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Robust Topology Optimization of Resonant Structures Considering Uncertainty in the Excitation Frequency

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Abstract. This article addresses robust continuous topology optimization of resonant structures under uncertain excitation frequency. The harmonic dynamic response is evaluated by a density-weighted norm and the optimization problem is solved by using the Method of Moving Assymptotes. The formulation is assessed by using a common benchmark problem. Results show that the proposed formulation leads to the design of resonant structures with improved robustness. Results also show that the mechanism used by the optimizer to improve robustness depends on the magnitude of the target excitation frequency: at lower frequencies, a low-energy resonance is used to create an interval around the target frequency with minimized dynamic response variations whereas at higher frequencies, a pair of high-energy resonances is located at the neighborhood of the target excitation frequency to create the same effect and improve the dynamic robustness.

Keywords: Topology optimization; Harmonic vibration; Maximization; Resonance.

1 Introduction

A formulation for the robust design of resonant structures considering uncertainties in the excitation frequency is proposed to maximize the dynamic response while minimizing its variance around a given target frequency. In general, due to the nature of dynamic problems and specially at resonance, structures are highly sensitive to the excitation frequency as they are only resonant at very narrow intervals of the frequency spectrum. However, for some applications, such as energy harvesting from vibration, a smoother dynamic response behavior around resonance could improve power generation in scenarios where the excitation frequency is not precisely synchronized to the resonance.

Such improved dynamic behavior can be achieved by using the robust approach to design structures less sensitive to changes in operation parameters, as the excitation frequency, with the lowest possible penalty in the dynamic response amplification at a given target frequency.

Regarding the maximization of harmonic response, the first work on this area is [1] where the problem of disconnections from the boundary conditions is reported. Static compliance has been added as a constraint to solve the issue. The work of [2] reports spurious modes in void regions and proposes the use of external dampers to ensure proper connection to the essential boundary conditions. In [3], an objective function was proposed with two different parts. The first one ensures the maximization of the output displacement, thus increasing the energy generated, and the second one ensures minimization of perpendicular stiffness such that the device would not touch the surroundings. This second part of the objective function ends up increasing the static stiffness and ensures a well connected structure.

Concerning the maximization of the dynamic displacement, both the input power and the dynamic stiffness have been evaluated by [4] for the design of resonant structures, where the static compliance was added to the formulation to aid in the structure connection to the supports. Furthermore, a complete analysis of several different ways of using the dynamic compliance in harmonic response maximization problems is discussed in [5], where it is shown that minimizing the dynamic compliance results in anti resonance for designs targeting frequencies above the first resonance, leading to convergence problems.

More recently, [6] proposed a density-weighted norm able to precisely identify resonances even for large damping ratios as well as hinder the presence of non-physical modes in void regions. However, addition of static compliance is still used for ensuring connectivity. Since the mentioned capabilities are perfectly aligned to the goals herein discussed, this is the formulation used in this work.

Recently, the robust design of non-resonant structures was evaluated by the authors [7]. Monte Carlo Simulation method, considering a particular method of stratified sampling was used to model the uncertainties and to evaluate both the expected value and the standard deviation of the dynamic displacements. The method therein used for minimization of dynamic displacements is now used for maximization, such that this article can be considered as an extension of this previous work, as will be further discussed.

Thus, based on the literature overview, but specially on [7], a formulation is proposed for the design of resonant structures considering uncertainty in the excitation frequency. The method consists in using the density-weighted norm for describing the dynamic displacements under each excitation realization and a modified Monte Carlo Simulation method with stratified sampling [8] for providing probabilistic data for the optimization process. Static compliance is added to the formulation to ensure structural connectivity, given the fact that the density-weighted norm cannot, alone, guarantee it.

2 Proposed formulation

As discussed in the previous section, the main objective of this work is the design of robust resonant structures with respect to uncertainties in the excitation frequency.

Classical SIMP approach is used for plane stress problems in this work, with the mass parametrization proposed by [9]. Simple spatial filter and projection technique are used to avoid intermediate results and mesh dependency [10].

