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Limit Plastic Analysis of a Structural Concrete Block Wall

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Abstract. At this paper, it is searched the maximum collapse load of a structural concrete block wall. Simulations are made considering the removal of resistant material, such as the installation of a door or openings motivated by modifications at the architectural project. A mathematical programming using the Coulomb and Von Mises criteria is used at the limit plastic analysis assuming the basic hypothesis of associated plasticity. It is used a polyhedral representation of the yielding surface studying the convergence of the results in relation of the chosen number of planes at each representation. It is used the hybrid finite elements formulation. Numeric examples are shown for the structural concrete block wall case, considering different finite elements meshes and the obtained results are compared with those of the analytical analysis that exists at the criteria adopted by the Brazilian concrete block structure project Standard - NBR 10837. Keywords: structural concrete block, hybrid finite elements, limit plastic analysis. Theme: research and testing.

Keywords: structural masonry, structural concrete block, hybrid finite elements, plastic boundary analysis.

1 Introduction

The following analysis aims to simulate an intervention that is quite possible to occur without technical supervision in structural masonry buildings: the removal of part of a retaining wall during an apartment renovation.
It is proposed to analyze what are the structural consequences to the wall due to this removal, through numerical simulation based on limit plastic analysis software, which employ hybrid finite element models. The plasticizing surface adopted is that of modified Mohr Coulomb, resisting only compression. The value of the ruin load or the collapse load factor is obtained by optimization process through the linear mathematical programming available in the "LINDO" software. This process determines a collapse load factor for a set of points of the structure, which reaches the maximum resistance value until it forms a plastic collapse mechanism (ruin). The hypothesis is that there is a sufficient resistance reserve in the blocks, which ensures the resistant capacity of the wall after the intervention, specifically in this case. Other situations and geometries should be individually studyd. It should be noted, however, that this approach uses software only as an analysis tool, not proposing to explore the mathematical models involved. It refers to works such as Buzar (2004) or Santos da Silva (2003) for more in-depth information on this topic.

2 Hybrid Finite Element Formulation

The theory of finite elements based on hybrid functionals is briefly described below. Hybrid finite elements have one or more primary fields that are defined only in the interface or outline of the element. Hybrid variational principles represent an important extension of the classical principles of mechanics. This extension is an attempt to strengthen finite element models.

The first hybrid element was quite limited because it was not able to treat nonlinear and dynamic problems. However, these limitations were gradually overcome with the understanding and evolution of the basic concepts. The adoption of hybrid finite elements in this work was motivated by the fact that the four-node quad hybrid finite element is probably the four-node element more accurate in a wide range of tension problems and flat deformation (Zienkiewicz & Taylor 1995).

2.1 Equilibrium equation

The functional used in obtaining the hybrid element is obtained by the sum of two other functional ones that contain the functional inside (domain) and the interface potential (contour). The expression (1) represents the functional used in hybrid elements (Felippa, 2000), (Pian & Tong, 1969):

$$\pi_C^u(\sigma_{ij}, u_i) = -U_C + W_d = -\frac{1}{2}\int_V \sigma_{ij} D_{ijlk} \sigma_{kl} dV + \int_S u_i \sigma_{ij} n_j dS - \int_{St} u_i \hat{t}_i dSt \tag{1}$$

where is the hybrid multiple field functional $\pi_C^u(\sigma_{ij}, u_i)$ (σ_{ij} and); UC is complementary energy in terms of tensions; Toilet is the potential job; it is the tensioner of tensions; D is the tensor of the constitutive relationship; u is the displacement vector; V is the volume; S is the surface; St is the part of the surface where there are loads and is the vector of prescribed surface forces. $\dot{u}$ σ_{ij} $\hat{t}$

The functional presented in the expression (1) can be applied in the construction of the hybrid finite element in Figure 1.

