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BIODYNAMIC MODELING OF THE HUMAN ACTIONS IN THE DYNAMIC ANALYSIS OF FOOTBRIDGE

Vitor A. Gonçalves[1], Anderson de S. M. Gadéa [1], Geraldo J. B. dos Santos[1]

[1] *Civil and Environmental Engineering Postgraduate Programme, State University of Feira de Santana*
Av. Universitária, s/n - Km 03 da BR 116, Campus Universitário, 44.031-460, Feira de Santana/BA, Brasil
eng.vitoradorno@gmail.com, belmonte@uefs.br, gadea@uefs.br

Abstract. Civil structures such as footbridges and floors are commonly subject to dynamic loads due to human activities such as walking, running and jumping. Currently there are several light and slender structures with low natural frequencies, which are susceptible to human actions, generating discomfort and structural safety risks problems. In most cases the human induced loads on footbridges are considered as equivalent static loads or as moving loads. This paper presents a biodynamic modeling of human walking or running on footbridges, considering people as a simple spring-mass-damper system. The dynamic analysis was based on the finite element method and implemented using Scilab open-source software for numerical computation. The results showed that crowd load can significantly change the responses in relation to the moving load because of the addition of mass and damping to the system and due the dynamic interaction between structure and people.

Keywords: Biodynamic models, dynamics, pedestrian load, footbridges.

1 Introduction

Architectural trends for innovative projects associated with technological advances both in the execution process and in the development and use of materials in civil construction have generated lighter and slender structures. These dispositions can be observed in modern pedestrian walkways, resulting in a greater susceptibility of these structures to vibrations caused by dynamic efforts [1]. Researchers have investigated the problem of excessive vibrations in structures [2, 3] and questioned the models used to analyze the problem. A modeling that has received a lot of attention from scientists nowadays is the use of Biodynamic Models of human loading, which has led to satisfactory results [4, 5]. There are reports of increase in accelerations [6] and changing in dynamic properties of the structure (damping ratio and natural frequency) [7, 8].

The most common methods of predicting the behavior of a footbridge-like structure are 3: equivalent static load, mobile dynamic load and the biodynamic. The first appears in some norms and even in them appears in specific situations. The second also appears in standards [9] and, unlike the first, considers the parameters of the structure, being able to perceive dynamic phenomena such as resonance and beat. The third has been studied in recent years and seeks to find results closer to reality, considering that it adds to the modeling the consideration of the interaction between pedestrians and structure and how this influences the response.

In this work, the comparative results between moving load models and moving biodynamic models will be presented, through numerical modeling based on finite elements implemented in a computational tool developed by the authors using the open language software Scilab. For this, data from footbridges studied by Costa [5], two models of the dynamic loading function and one other model of biodynamic parameters will be used. Four structures derived from the cross section studied by Costa will be submitted to the methods presented in three different crowd densities, results are later shown in this work.

2 Moving loads approach

It is possible to represent the load carried by the human as a time and space varying load function. The space-varying parcel can be treated as uniform motion or uniformly varied motion. Through the Dirac delta operator, the parcels in time and space can be separated. Considering, for example, a uniform motion [9]:

$$P(x,t) = \int F(t) \cdot \delta(x - v_p \cdot t)\, dt$$

Where P(x,t) is the vertical force applied by the pedestrian on the footbridge floor, F(t) is the time-varying portion of the force, δ is the Dirac delta operator, x is the position of the pedestrian in the structure, v_p its speed and t is the time instant. The time-varying portion is commonly represented by a 3-term Fourier series, as shown in equation (2), but there are other variations. A series of models and the coefficients proposed by each author can be found in Zivanovic [1].

$$F(t) = G + \textstyle\sum_{i=1}^{n} G\, \alpha_i \, sin\,(2\pi i f_p - \varphi_i) \tag{2}$$

Where G is the pedestrian's weight, α_i is the i^{th} dynamic load factor, φ_i is the i^{th} phase angle between the i^{th} and the first harmonic, and f_p is the pedestrian's step frequency. The first loading model used in this work follows equation (2) and was proposed by Bachmann *et. al* [10], in which the coefficients used are available.

