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Numerical Simulation of Oil and Water Displacements in Petroleum Reservoirs using a Multipoint Flux Approximation Method Coupled to a Flux Corrected Transport with a Flow Oriented Formulation

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Abstract. The numerical modeling of multiphase and multicomponent flow in oil reservoirs poses a great challenge and demands the development of robust and computationally efficient numerical formulations. Common reservoir simulators are based on the combination of the classical Two Point Flux Approximation (TPFA) for the discretization of diffusive fluxes and the First Order Upwind (FOU) method for the discretization of the advective fluxes in the fluid flow equations. In certain cases, particularly for high mobility ratios between the injected and the resident fluids, the numerical solution may strongly depend on the alignment between the flow and the computational grid, this is known as the grid orientation effect (GOE). This effect is linked to the anisotropic distribution in the truncation error associated to the numerical approximation of the transport term. Another problem occurs when non monotonic solutions are obtained whenever using highly distorted meshes. Besides, the standard TPFA method may not converge at all to an adequate solution when the grid is non k-orthogonal. In this context, in the present paper, our main goal is present a full cell centered finite volume formulation for the numerical simulation of oil-water displacements in oil reservoirs using a segregate formulation in general unstructured and non k-orthogonal 2D meshes. For the discretization of the diffusive fluxes, we use a Multipoint Flux Approximation based in Harmonic Points (MPFA-H) and for the discretization of the transport term, we present a modified version of the 2nd order Flux Corrected Transport (FCT) approach to reduce artificial numerical diffusion and we also use a Flow Oriented Scheme (FOS) in the computation of the low-order approximations used in our modified FCT scheme. The FOS philosophy consists in using weights that are properly adapted to the flow direction, turning our scheme into a truly multidimensional approximation in order to reduce GOE. Our strategy was tested against one benchmark problem available in literature, producing very accurate results with reduced artificial numerical diffusion and GOE.

Keywords: Numerical Simulation, Grid Orientation Effect (GOE), Flux Corrected Transport (FCT), Multipoint Flux Approximation based in Harmonic Points (MPFA-H).

1 Introduction

The reservoir simulation is an important tool to engineers and researchers that help to understand recovery mechanisms therefore it has become a major source of decision-making [1]. Nevertheless, the numerical modeling of multiphase and multicomponent flow in oil reservoirs demands the development of robust and efficient computational tools because is a very complex task. Most of commercial reservoir simulators are based on a combination of the classical Two Point Flux Approximation (TPFA) for the discretization of diffusive fluxes and the First Order Upwind (FOU) method for the discretization of the advective fluxes in the fluid flow equations. The standard TPFA method has a serious numerical limitation when using non K-orthogonal grids or when using anisotropic full permeability tensors even been a robust method for K-orthogonal grids, therefore the numerical solution may not converge to the expected solution in these situations. The FOU method is heavily influenced by the grid orientation effect (GOE) especially when there are high mobility ratios between the injected and reservoir fluids. The GOE is related to the anisotropic distribution in the truncation error linked to the numerical approximation of the transport term, thus the numerical solution may be deeply dependent on the alignment between the flow and the computational grid. Hence, it is required a robust formulation adequate to correctly express the anisotropy and heterogeneity for the fluxes in order to diminish these problems.

In this work, we proposed the adoption of a Multipoint Flux Approximation based in Harmonic Points (MPFA-H) scheme developed by [3] to solve the elliptic equation and for the discretization of the transport term a modified version of the 2nd order Flux Corrected Transport (FCT) and we also use a Flow Oriented Scheme (FOS) developed by [13] in the computation of the low-order approximations used in our modified FCT scheme to solve the hyperbolic equation. The MPFA-H is able to deal with any star-shaped polygonal meshes, respects the linearity preserving criterion and provides accurate results even when dealing with anisotropic permeability tensors. A linear combination of the one-sided fluxes of the two control volumes adjacent to each face is used to express the unique flux on each control surface of the discrete domain. These fluxes are expressed by one cell centered unknown and points defined on the faces, which are the harmonic points [12]. To achieve a conservative scheme, the unique flux over the face IJ is written as a convex combination of the one-sided fluxes [9]. The discretization of the MPFA-H is explained in more detail in [4].

