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Inference of the modal parameters of a walkway through the clustering and bootstrap techniques association

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Abstract. Monitoring the behavior of a structure in operation is essential for the early detection of changes that may compromise its integrity. This is possible from the identification of its modal parameters, estimated through the modal analysis of the response signals (continuous in time) of the structure to the actions that act on it. Because they are estimated, the parameters obtained have uncertainties that significantly interfere with the reliability of structural monitoring, and therefore their quantification is essential. The principles of systems identification and experimental estimation have provided, in the last decades, innovative tools for the understanding and control of vibrations, design optimization, performance evaluation and structural integrity (Rainieri and Fabbrocino [1]). Several studies have also ratified the efficiency of continuous monitoring of structures in assessing their integrity (CHENG *et al.* [2] and SAISI *et al*. [3]). Thus, there is a need for study and continuous improvement of monitoring and structural identification systems in order to ensure the safety of existing structures. The objective of this work is to apply, in response signs of a pedestrian walkway submitted to people walking, a modal analysis methodology that provides more robust estimates and with lower levels of uncertainty of its modal parameters, through the association of Data- driven Stochastic Subspace Identification (SSI-DATA), bootstrap and clustering techniques. Finally, the results found for the uncertainties of the estimated parameters are discussed.

Keywords: uncertainty, bootstrap method, clustering method.

1 Introduction

The monitoring of buildings and large works is a relevant engineering tool for the analysis of structural integrity, and usually uses natural frequencies and modal shapes of structures as main parameters. The estimates of these parameters can be obtained through the modal analysis of continuous signals in time, which correspond to the response of the structure to the loads that act on it.

Aragão Filho [4] cites some of the techniques that calculate the estimates of modal parameters, such as the Frequency Domain Decomposition (FDD), Rational Fraction Polynomial (RFP), Least-Squares Complex Exponential Method (LSCE), Eigensystem Realization Algorithm (ERA) and the Data-driven Stochastic Subspace Identification (SSI-DATA). The use of these estimated parameters causes the modal analysis to have uncertainties, which can be better defined through signal pre-processing, statistical inference bootstrap and clustering techniques.

The objective of this work is to apply, in response signals from a pedestrian walkway subjected to people walking, a modal analysis methodology that provides more robust estimates with lower levels of uncertainty of its modal parameters, through the association of a strategy signal pre-processing with SSI-DATA, bootstrap and clustering techniques.

The first section of this article presents a brief introduction about the proposed theme. The second section briefly presents the signal pre-processing strategy, SSI-DATA, bootstrap and clustering techniques. The third section presents the structure and the signal acquisition system used. The fourth section addresses the proposed methodology of modal analysis of the acquired signal. The fifth section presents the results obtained with the application of the methodology discussed in the fourth section. In the sixth section, final considerations are made about the results found for the estimated parameters.

2 Theoretical fundamentals

2.1 Signal pre-processing

The pre-processing of a signal aims to increase the efficiency in the estimation of modal parameters and can be performed in several ways. In this work, the signal pre-processing strategy proposed by Lefone *et al.* [5] in the analysis of a numerical model of a double-supported beam. The pre-processing divided a 120-minute signal into 10 segments of 12 minutes each.

The first step of signal segmentation consisted of identifying the 10 largest positive peaks in the entire signal, for each monitored degree of freedom. After that, a degree of freedom was defined as a reference ($DOFr_{ef}$) and, from the position of the largest positive peaks of its signal, the 10 12-minute segments were formed with the peaks positioned in the center of each segment.

2.2 Bootstrap technique

The bootstrap technique of statistical inference is a tool that can be used to calculate the uncertainties inherent in the estimates of modal parameters. According to Efron [6], this technique is a resampling process that estimates parameters and obtains the confidence interval of random variables of unknown probability distribution from a limited availability of data. In this work, the bootstrap technique will be used with the same parameters adopted by Lefone *et al*. [7], who used 10 segments of the signal to compose each bootstrap resampling, defined from a random combination, with replacement. Each resampling was obtained from the simple average of the signal segments that compose it. A total of 50 bootstrap resamplings were used for each reference degree of freedom.

2.3 SSI-DATA technique

The SSI-DATA technique was proposed by Van Overschee and De Moor [8], obtaining a derivation proposed by Peeters [9]. This technique organizes the response signal into a Hankel matrix (symmetric matrix in which the elements of each anti-diagonal are equal), subdivided into two matrices: "past" data and "future" data.

After forming the Hankel matrix, the State matrix is estimated from the projection matrix P of the line-space formed by the lines of the "future" data matrix on the line-space formed by the lines of the data matrix. "past", and its decomposition into singular values.