The second model used in this work was proposed by Varela [11]. The author proposed an adjustment in the Fourier series to take into account the peak loading produced by the heel impact. Equations (3) to (7) coupled are the results found by the author. Figure (1) illustrates the Bachmann and Varela models for a pedestrian with a weight of 700 N and a step frequency of 2 Hz.

$$\boldsymbol{F}(\mathrm{t}) = \left(\frac{f_{mi} \cdot G \cdot (1+\sum \alpha_i) - G}{0{,}04 * T_p}\right) \cdot t + G \qquad se\ 0 \leq t \leq 0{,}04T_p \tag{3}$$

$$\boldsymbol{F}(\mathrm{t}) = f_{mi} \cdot G \cdot (1+\sum \alpha_i) \cdot \left[\frac{\left(\frac{1}{f_{mi}} - 1\right)(t - 0{,}04T_p)}{0{,}02 \cdot T_p} + 1\right] \qquad se\ 0{,}04T_p \leq t \leq 0{,}06T_p \tag{4}$$

$$F(t) = G \cdot (1 + \sum \alpha_i) \qquad se\ 0{,}06T_p \leq t \leq 0{,}15T_p \tag{5}$$

$$\boldsymbol{F}(\mathrm{t}) = G\left[1 + \sum \alpha_i \,\mathrm{sen}\left(2\pi i f_p\left(t + 0{,}1T_p\right)\right) - \varphi_i\right] \qquad se\ 0{,}15T_p \leq t \leq 0{,}90T_p \tag{6}$$

$$F(t) = 10[G - G \cdot (1 - \alpha_2 + \alpha_4)] \cdot \left(\frac{t}{T_p} - 1\right) + G \qquad se\ 0{,}90T_p \leq t \leq T_p \tag{7}$$

Where f_{mi} is the heel impact augmentation factor, taken in this work as 1.12, and T_p is the function period.

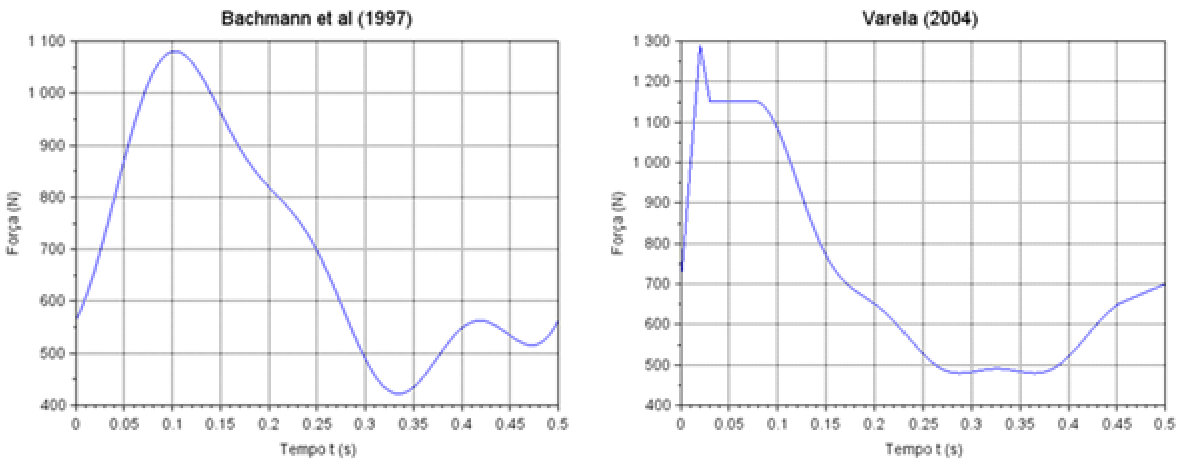

Figure 1. Time-varying portion of the load function according to Barchmann and Varela.

3 Biodynamic approach

The biodynamic models differ from the force model when considering the exciter source of the load, in this case the pedestrian, as an integral part of the structure, having mass, damping and stiffness. Knowing that the walking process is formed by a complex set of ligaments and muscles, it is possible to consider the human body as a system of one or more degrees of freedom that moves through the structure, causing the excitations previously attributed to directly applied forces. For this work, the pedestrian will be considered as a single degree of freedom (SDoF), illustrated in Figure (2).

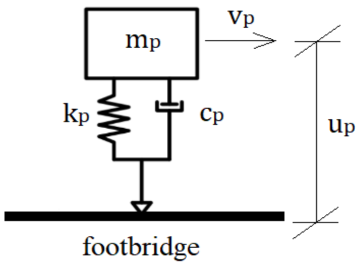

Figure 2. One degree of freedom system.