In the literature, it is frequent the use of flow-oriented schemes (FOS) to lower the grid orientation dependence. We can cite some works related to the context of petroleum reservoir simulation such as [5], [7], [8], [13], [14]. The FOS is able to reduce the GOE mainly in non-orthogonal grids. This scheme is described by the use of a convex combination of the water fractional fluxes values succeeding the approximate flow orientation throughout the computational domain. As far as the authors know, it's the first time that is proposed a multipoint flux approximation using harmonic points scheme to solve the elliptic equation associated to a modified version of the 2nd order Flux Corrected Transport (FCT) using a truly multidimensional formulation in the computation of the low-order approximations.

2 Mathematical Formulation

In this section, the basic governing equations for the two-phase flow of water and oil in petroleum reservoirs will be described. We assume incompressible fluid flow and rock, in the absence of capillarity, gravity and thermal effects. Also, we adopt that the reservoir rock is fully saturated by water and oil and by using the mass conservation equation and the Darcy's law [13] we achieve the pressure and saturation equations, respectively, as follows:

$$\nabla . \vec{v} = Q \quad with \quad \vec{v} = -\lambda \overline{K} \nabla \mathrm{P} \tag{1}$$

$$\varphi \frac{\partial (S_i)}{\partial t} = \nabla . \vec{F}(S_i) + Q_i \quad with \quad \vec{F}(S_i) = f_i \vec{v} \tag{2}$$

In Eq.1, Q means the total fluid injection or production specific rate with $Q_i = q_i / p_i$ with $Q = Q_w + Q_o$. $\overline{K}$ and ∇P represents, respectively, the absolute rock permeability and the pressure gradient. In addition, λ is the total mobility which is given by $\lambda = \lambda_w + \lambda_o$. In Eq.2, f_i is the fractional flux function which can be expressed by $f_i = \lambda_i / \lambda$ and $\vec{F}(S_i)$ is the advective flux function. Besides, S_i, φ, p_i means, in this order, saturation of each phase i (i=w for water and i=o for oil), the rock porosity, density. Finally, to complete the mathematical model adequate boundary and initial conditions are required, such as:

$$\begin{array}{llll} p(\vec{x},t) = g_D & on & & \Gamma_D \times [0,t] \\ \vec{v}.\vec{n} \quad = g_N & on & & \Gamma_N \times [0,t] \\ S_w(\vec{x},t) = \bar{S}_w & on\ \Gamma_I \times [0,t] & or & \Gamma_D \times [0,t] \\ S_w(\vec{x},0) = \bar{S}^0{}_w & & on & \Omega \times t^0 \end{array} \tag{3}$$

where Γ_D, Γ_N are, respectively, the Dirichlet and Neumann conditions and Γ_I represent the injector wells. Besides, g_N and g_D means, in this order, the prescribed fluxes and pressures. The prescribed water saturation attributed at the injection wells is represented by $\bar{S}_w$ and $\bar{S}^0{}_w$ is the initial water saturation field.

3 Numerical Formulation

In this section, we present the numerical formulations used to approximate the equations that describe the fluid displacements in oil reservoirs. Then, by integrating the Eqs.1 and 2 on each control volume of computational domain and using the mean value and Gauss's divergence theorem, we achieve the elliptic pressure equation and hyperbolic saturation equation, respectively:

$$\sum\nolimits_{IJ \in \Gamma_k} \vec{v}_{IJ}.\vec{N}_{IJ} = \bar{Q}_{w(k)}\Omega_k \tag{4}$$

$$\bar{\varphi}_k \frac{\Delta \bar{S}_{w(k)}}{\Delta t}\Omega_k = -\sum\nolimits_{IJ \in \Gamma_k} \left(f_w\right)_{IJ} \vec{v}_{IJ}.\vec{N}_{IJ} + \bar{Q}_{w(k)}\Omega_k \tag{5}$$

where $\bar{\varphi}_k$ and $\bar{S}_{w(k)}$ are, in this order, the porosity and representative mean values of the water saturation to a generic control volume (k). The fractional flux and the Darcy's velocity associated to edge IJ are represented by $\left(f_w\right)_{IJ}$ and $\vec{v}_{IJ}$, respectively, and $\overline{Q}_{w(k)}$ is sink/source term.