The verification of the singular values of greater magnitude of the projection matrix allows to infer the order of the system. By varying the number of poles (reducing the number of lines in the Hankel matrix), we obtain the pole stabilization diagram and with it the modal parameters.

The number of lines *i* of each data block ("past" and "future") of the Hankel matrix is one of the fundamental parameters of the technique and must be greater than the maximum order of the system to be identified (Van Overschee and De Moor). [8]). Van Overschee and De Moor [8] state that, in a stochastic system, the bias of their estimates is inversely proportional to the number of rows *i*.

2.4 Clustering techniques

Clustering techniques consist of grouping estimates of modal parameters that are similar to each other, being widely used in structural identification research, as presented by Tronci *et al.* [10]. These groups are formed from the distance (or dissimilarity) between elements of the available data set, which can be calculated in several ways. Among them we have the Euclidean distance, used by Alves [11] and Lefone *et al.* [12] and represented in eq. (1), the distance proposed by Magalhães *et al*. [13] represented in eq. (2), and the distance proposed by Cardoso *et al.* [14] represented in eq. (3). The last two employ the parameters natural frequency and Modal Assurance Criterion

(MAC). The terms f_i and f_j, in eq. (1), eq. (2) and eq. (3), correspond to estimates of the natural frequencies of modes *i* and *j*. Cardoso *et al*. [14] defined the value of 5 Hz for the constant *c*, in eq. (3). The MAC calculation is shown in eq. (4), in which ϕ_i and ϕ_j are the vectors of the modal forms of modes *i* and *j*.

Clustering techniques are also classified into hierarchical and partitioning. Hierarchies are further divided into agglomerative and divisible. Agglomeratives consider that each available data element are distinct clusters that are joined using an agglomeration technique. The final number of clusters is determined by a threshold distance for cluster agglomeration. In divisible, the entire available dataset represents a single cluster that is successively divided into smaller clusters until a pre-defined maximum number of clusters is reached. In this work, the hierarchical agglomerative clustering technique was adopted, and the value of 1 Hz as the threshold distance ($d_{threshold}$) for cluster agglomeration.

$$d_{i,j} = \sqrt{\left(f_i - f_j\right)^2}. \tag{1}$$

$$d_{i,j} = \left|\frac{f_i - f_j}{f_j}\right| + \left(1 - MAC_{i,j}\right). \tag{2}$$

$$d_{i,j} = \left|f_i - f_j\right| + \left(1 - MAC_{i,j}\right) * c. \tag{3}$$

$$MAC_{i,j} = \left(\frac{|\phi_i^* \phi_j|}{\|\phi_i\| \|\phi_j\|}\right)^2. \tag{4}$$

3 Description of the structure and signal acquisition system

The present work used the response signals obtained by experimental tests carried out by Aragão Filho [4] on a pedestrian walkway subjected to two people walking in a straight line and in the same direction. The walkway is a mixed structure, with I-profile metal beams for the transvers and stringers, and a reinforced concrete deck measuring 12,20 m x 2,20 m x 0,10 m. On the board, there are 26 load cell modules made from MDF boards.

Aragão Filho [4] monitored the response of the footbridge by installing three 1*g* and two 2*g* resistive accelerometers, positioned as shown in Fig. 1 to obtain good sensitivity to the first five vibration modes. The acquisition frequency was 200 Hz, with an anti-aliasing filter of 30 Hz, with a total acquired time of 300 seconds. The intervals obtained by Aragão Filho [4] for the natural frequencies and the damping rates, estimated from the LSCE and ERA techniques for heel drop tests, are presented in Tab. 1.

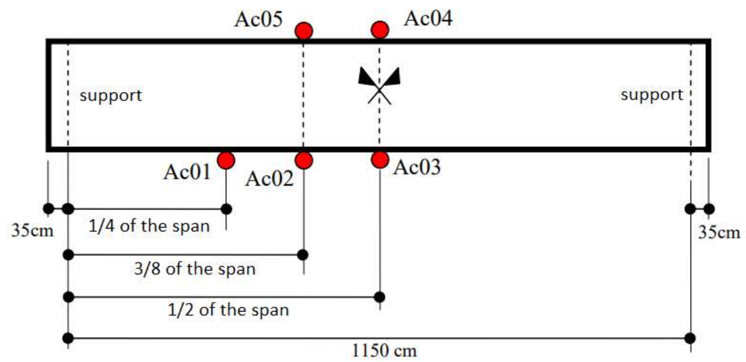

Figure 1. Layout in plan of accelerometers on the walkway (Aragão Filho [4]).