The objective function is comprised of two terms. The first term is the robust term

$$R(\boldsymbol{\rho},\Omega) = \frac{\gamma_1 E[N_{mwdB}(\boldsymbol{\rho},\Omega)] + |\gamma_1|\varphi Std[N_{mwdB}(\boldsymbol{\rho},\Omega)]}{E[N_{mwdB}(\boldsymbol{\rho},\Omega)]^0 + \varphi Std[N_{mwdB}(\boldsymbol{\rho},\Omega)]^0}, \tag{1}$$

where $E[N_{mwdB}(\boldsymbol{\rho},\Omega)]$ and $Std[N_{mwdB}(\boldsymbol{\rho},\Omega)]$ are the expected value and standard deviation of the density-weighted norm N_{mwdB}, [6], as a function of the vector of relative densities $\boldsymbol{\rho}$ and the uncertain angular frequency Ω. The weighting factor φ is used to set the relative importance of the standard deviation with respect to the expected value. Weight γ_1 multiplies both the expected value and the standard deviation in Eq. (1), however, $Std[.]$ is multiplied by $|\gamma_1|$ to ensure that the standard deviation is always minimized regardless of the signal of γ_1 (in this work, it is always negative). This weight is used to set the relative importance with respect to the second term in the final objective function, as it will be explained in the following. The objective function is defined as a linear combination of two terms: the robust design formulation presented and the static compliance. The problem is defined as

$$\begin{aligned}
\underset{\boldsymbol{\rho}}{\text{minimize}} \quad & \Phi(\boldsymbol{\rho},\Omega) = R(\boldsymbol{\rho},\Omega) + (1-|\gamma_1|)\frac{\Upsilon_S(\boldsymbol{\rho})}{\Upsilon_S^0} \\
\text{subject to} \quad & \mathbf{K}_D(\boldsymbol{\rho},\Omega)\,\mathbf{U}(\boldsymbol{\rho},\Omega) = \mathbf{F}, \\
& \mathbf{K}(\boldsymbol{\rho})\,\mathbf{U}_S(\boldsymbol{\rho}) = \mathbf{F}, \\
& V(\boldsymbol{\rho}) = \sum_{e=1}^{ne} \rho_e V_e^0 \leq \bar{V}, \\
& \boldsymbol{\rho}_l \leq \boldsymbol{\rho} \leq \boldsymbol{\rho}_u,
\end{aligned} \tag{2}$$

where $\mathbf{K}_D$ is the harmonic stiffness matrix, $\mathbf{K}$ the static stiffness matrix, $\mathbf{F}$ the force vector, $\mathbf{U}$ is the complex vector of harmonic displacements and $\mathbf{U}_S$ is the vector of static displacements, used to compute the static compliance Υ_S. Υ^0 is the initial value, used for scaling this term. Both equilibrium equations are identically satisfied during the finite element computations, such that the only functional constraint is the (deterministic) volume $V(\boldsymbol{\rho}) = \sum \rho_e V_e^0 \leq \bar{V}$. The optimization problem defined in Eq. (2) is solved using the MMA method [11]. The Local Averaged Stratified Sampling method, LASS, is used to compute both the expected value and standard deviation [8].

3 Results

Even though the formulation herein described allows the use of any kind of probability distribution, a truncated normal distribution is used in this work to model the uncertainties, as suggested in [7]. The motivation for this definition is that, theoretically, a normal distribution can result in the evaluation of negative realizations or even unrealistic high frequencies, which would have no physical meaning in the context of this work. The truncation is defined such that at least 99.7% of the realizations of the non-truncated distribution is represented by the truncated distribution. Thus, the interval of interest is defined as $\pm\Delta\omega$ around the mean $\bar{\omega}$, which is the target excitation frequency. The uncertain frequency Ω, therefore, is modeled as

$$\Omega \sim \mathcal{N}\left(\omega_l, \omega_u, \bar{\omega}, \eta\right), \tag{3}$$

where ω_l and ω_u are the lower and upper bounds of Ω, $\bar{\omega}$ is the mean angular frequency value and η is the standard deviation. The referred bounds are written as

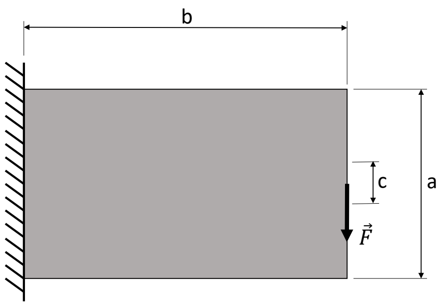
$$\omega_{u,l} = \bar{\omega} \pm \Delta\omega, \tag{4}$$

where $\bar{\omega}$, η and $\Delta\omega$ are defined for each one of the test cases.