In this work, we use the implicit pressure explicit saturation (IMPES) strategy to solve the pressure and saturation equations in a segregated procedure. In the IMPES strategy, known an initial saturation distribution, we calculate the total mobility throughout the domain and we compute the unknown pressure field and thus the unknown saturation field is computed. Thenceforth, the new saturation field is used to update the mobilities which is take into account when the pressure field is evaluated. This process happens until the final time of the simulation.

3.1 Implicit Pressure Discretization using MPFA-H

This method was developed in the work of [3], in which it's used the harmonic points strategy originally inspired in the work of [2], [11] [6] and [15] these auxiliary points are calculated from the harmonic points concept [12]. This scheme guarantees monotone solutions on arbitrary anisotropic permeability tensors and any star-shaped polygonal meshes.

3.2 Numerical Discretization of the Saturation Equation

In the present work, in order to discretize the saturation equation, we have devised to proposed a Flux Corrected Transport method based in the work of [5]. The FCT is one high-order scheme originally developed by [10], [16] proposed a multidimensional formulation to the method. The core of the FCT is a high-order solution obtained by the correction of a low-order monotonic solution through an antidiffusive flux. It's necessary the limitation of the antidiffusive flux focus on prevent the appearance of a new local extrema, therefore the local extremum diminishing criterion is frequently used to obtained the flux limiters [5].

The FCT method consists in five steps as follows:

1. Compute a low-order monotonic solution which is overly diffusive;
2. Compute a higher-order flux which may result in oscillations in the solutions;

3. Define the antidiffusive flux as the difference between the fluxes in the high-order discretization and fluxes in the low-order discretization.
4. Limit the antidiffusive flux aiming to obtain a corrected antidiffusive flux, then there will not be any overshoots or undershoots in the solution.
5. Add the corrected antidiffusive flux contribution to obtain the updated high-order saturation field.

The application of this method can be seen in [5]. In this present work, we propose the use of a Flow Oriented Scheme (FOS) strategy in the low order approximation. The FOS is a multidimensional formulation which promotes a better solution in the presence of the grid orientation effect (GOE). This strategy used in this work was developed by [13] and used by [7].

4 Numerical Experiment

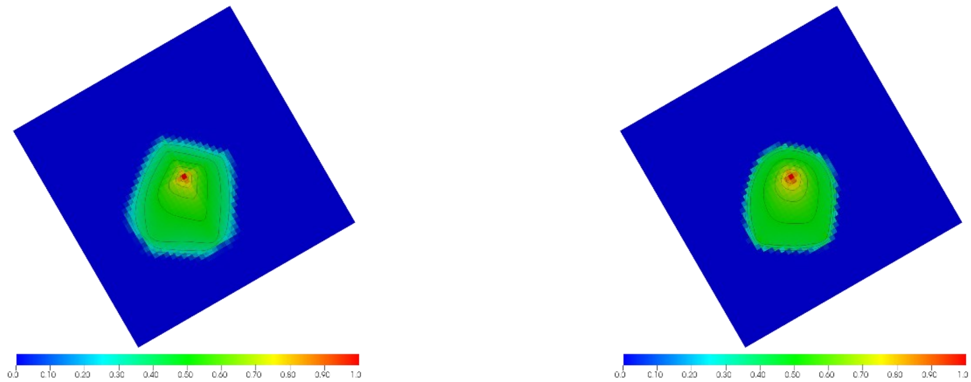
This problem was proposed by [8] and we can use to analyze the GOE in the saturation front when the computational grid is rotated. The domain is delimitated by a square [-0.5; 0.5]², initially it is fully saturated with oil. It has been discretized with a 51×51 uniform quadrilateral grid and rotated by α = 30° that is defined with the horizontal line. The permeability tensor is defined by K1 = I and K2 = 10^{-6}I, I is the second order identity tensor. The porosity is constant throughout the domain. The viscosity ration between the two phases is designated by $M = \mu_o/\mu_w = 100$. The injector well is placed at the domain's center and the producer wells are symmetrically positioned at the points given by the coordinates [+0.3 cos(30°), -0.3 sin(30°)]. The pressure at the producer wells is equal to zero $(\bar{p}_{prod} = 0)$. The water saturation and water flow-rate are defined by $\overline{S_w}_{inj} = 1$ and $\bar{Q}_{inj} = 1$ in this order. The Brooks-Corey model define, respectively, the water and oil relative permeabilities where $k_{rw} = {S_w}^4$ and $k_{ro} = (1 - S_w)^2$. The total mobility is obtained by an average weighting based on the volumes that sharing a common face.