The biodynamic parameters (mass, damping and stiffness) are found from experiments with pedestrians and analysis of frequency domain acceleration signals. Costa [5], Silva and Pimentel [11] and Toso *et al.* [12] are among the researchers who used linear regression functions to propose functions that output the aforementioned parameters. The biodynamic model proposed by Costa makes use of equations (8) to (10).

$$m_p = 12.94 + 0.874 \cdot M - 9.142 \cdot \mathrm{f_p} \tag{8}$$

$$k_p = 360.3 \cdot m_p - 1282.5 \tag{9}$$

$$\xi_p = -20.818 \cdot f_{ma} + 87.513 \tag{10}$$

Where M is the pedestrian's mass, f_{ma} is the pedestrian's damped natural frequency and m_p, k_p and ξ_p are the model's mass, stiffness and damping rate, respectively. The SDoF coupling of a moving pedestrian in the structure follows the model number presented by Toso [14]. Equations (11) and (12) presents the equilibrium equations in summary form of the structure and the SDof system.

$$[\boldsymbol{M}] \cdot \ddot{D} + [\boldsymbol{C} + \boldsymbol{C}^*] \cdot \dot{D} + [\boldsymbol{K} + \boldsymbol{K}^*] \cdot D - k_p \cdot \mathrm{y} \cdot N^T - c_p \cdot \dot{\mathrm{y}} \cdot N^T = \boldsymbol{F} \tag{11}$$

$$m_p \cdot \ddot{\mathrm{y}} + c_p \cdot (\dot{\mathrm{y}} - N \cdot \dot{D}) + k_p \cdot (\mathrm{y} - N \cdot D) = 0 \tag{12}$$

Where $\boldsymbol{M}$, $\boldsymbol{K}$, e $\boldsymbol{C}$ are the structure global matrices of mass, stiffness and damping, the terms m_p, c_p and k_p are the biodynamics parameters, v_p is the pedestrian speed, y and its derivatives are the displacement, speed and acceleration of the contact point between the SDoF and the structure. D and its derivatives are the displacement, speed and acceleration vectors of the discretized finite element system and N is the vector of the interpolation functions. Finally, the matrixes $\boldsymbol{C}^*$ and $\boldsymbol{K}^*$ are associated with the biodynamic model, according to Toso [14]:

$$\boldsymbol{C}^* = c_p \cdot N^T \cdot N \tag{13}$$

$$\boldsymbol{K}^* = k_p \cdot N^T \cdot N \tag{14}$$

4 Finite Element Analysis

4.1 Software development

To perform the dynamic analysis the authors developed a computational tool based on finite elements using the open-source software Scilab. The algorithm, after receiving data from the discretization of the problem in nodes, elements, materials and sections, performs the solution of the problem using Euler special frame elements. The user can also choose the type of mass hue (Consistent or Lumped), add loading functions and perform time integration by modal superposition or direct integration methods of Newmark or 4^{th}-order Runge-Kutta. The damping matrix formulation follows the Rayleigh formulation and the modal analysis of the structure in free vibration is performed via subspace iteration. An overview of the methods used can be found in Bathe [15]. The software was verified via comparisons with SAP2000, Bathe [15] and Costa [5]

The implementation of the pedestrian-structure interaction was carried out from the interpolation of the biodynamic parameters within the elements of the discretized structure as the mass-spring-damper system moves in the structure. The interpolation was carried out using the same vectors (Hermit interpolating functions) used to simulate the displacement of the force vector in space and find the displacements within each element of the structure. To solve the dynamic problem, it is proposed that for each instant of time the mass, stiffness and damping matrices of the structure are updated in order to introduce the biodynamic parameters of each of the pedestrians in contact with the structure.