The number of bins used in the pre-processing of the LASS method was found after a careful evaluation of the errors for the expected value, standard deviation and the Inf norm of gradient for the worst case (large η and small φ), i.e., with large output standard deviation of $N_{m\omega dB}$, as described in [7]. Based on the results of the referred study, $N_{bins} = 30$ is used to perform all the optimizations in this article. All optimized results are verified using the full MCS method with 1×10^6 realizations.

The test case considered in this work is the same found in [5–7] and it is shown in Fig. 1. The load is homogeneously applied along edge c and the relative densities of all elements on this region are kept constant and equal to 1.0 during the optimization.

Figure 1. Problem Definition.



The maximization comprises all vertical degrees of freedom of the nodes located at the edge c. Relevant data used in this test case are $a = 0.5$ $[m]$, $b = 1.0$ $[m]$, $c = 0.1$ $[m]$, Young Modulus 210 $[GPa]$, Poisson 0.3, mass density 7860 $[kg/m^3]$, Harmonic Load $F = 10000$ $[N]$. A mesh of 140×70 elements four-node incompatible bilinear isoparametric elements is used, due to its capability to represent bending behavior, specially in slender reinforcements. A filter radius of 0.02m is used in all examples. Parameters γ_1 and $\bar{V}$ are defined as -0.75 and $0.5|\Gamma|$, respectively, where Γ is the total area of the design domain. Thus, $1 - |\gamma_1| = 0.25$, making the objective function dominated by the dynamic norm, as intended. Exponents from the density-weighted norm are defined as $m = 8.0$ and $w = 2.0$. A high value of the exponent m, i.e. above 2.0, helps the dynamic norm to identify the resonances [6].

Although the formulation is developed using the concept of angular frequency ω, all the frequencies in this section are shown as f Hz. Thus, following the work of [6], mean frequencies of $\bar{f} = 365$ and $\bar{f} = 1135$ Hz are used to assess the behavior of the formulation at different regions of the frequency response. A single deviation of $\eta = 20$ Hz is used in all examples with $\Delta\omega = 60$ Hz, to account for at least 99.7% of the original frequency (not truncated) content. Topologies obtained for the deterministic maximization of the harmonic response are shown in Fig. 2 for reference. Parameter φ is used to study the formulation.

Figure 3 depicts the topologies obtained at $\bar{f} = 365$ Hz for different values of φ. Deterministic-like topologies were obtained by the use of lower values of φ, as expected. However, for $\varphi = 50$, a different design with a different overall shape, disposition of reinforcements and visible different stiffness in the connection arrangement to the loaded edge is obtained. A comparison of the frequency responses is shown in Fig. 4, where it can be noticed a radical change in response for $\varphi = 50.0$, where a low-energy resonance appears close to the target frequency.

Topologies obtained for $\bar{f} = 1135$ Hz are shown in Fig. 5. A very interesting result is the loss of symmetry as φ increases. As shown in Fig. 6, robustness is achieved by the presence of two additional modes close to the resonance frequency, as a result of the lack of symmetry.

Figure 2. Topologies obtained with the deterministic approach for the test case depicted in Fig. 1.

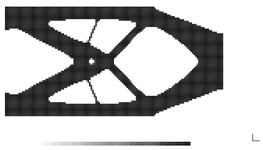
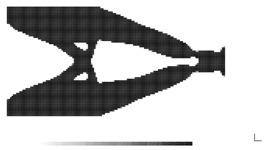
365.0 Hz	1135.0 Hz
	

Figure 3. Topologies obtained for maximization at 365 Hz, for different values of φ.