The symmetry of the location of the wells implies the saturation front should be ideally the same even if the computational grid rotates by the angle previously defined. Besides, the water-cut curves referent to both producer wells should be exactly the same. Hence, any difference between these curves proves the GOE existence.

In this paper, the authors use the MPFA-H to solve the pressure equation and the flow-oriented variants Tight Multidimensional Upstream (TMU) approached by [8] and Smooth Multidimensional Upstream (SMU) proposed by [5] in low-order approximation of the FCT method to solve the saturation equation.

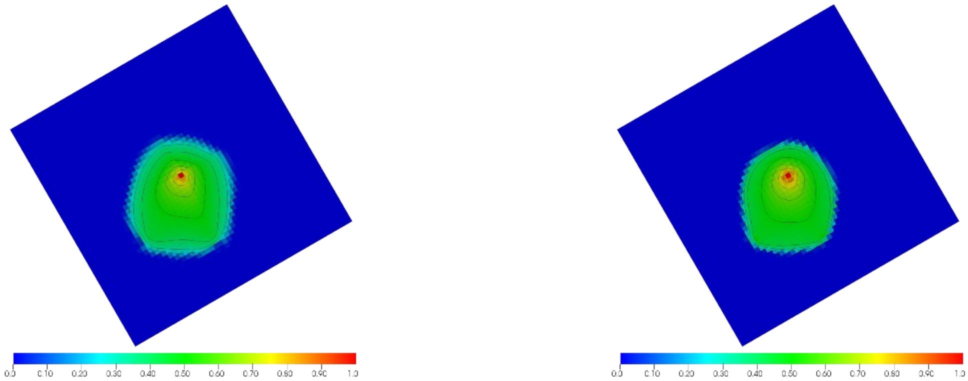
In Fig.1, we present the saturation fields for the domain rotated by α = 30° using the IMPES strategy, in which the pressure method was the Multi-Point Flux Approximation based in Harmonic Points (MPFA-H) using standard formulation. In Fig.2 and Fig.3, we show the results using the multidimensional formulation. The saturation method was the same in Fig.1, Fig.2 and Fig.3 that is denoted in the subtitle of the figures.

In this case, there is no symmetry of the flow with respect to the grid orientation because of the rotation of the domain. So, it is expected the occurrence of the GOE and consequently are expected differences in the breakthrough time for each producer well as seen in Fig.4.



