i(t)$

$$ds_i(t) = z_{bi}(t) - z_{wi}(t), \tag{18}$$

where $z_{bi}(t)$ is the vertical displacement of body at connection with the suspension i, $z_{wi}(t)$ is the vertical displacement of wheel at same suspension position i.

The constraint function related to the tire deflection is given by

$$g_2 = \frac{|dt_{min}|}{\bar{dt}} - 1, \tag{19}$$

where $\bar{dt}$ is the maximum value that $|dt_{min}|$ can assume for an acceptable setup and

$$dt_{min} = \min(dt_i(t))\ \forall i = 1, 2, ..., n_t, \tag{20}$$

where n_t is the number of tire elements and $dt_i(t)$ is given by

$$dt_i(t) = z_{wi}(t) - z_{ri}(t), \tag{21}$$

where $z_{wi}(t)$ is the vertical displacement of wheel i, $z_{ri}(t)$ is the vertical displacement of road at same wheel position i.

For an oversteer behavior, it is necessary that the rear torsional stiffness is higher than frontal torsional stiffness [17].

Two static analysis are performed to estimate the difference between the frontal and rear torsional stiffness:

- Applying a torque T at the rear nodes related to the fixation of suspension system at the structure;
- Applying a torque T at the frontal nodes related to the fixation of the suspension system at the structure.

Thus it is possible to obtain the difference between the right and left displacement for rear nodes in the first analysis and for the frontal nodes in the second analysis. The relation between frontal and rear torsional stiffness is given by

$$\gamma = \frac{|z_{b1} - z_{b3}| - |z_{b2} - z_{b4}|}{|z_{b2} - z_{b4}|}, \tag{22}$$

where z_{b1}, z_{b2}, z_{b3} and z_{b4} are the vertical displacement of the body in the front left, rear left, front right, and rear right positions, respectively.

The oversteer constraint is then defined as

$$\begin{cases} g_3 = -\frac{\gamma}{\underline{\gamma}} + 1, & if\ \underline{\gamma} > 0 \\ \\ g_3 = -\frac{\underline{\gamma}}{\gamma} + 1, & if\ \underline{\gamma} \le 0 \end{cases}, \tag{23}$$

where $\underline{\gamma}$ is the minimum value assigned for the relation between frontal and rear torsional stiffness.

The optimization problem can be written as

$$\begin{cases} \min\limits_{\mathbf{x}} & \epsilon(\mathbf{x}) \\ S.t. & \\ & \mathbf{M}\ddot{\mathbf{U}}_j(t) + \mathbf{C}\dot{\mathbf{U}}_j(t) + \mathbf{K}\mathbf{U}_j(t) = \mathbf{F}_j(t) \qquad \forall\, j \in\ 1, 2, ..., o \\ & g_l^j(\mathbf{x}) \le 0 \qquad \forall l = 1, 2, ..., m\ ,\ j = 2, 3, ..., o \\ & \underline{\mathbf{x}} \le \mathbf{x} \le \bar{\mathbf{x}} \end{cases} \tag{24}$$

where o is the total number of operational conditions, beginning from $j = 2$ because it is not evaluated for suspension and tires deflection for uniform rectilinear movement, which is the less severe condition. m is the total number of functional constraints $\underline{\mathbf{x}}$ and $\bar{\boldsymbol{x}}$ are the lower and upper side constraints.

As the number of design variables is small, a populational zero order method is a viable option to solve the optimization problem stated in Eq. (24). In this work, it is used the Particle Swarm Optimization method, first proposed by [18] and uses a swarm of N particles, or candidate solutions, to explore the design space. The traditional PSO algorithm does not consider functional constraints. Thus, the modification proposed by [19] for genetic algorithms is used herein.

3 Results

The car model used in this study was provided by the UDESC Baja team (Velociraptor), which is shown in the figure 1.

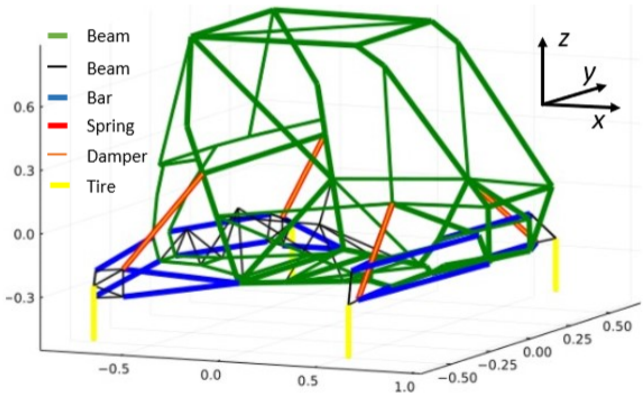

Figure 1. Car model

Table 1 shows the results of 10 independent runs of the optimization algorithm. Results shown in Tab. 1 present small variation (1.17%) of the optimized objective $f(\mathbf{x})^*$. The stopping criteria is the number of iterations without improvement (20). The run number 4 is the best solution among the 10 runs.