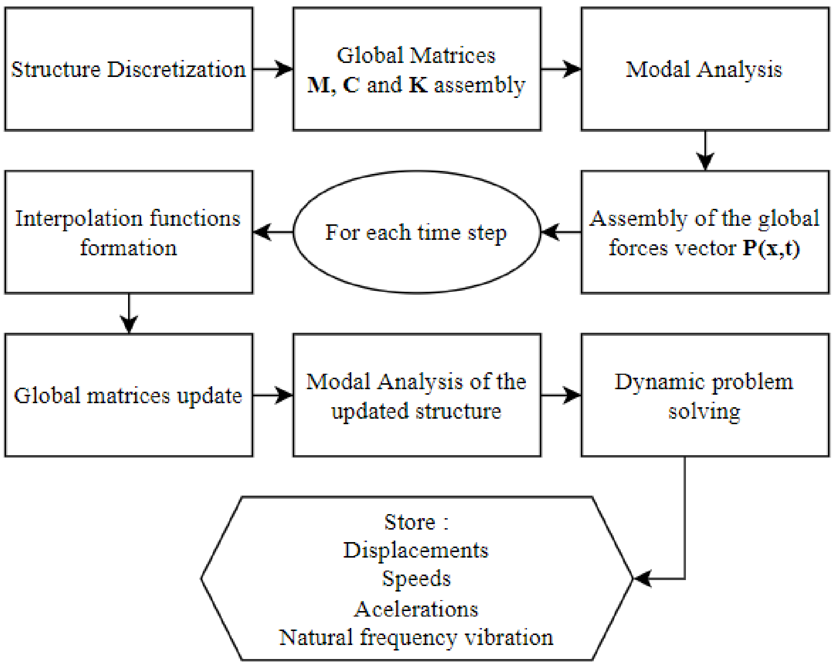

Figure 3. Summary flowchart of the algorithm developed by the authors.

4.2 Analyzed structures

The structure analyzed will be based on the mixed steel and contract beam studied in [5]. Using data from the homogenized section presented, the structures described in Table 1 will be simulated. All of them were discretized by 9 equally spaced nodes and 8 elements. It is important to inform that in all structures the boundary conditions are present at nodes 1 and 9, respectively at start and the end of the simulated footbridges. The structures vary in fundamental frequency, where at Structure 01 the loading is resonating with the first vibration mode, at Structure 02 the loading frequency is lower than the first vibration mode and at Structure 03 the loading frequency is higher.

Table 1. Analyzed Structures.

Input data	Structure 01	Structure 02	Structure 03
Length	35 (m)	35 (m)	60 (m)
Cross-section specific mass	23.371 (ton/m³)	23.371 (ton/m³)	23.371 (ton/m³)
Fundamental frequency	2.02 Hz	4.59 Hz	1.56 Hz
Damping ratio	0.50%	0.50%	0.50%
Cross-section area	0.0783 m²	0.0783 m²	0.0783 m²
Inertia	$2.17 \cdot 10^{-2}$ m^4	$2.17 \cdot 10^{-2}$ m^4	$2.17 \cdot 10^{-2}$ m^4
Elasticity modulus	210 GPa	210 GPa	210 GPa
Boundary Conditions	Supported	Embedded	Embedded

4.3 Pedestrian information

In order to obtain reliable comparisons between the models, all pedestrians have the same characteristics of mass (80 kg), step frequency (2.0 Hz) and speed (1.6 m/s). For each simulation 100 people will walk through the structures, with a pedestrian density of 0.6 person/m² (typical value). For the first and second structures, the densities of 1.0 person/m² (probable value) and 1.5 person/m² (maximum value found on the Millennium Bridge opening day) are simulated to evaluate the changes of the system fundamental frequency and acceleration [1].

5 Numerical Results

The first analysis to be presented is the comparison between the displacements and accelerations obtained for the moving force models (MFM) and moving biodynamic models (MBM). The Table (2) shows the results for a crowd density of 0.6 persons/m², where Structure 01 to 03 are the structures, *a* is the acceleration and *u* is the displacement of the structure central node. In every scenario the Biodynamic model has both displacement and acceleration results are lower than the forces model. This occurs because the introduction of biodynamic parameters promotes an increase in the damping coefficient of the structure, dispensing more quickly the energy associated with the movement.

Another interesting analysis that we can take from this table is the sudden reduction that occurs in Structure 1. Initially, the frequency of the pedestrian step would be in resonance with the fundamental frequency of the structure, seen in Table 1. It happens that, when having its fundamental frequency of vibration reduced due to the presence of pedestrians, which can be seen in Figure 4, resonance ceases to occur, and a beat starts, which generates smaller displacements than the resonance.

Table 2. Results for crowd density of 0.6 persons/m².