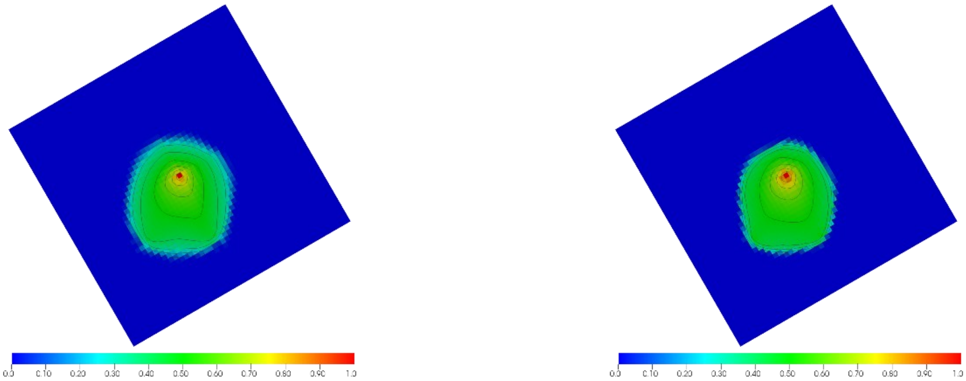
(a) Standard Upwind (b) FCT

Figure 1. Saturation field using standard formulation



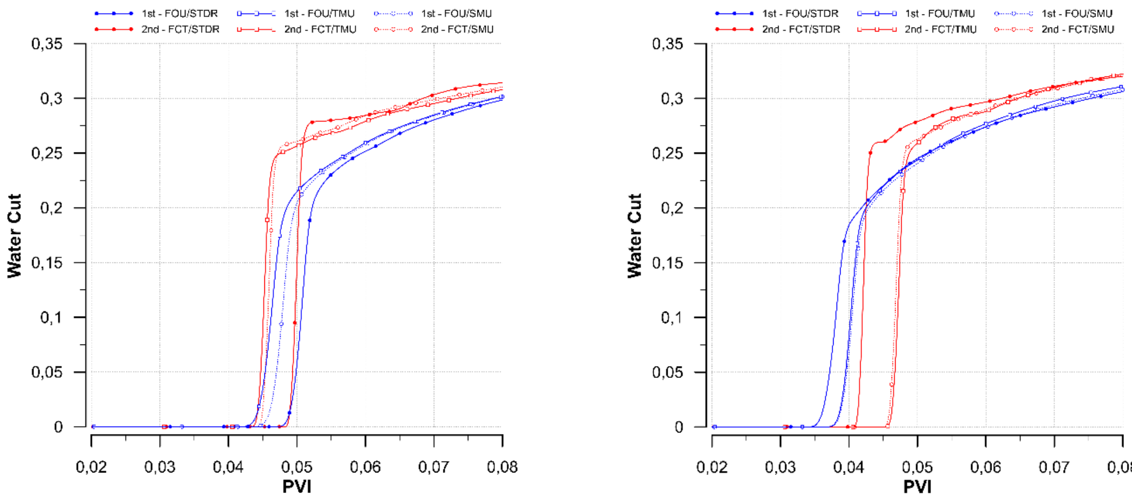
(a) Upwind – SMU (b) FCT - SMU

Figure 2. Saturation field using Smooth Multidimensional Upstream (SMU)



(a) Upwind – TMU (b) FCT - TMU

Figure 3. Saturation field using Tight Multidimensional Upstream (TMU)



(a) Producer Well 1 (b) Producer Well 2

Figure 4. Water-cut curves obtained with standard and multidimensional formulation

How expected the Flux Corrected Transport (FCT) provided the best results due to the higher-order resolution of this scheme. Besides, in this case, the Standard Upwind was the most sensitive scheme to the GOE, thus it is possible notice the multidimensional formulation is able to reduce the grid orientation effect. Furthermore, the SMU approach of compute the adaptative normalized weight performed slightly better than TMU.

5 Conclusions

In this paper we presented a robust MPFA-H method coupled with a second order flux corrected transport (FCT) with a flow-oriented scheme (FOS) to calculate the low order approximations aiming simulate two-phase flows of water and oil displacements in petroleum reservoirs. We compared the FCT with a FOU and showed the robustness of our method. Also, the results using the multidimensional formulation either TMU or SMU used to solve the saturation equation proved to diminish the GOE of the numerical solutions and consequently generated better results than the standard formulation which is strongly sensitive to the GOE.

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