Table 1. Natural frequencies and damping rates estimated from LSCE and ERA techniques for the heel drop tests (Aragão Filho [4])

Vibration mode	Natural frequency (Hz)			Damping rate (%)		
	Min	Mean	Max	Min	Mean	Max
1st of bending	3,20	3,22	3,23	0,87	1,06	1,23
1st of torsion	9,76	9,82	9,92	0,93	1,22	1,56
2nd of bending	12,05	12,09	12,12	1,50	1,63	1,79
2nd of torsion	23,27	23,33	23,42	0,59	0,72	0,84
3rd of bending	24,67	24,77	24,88	0,65	0,74	0,85

4 Proposed methodology

The methodology of the present work is based on improving the methodology proposed by Lefone *et al.* [12], using a new way of calculating the distance between estimates, instead of the Euclidean distance. Furthermore, the present work will apply the methodology in a real structure, unlike Lefone *et al.* [12] who used a numerical simulation of a double-supported beam.

A new way of calculating the distance between estimates is proposed, through a merger between the proposals of Magalhães *et al.* [13] and Cardoso *et al.* [14], presented in eq. (5) and keeping the value of 5 Hz for the constant *c*.

$$d_{i,j} = \left|\frac{f_i - f_j}{f_j}\right| * c + \left(1 - MAC_{i,j}\right). \tag{5}$$

The first step consists of carrying out the pre-processing of the signal proposed by Lefone *et al.* [5], dividing it into 10 segments of 30 seconds, for each DOF_{ref}. In the case of the footbridge, 5 DOF_{ref} were worked on because the monitoring took place with 5 accelerometers. After that, for each DOF_{ref}, 50 bootstrap resamplings are calculated as performed by Lefone *et al.* [7]. Subsequently, estimates of the modal parameters of the 50 bootstrap resamplings are obtained through the SSI-DATA technique, for each of the 3 values of the number of rows *i* of the blocks of the Hankel matrix adopted by Lefone *et al.* [12]. With all estimates calculated, the agglomerative hierarchical clustering technique is applied, with $d_{threshold}$ of 1 Hz for agglomeration of estimates and formation of clusters. The 5 largest valid clusters correspond to the first 5 vibration modes of the structure. The steps of the proposed methodology are presented, in simplified form, in Fig. 2.

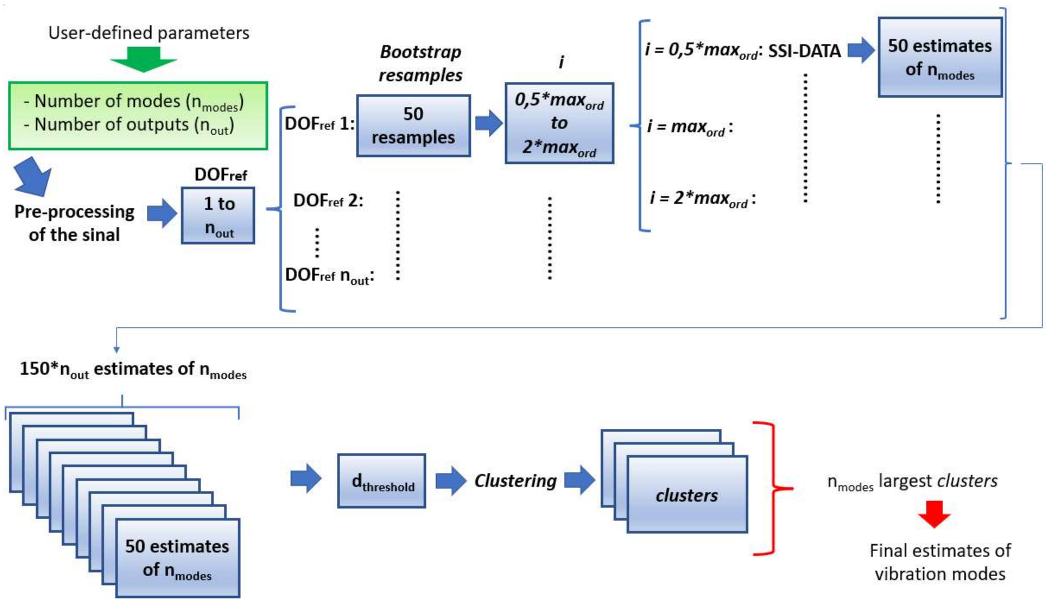

Figure 2. Proposed methodology steps (Lefone *et al.* [12]).

5 Results

The estimates of the natural frequencies of the analyzed structure, as well as their confidence intervals, are presented in Tab. 2. The confidence intervals of the estimates of the modal parameters were calculated in the same way as the previous simulations, considering that the values of the calculated estimates correspond to a normal distribution with a confidence level of 95%.