Table 1. Results after running the PSO 10 times

	k_{sf} $[N/m]$	k_{sr} $[N/m]$	c_f $[Ns/m]$	c_r $[Ns/m]$	k_{tf} $[N/m]$	k_{tr} $[N/m]$	$f(\boldsymbol{x}*)$
1	5775,6	5725,6	658,2	1278,1	28000,0	28000,0	43,10%
2	6642,9	5454,1	1000,0	1227,9	28000,0	28000,0	43,81%
3	5977,1	5541,4	722,8	1164,3	28000,0	28000,0	43,08%
4	5539,1	5771,6	726,4	1051,6	28000,0	28000,0	42,98%
5	5402,5	6069,1	615,9	1703,9	28000,0	28000,0	43,56%
6	5000,0	7640,9	951,2	1060,3	28000,0	28000,0	43,85%
7	5405,0	6018,7	633,8	1460,6	28000,0	28000,0	43,22%
8	5564,1	6163,3	518,3	1189,3	28000,0	28000,0	43,51%
9	5311,1	6575,3	587,2	1938,9	28000,0	28000,0	44,15%
10	5000,0	7574,4	1000,0	1301,2	28000,0	28000,0	43,94%

The behavior of the optimized solution, compared with the initial one, during the transient analysis for each operational condition is shown in the figure 2. It is possible to verify a significant decrease of amplitudes of the body acceleration and the normalized tires deflections, which reduce the integrals therms of equations 8 and 10 and, consequently, the comfort and safety criteria values. The individual contributions to the objective function are

$\epsilon_c = 0.338$, Eq. (9), and $\epsilon_s = 0.522$, Eq. (13). Thus the optimized solution provides an improvement of 66.2% of comfort and 47.8% of safety.