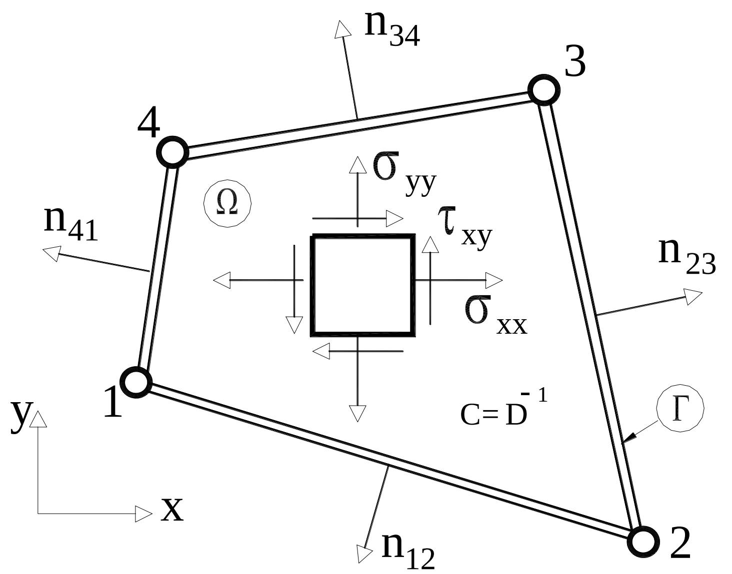


Figure 1. Bilinear quadruplet element of flat tension (Felippa, 2000).

The relationship between the

$$\boldsymbol{\sigma} = \boldsymbol{\Psi\alpha}$$

being the matrix that interpolates the stresses, **and α** are the parameters of stresses.

The functional expression (1) can be written for the entire domain and outline of the discretized problem as

$$\pi_C^u = \boldsymbol{\alpha}^{\mathbf{T}} \mathbf{G}\dot{\mathbf{u}} - \mathbf{P}^T \dot{\mathbf{u}}$$

where **G** is given by

$$\boldsymbol{G} = \int_{\Gamma^{(e)}} \boldsymbol{T}^T \boldsymbol{\Phi}\, d\Gamma$$

Since the displacement rates in the problem are discretized, the vector of nodal loads is the matrix of shape functions for the linear contour element described on the surface of the hybrid hybrid element of flat stresses $\dot{u}$ Ρ Φ ***and T is*** an array (8 x 7) organized into four sub-matrices (2x7) that are $\boldsymbol{N}_{1212}\boldsymbol{\Psi}$, $\boldsymbol{N}_{2323}\boldsymbol{\Psi}$, $\boldsymbol{N}_{3434}\boldsymbol{\Psi}$, and $\boldsymbol{N}_{4141}\boldsymbol{\Psi}$.

Making stationary in relation to the displacement rate, one has π_C^d

$$\frac{\partial \pi_C^{\boldsymbol{u}}}{\partial \dot{\mathbf{u}}} = \mathbf{G}\boldsymbol{\alpha} - \mathbf{P}^{\boldsymbol{T}} = 0$$

or

$$\mathbf{G}\boldsymbol{\alpha} = \mathbf{P}^{\boldsymbol{T}}$$

The expression (6) is a balance relationship between the nodal loads and the stress parameters, g being a equilibrium matrix in terms of the stress parameters, integrated in the contour of the hybrid finite element. $\mathbf{P}$ α

3 MOHR-COULOMB RESISTANCE CRITERION

In the present work, the Mohr-Coulomb criterion is assumed to be valid. In this criterion, the shear stress at the rupture or flow of the material is a function of material properties, such as cohesion and friction angle, and varies linearly with the normal stress acting (Chen, 1982). Thus the shear forces grow with the increase of normal stresses to the rupture plane, i.e.,τ

$$\tau = c + \sigma \tan\phi \tag{7}$$

where is the shear stress in the rupture plane, c is the cohesion of the material and is the internal friction angle.$\tau\phi$

The Mohr-Coulomb criterion ignores the effect of the intermediate main tension and the equation can be written in the form of the main tensions as

$$\frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin\phi + c\cos\phi \tag{8}$$

where σ_1 and σ_3 are the highest and lowest main stresses, respectively.