Table 2. Estimated modal parameters values.

Parameters	Vibration modes (formed clusters)				
	1st of bending	1st of torsion	2nd of bending	2nd of torsion	3rd of bending
Number of elements	235	70	442	21	137
Natural Frequency (Hz)	3,26	9,87	12,11	23,43	24,87
Confidence interval (Hz)	0,003	0,016	0,003	0,046	0,005
Damping ratio (%)	1,55	2,05	1,41	5,68	0,66
Confidence interval (%)	0,14	0,14	0,03	5,77	0,02

The analysis of the signal's autospectrum allowed us to verify that the first 5 modes of the structure have natural frequencies below 30 Hz, making it possible to eliminate clusters with frequencies above this value and obtaining the 5 largest final clusters formed, shown in Tab.2.

The estimates of the averages of the natural frequencies of each cluster presented values very close to the values obtained by Aragão Filho [4] (Table 1), with significantly lower confidence intervals. The minimum reduction was 48.89% for the 2nd torsion mode, and the maximum of 95.45% for the 3rd bending mode.

Regarding the average estimates of the damping rates of each cluster, those related to the 2nd and 3rd bending modes presented values very close to one of the limits of the confidence range of Tab. 1 (variation of -6.0% and 1.54%, respectively). The estimates referring to the 1st bending mode and the 1st torsion mode were higher in 26.02% (bending) and 31.41% (torsion). The estimate of the 2nd torsion mode was much higher than the limits obtained by Aragão Filho [4]. The confidence intervals obtained were smaller, with a reduction from 26.32% (1st bending mode) to 81.82% (3rd bending mode), except for the 2nd torsion mode, which had a much higher confidence interval.

The higher difference in the estimates of the damping rate of the 2nd torsion mode, in relation to the result obtained by Aragão Filho [4], may be because it is a more difficult mode to be excited by the action of two people walking. The cluster that represents this mode of vibration is the one with the least number of elements.

The modal shapes of the estimates existing before, by frequency range, and after the formation of clusters are shown in Fig. 3 to Fig. 7. The abscissa axis of these figures represents the accelerometers distributed along the walkway, as shown in Fig. 1. The mean modal shapes of the formed clusters are compatible with the modes they represent.

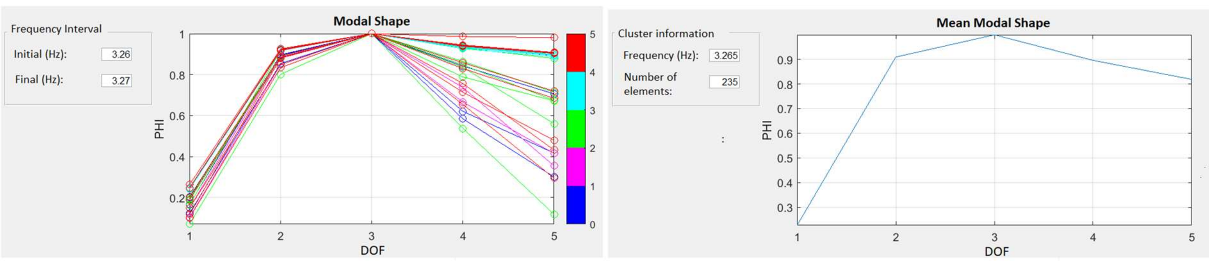

Figure 3. Modal shape of the 1st bending mode (formed cluster).

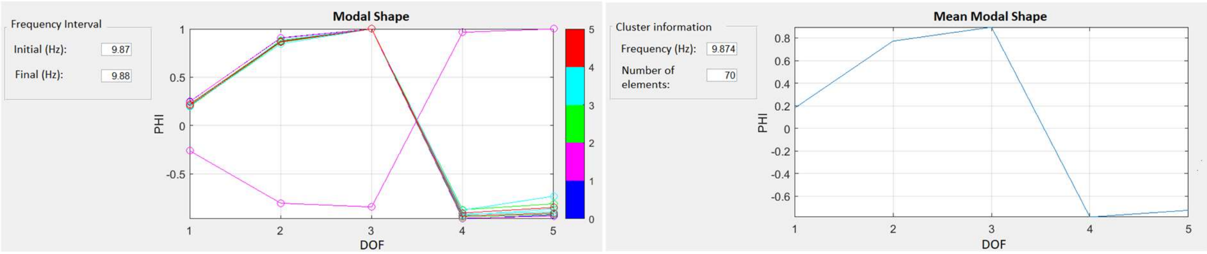

Figure 4. Modal shape of the 1st torsion mode (formed cluster).