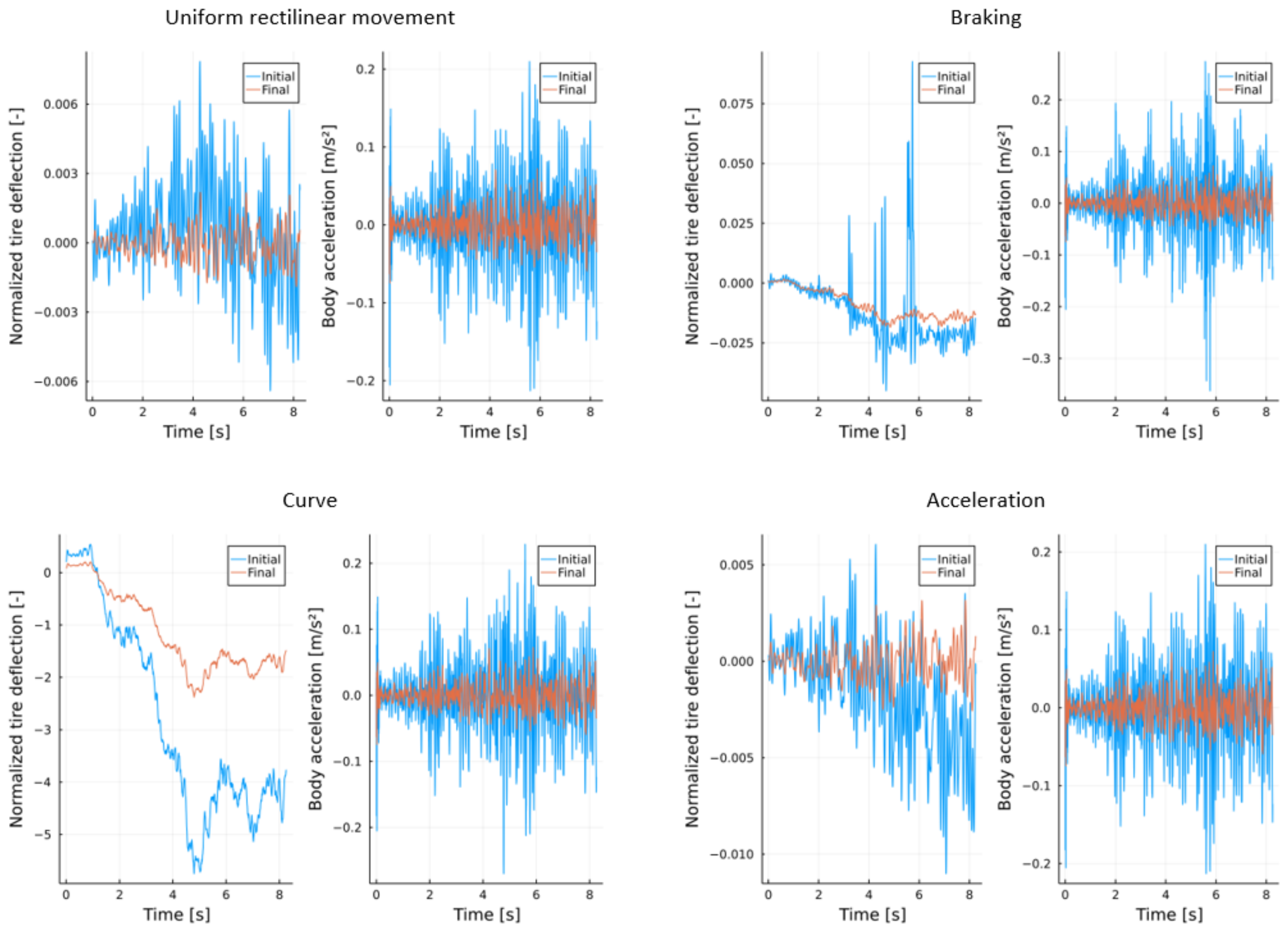

Figure 2. Normalized tire deflection and body acceleration

The functional constraints $g_1 = -0.004$, related to the suspension deflection, and $g_3 = -0.010$, related to the oversteer behavior are active, with values close to 0. The functional constraint related to the tire deflection ($g_2 = -0.435$) is not active, what is expected once the side constraints $\underline{\mathbf{x}}_5 = 28000\ N/m$ and $\underline{\mathbf{x}}_6 = 28000\ N/m$, which are the tire stiffness (k_{tf} and k_{tr}), were active.

4 Conclusion

The parametric optimization of suspension parameters for a compromise between comfort and safety is discussed in this work. User can adjust these criteria by setting the weight factors according to a given priority and the weight factors associated to the operational conditions.

Results show an improvement of 57% with respect to the reference (initial) configuration. The parameters used in the PSO algorithm lead to a variation of 1.17% in the objective function after 10 independent runs, indicating a proper tune to this class of optimization problem. The best result (improvement of 57.02%) is associated to an improvement of 66.2% in comfort and 47.8 in safety. This result was limited by the active functional constraints associated with the maximum suspension deflection and oversteer behavior, and side constraints associated with the minimum tires stiffness.

A decrease in the dynamic forces on wheels (when compared to the static case) is observed in all operational conditions. This is related to the Safety Criteria. The same trend is observed in the vertical acceleration of the 4 points linking the body to the suspension element, for all operational conditions.

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References

[SAE] SAE. Baja nacional.

[2] M. Mahmoodi-Kaleibar, I. Javanshir, K. Asadi2, A. Afkar, and A. Paykani. Optimization of suspension system of off-road vehicle for vehicle performance improvement. *Journal of Central South University*, vol. 20, pp. 902–910, 2013.

[3] G. Rill. *Road Vehicle Dynamics: Fundamentals and modeling*, volume 11. FL: Taylor & Francis Group, 2012.

[4] D. Peng, G. Tan, K. Fang, L. Chen, P. K. Agyeman, and Y. Zhang. Multiobjective optimization of an off-road vehicle suspension parameter through a genetic algorithm based on the particle swarm optimization. *Mathematical Problems in Engineering*, vol. 2021, pp. 1–14, 2021.

[5] G. Geng, Y. Yu, L. Sun, and H. Li. Research on ride comfort and driving safety under hybrid damping extension control for suspension systems. *Applied Sciences*, vol. 10, pp. 1442, 2020.

[6] S.-J. Shao, D. Jing, and C.-B. Ren. Multiobjective optimization of nonlinear active suspension system with time-delayed feedback. *Mathematical Problems in Engineering*, vol. 2020, pp. 1–11, 2020.

[7] B. Gadhvia, V. Savsania, and V. Patela. Multi-objective optimization of vehicle passive suspension system using nsga-ii, spea2 and pesa-ii. *Procedia Technology*, vol. 23, pp. 361–368, 2016.

[8] G. Papaioannou and D. Koulocheris. Multi-objective optimization of semi-active suspensions using kemoga algorithm. *Engineering Science and Technology, an International Journal*, vol. 22, pp. 1035–1046, 2019.

[9] A. Polanco and L. Munoz. Effect of the change of inertial, elastic and dissipative parameters on the ride comfort of a road vehicle, 2013.

[10] Y. Shen, L. Chen, Y. Liu, and X. Zhang. Modeling and optimization of vehicle suspension employing a nonlinear fluid inerter. *Shock and Vibration*, vol. 2016, pp. 1–9, 2016.

[11] S. J. Idehara and M. R. R. Sabka. Influence of model parameters on vehicle suspension control. *International Journal of Research in Engineering and Technology*, vol. , 2018.

[12] M. H. B. D. Mello and S. J. I. . Measurement and analysis of vehicle comfort and road quality. *International Journal of Research in Engineering and Technology*, vol. , 2018.

[13] S. S. Rao. *The Finite Method in Engineering*. Elsevier, 2011.

[14] F. Valentini. Robust topology optimization of continuum structures subjected to uncertainties in the excitation frequency, 2021.

[15] T. J. R. Hughes. *The Finite Element Method: Linear Static and Dynamic Finite Element Analysis*. Dover Publication, 2000.

[16] ISO. Guide for evaluation of human exposure to whole-body vibration, 1978.

[17] T. D. Gillespie. *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers, 1992.

[18] J. Kennedy and R. Eberhart. Particle swarm optimization. In *Proceedings of ICNN'95 - International Conference on Neural Networks*, volume 4, pp. 1942–1948 vol.4, 1995.

[19] K. Deb. An efficient constraint handling method for genetic algorithms. *Computer Methods in Applied Mechanics and Engineering*, vol. 186, pp. 311–388, 2000.