In general, the Mohr-Coulomb rupture criterion in the strains space ($_x$, '$_y$' and '$_{xy}$' is given by

$$(\sigma_x - \sigma_y)^2 + (2\tau_{xy})^2 - \left[2c\cos\phi - (\sigma_x + \sigma_y)sen\,\phi\right]^2 \le 0 \tag{9}$$

The use of expression (9) in limit plastic analysis leads to obtaining a governing system in the form of a nonlinear programming problem (NLP). This expression can be linearized in order to work with governing systems in the form of linear mathematical programming problems (PLs). Using a linearized rupture surface (Santos *et al*, 1999a), (Santos *et al*, 1999b) the resistance conditions, at any point in the body, are expressed as

$$\boldsymbol{n}^T \boldsymbol{\sigma} \leq \boldsymbol{\sigma}^* \tag{10}$$

where ***n*** is the matrix of normality; and "⌡ * is the vector of plastic capacities.

The resistance conditions expressed in (10) are a function of ***the voltage ..*** Thus, in order to be possible to assemble the linear programming problem equivalent to the static criterion of the plastic limit analysis it is necessary to replace (2) in (10), reaching the expression (11), more detail can be found in (Buzar *et al*, 2003).

$$\mathbf{n}^T \Psi \boldsymbol{\alpha} \leq \boldsymbol{\sigma}^* \tag{11}$$

4 STATIC THEOREM OF PLASTIC ANALYSIS LIMIT

With the equilibrium ratio (6) and the resistance conditions (11), the linear programming problem associated with the static criterion is obtained as (where P_V is the vector of variable loads and P_f is the vector of fixed loads)

Maximize □ (12a)

Subject to

$$\begin{bmatrix} \mathbf{0} & \mathbf{n}^T \Psi \\ \mathbf{P}_v & -\mathbf{G} \end{bmatrix} \begin{bmatrix} \lambda \\ \boldsymbol{\alpha} \end{bmatrix} \begin{matrix} \leq \\ = \end{matrix} \begin{bmatrix} \boldsymbol{\sigma}^* \\ -\mathbf{P}_f \end{bmatrix} \tag{12b}$$

In the present work, the linear programming problems associated with the static theorem, obtained through the polyhedric representation of the rupture surface in hyperplanes (Sahlit, 1992), (Sahlit, 1993), (Smith, 1990) were solved using the commercial software LINDO (Linear INteractive DiscreteOptimizer) (Sschage, 1991).

5 EXAMPLE USED IN PLASTIC ANALYSIS STRUCTURAL CONCRETE BLOCK WALL BOUNDARY

For example, a four-story building was built, with four two-bedroom apartments per floor. This typology was chosen because it is very frequent in popular housing estates, where the possibility of an intervention without technical guidance is more likely to occur. The intervention is shown in Figure 2.