Structure and model		Bachmann		Varela	
		a (m/s^2)	u (mm)	a (m/s^2)	u (mm)
Structure 01	MFM	12.1	79.4	16	103
	MBM	0.022	3.91	0.058	4.04
Structure 02	MFM	0.36	1.51	0.483	1.64
	MBM	0.0324	0.83	0.058	0.86
Structure 03	MFM	0.784	10.7	1.15	11.5
	MBM	0.053	6.66	0.079	6.89

Setra [9] presents ranges associated with comfort levels based on the acceleration imposed on the system. In addition, a classification of frequency bands to evaluate the risk of resonance. Table 3 presents the classification of the comfort levels of the structures for the different models.

Table 3. Comfort levels according to *Setra* [16].

Structure and model		Bachmann	Varela
		a (m/s^2)	a (m/s^2)
Structure 01	MFM	unacceptable	unacceptable
	MBM	max	max
Structure 02	MFM	max	max
	MBM	max	max
Structure 03	MFM	mean	mín
	MBM	max	max

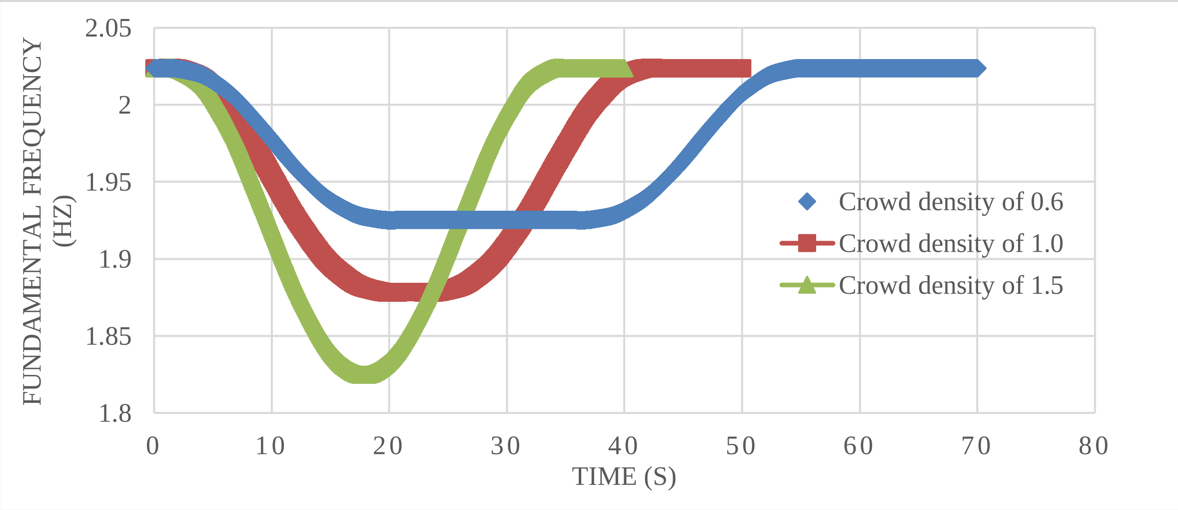

Figure 4. Influence of the crowd density in the natural frequency of the structure.

The increase in population density maintained the pattern of previous results, now with higher damping rates. Below are the results found for Varela model.

Table 3. Results of acceleration for crowd density of 1.0 and 1.5 persons/m².

Structure and model		Varela	
		1 pd/m²	1.5 pd/m²
Structure 01	MFM	17.22	108
	MBM	6.5	45
	%	83%	85%
Structure 02	MFM	1.34	3.5
	MBM	1.27	3.3
	%	13%	30%

6 Conclusions

This work shows a comparison between mobile force models and the mobile biodynamic model. In relation to the mobile forces model, the Varela model presents higher displacement and acceleration results than the Bachmann models, which was expected, since the consideration of the heel impact generates precisely this effect.

Regarding the biodynamic model, it was possible to observe that the pedestrian crossing leads to a significant change in the dynamic parameters of the structure, reducing the natural frequency of the structure by up to 13% in conditions of high pedestrian density. In the cases studied, this change was positive, since it avoided the resonance phenomenon, but the opposite could have happened. In addition, it is possible to prove the increase in damping in all cases, when comparing the models of mobile forces and biodynamic models. This increase in damping caused some structures to vary from comfort range, but the lack of experimental results made it impossible to assess whether this reduction in displacements and accelerations is more consistent with reality, but the fact that these results are in agreement with those obtained by other authors shows a pattern in the model.

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