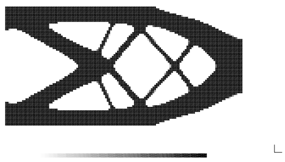
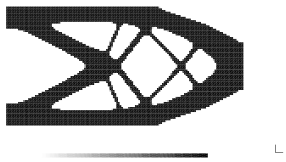
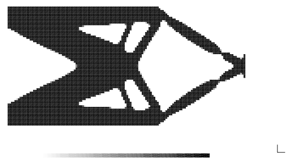
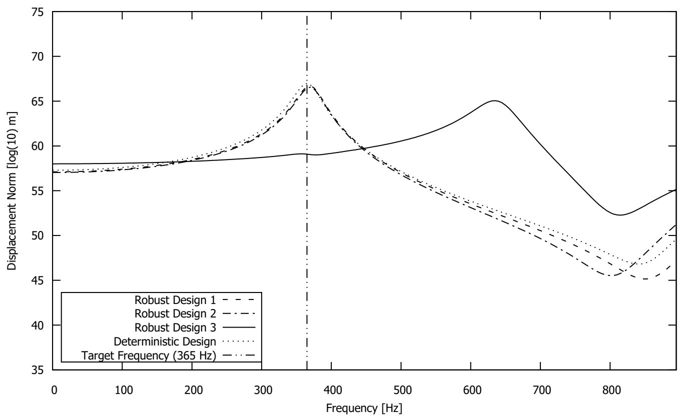
$\varphi = 10.0$	$\varphi = 20.0$	$\varphi = 50.0$
		

Figure 4. Frequency response of the deterministic design and robust design cases at 365 Hz, where Robust Design 1, 2 and 3 correspond to $\varphi = 10.0$, 20.0 and 50.0, respectively.



4 Conclusion

A formulation is proposed for the robust design of structures with maximum dynamic response with respect to uncertainties in the excitation frequency. The physical artifice employed by the optimization process to improve robustness was the development of resonances located close to the target frequency, leading to high dynamic displacements together with a robust dynamic behavior. Result at 365 Hz shows that the feasible solution was a low-energy resonance located close to the excitation frequency, whereas at 1135 Hz the mechanism was the formation of two high-energy resonances located before and after the target excitation frequency.

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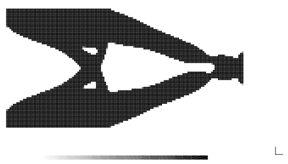
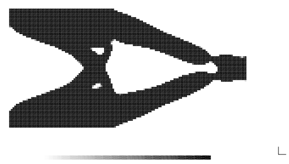
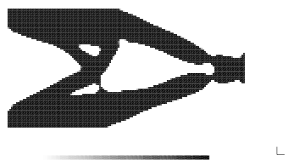
Figure 5. Topologies obtained for maximization at 1135 Hz, for different values of φ.

$\varphi = 10.0$	$\varphi = 20.0$	$\varphi = 50.0$
		

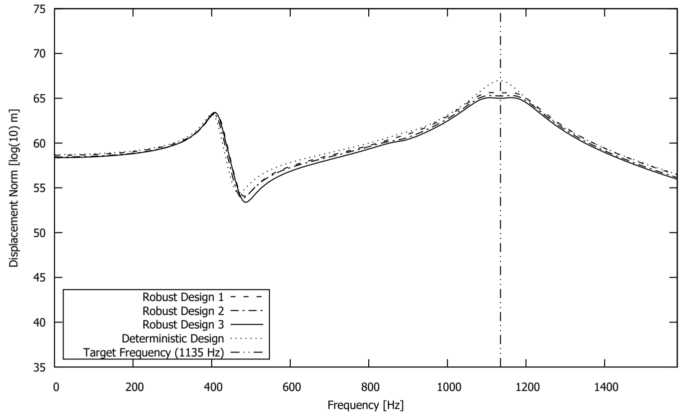


Figure 6. Frequency response of the deterministic design and robust design cases at 1135 Hz, where Robust Design 1, 2 and 3 correspond to $\varphi = 10.0$, 20.0 and 50.0, respectively.

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