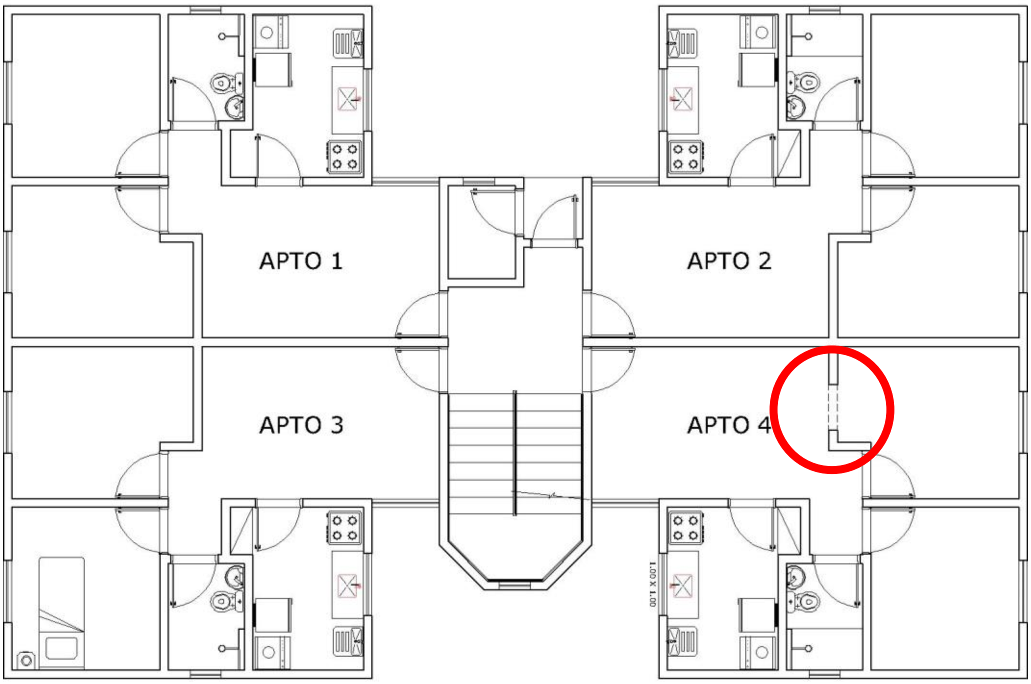


Figure 2. Intervention in popular standard building apartment. Source: Illustration of the author.

It simulates here the modification of the position of the bedroom door, with the intention of connecting it directly to the room, in a possible change of its use. It was assumed that this intervention occurred on the ground floor in order to simulate the most unfavorable situation.

The detailed elevation of the walls with the new opening is illustrated in Figure 3.

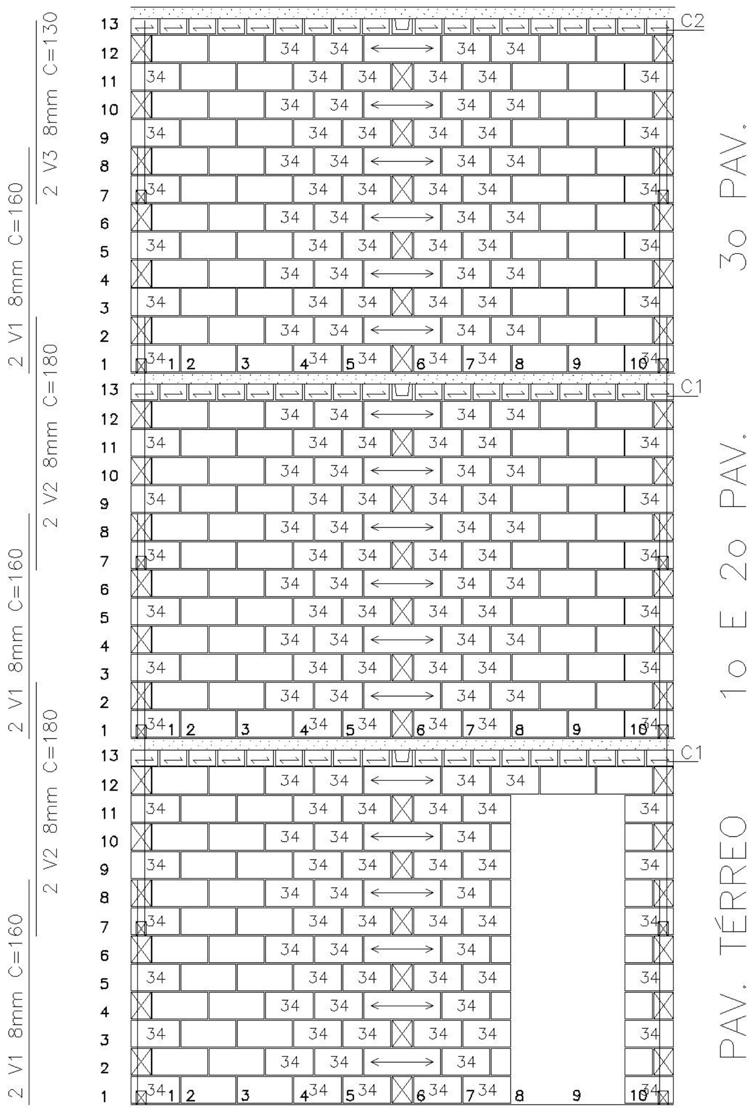


Figure 3. Detailed elevation of the wall where the intervention occurred. Source: Illustration of the author.