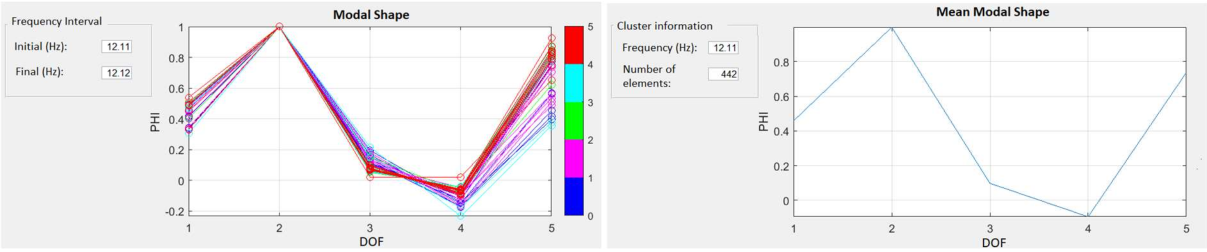

Figure 5. Modal shape of the 2nd bending mode (formed cluster).

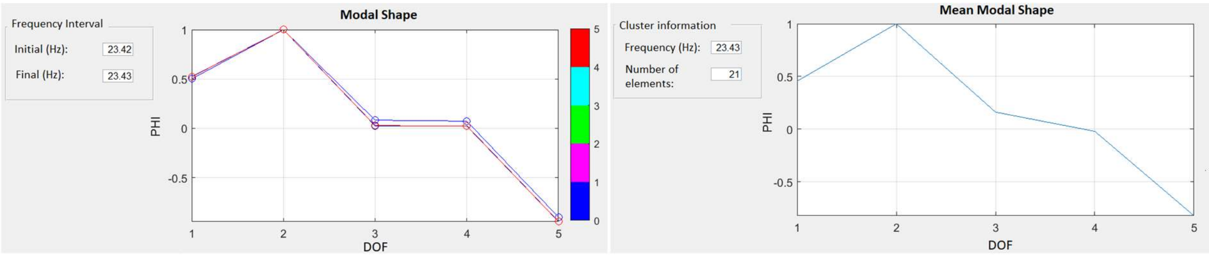

Figure 6. Modal shape of the 2nd torsion mode (formed cluster).

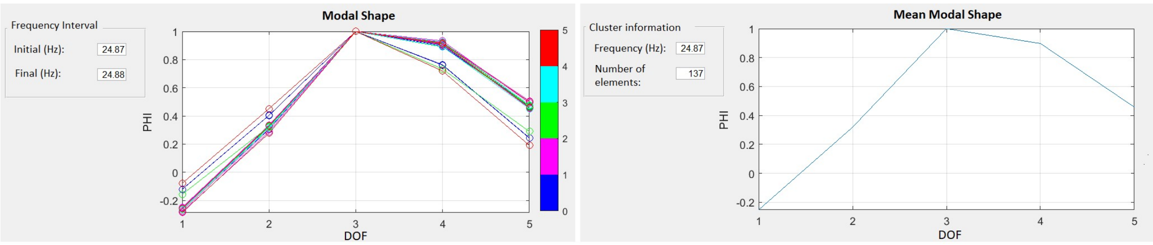

Figure 7. Modal shape of the 3rd bending mode (formed cluster).

6 Conclusion

The need to obtain previous estimates of impulse response functions (IRF) and the high sensitivity of the SSI technique to the numerical parameters of subspace projection and model order reduction motivated the proposition of a methodology that inferred a mass of parameter estimates modals and only then perform the grouping of the real vibration modes.

In this way, it is sought to give greater robustness and regularity to the estimates and, at the same time, and to estimate uncertainties by the scattering of the groups identified as representative of the vibration modes.

For the estimation of the IRF, a pre-processing strategy was used by windowing local maxima and booststrap resampling, multiplying the number of input functions. The SSI algorithm was also adapted to receive different adjustment parameters, which, cumulatively, allowed obtaining a significant mass of estimations.

The grouping of estimates by the proximity of the frequency and similarity of the modes using the Agglomerative Hierarchical Method consisted of the last step of the methodology, which, despite being an assisted machine learning process, gives substantial automatism to the process.

The proposed methodology presented robust estimates for the modal parameters of the footbridge under study, with lower levels of uncertainty in relation to the results obtained by Aragão Filho [4]. The results obtained demonstrate the feasibility of applying this methodology in the modal analysis of structures in operation through the SSI-DATA technique.

New works are still being developed seeking to improve efficiency in the grouping of estimates and uncertainty inferences.

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Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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