The study was done simply, considering the wall without problems of dissaprumo and treating it separately from the adjacent walls. In numerical analysis, the lateral buckling effect is disregarded. The lifting of the load is shown below:

SLAB LOADING

Own weight: 25KN/m^2
Finish: 10KN/m^2
Overload (flat): 15KN/m^2
TOTAL: 50 KN/m^2
Figure 4 illustrates the direction of loads:

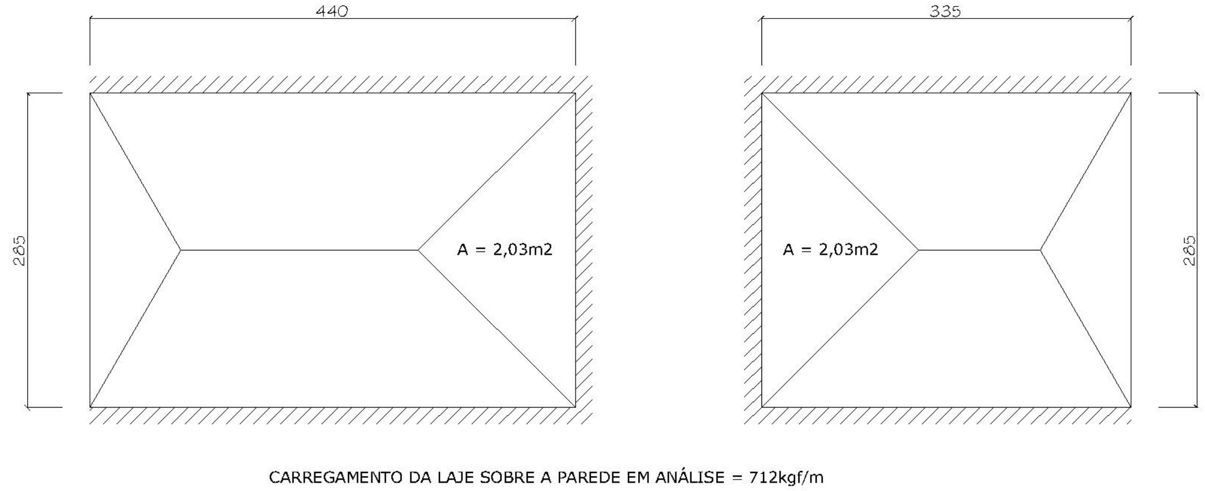
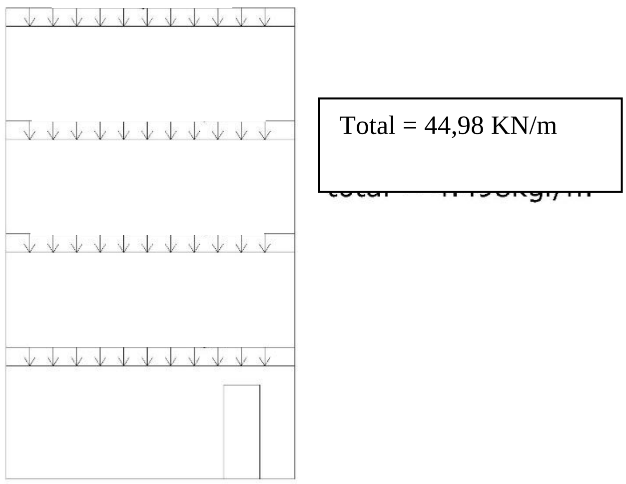


Figure 4. Targeting of loads on the wall under analysis. Source: Illustration of the author.

The specific weight of the hollow block of concrete is conventional as 14KN/^{m3}. The wall under analysis, 14cm thick, 2.85m long and 2.52m high results in its own weight of approximately 5.5 KN/m.

Figure 5 illustrates the accumulated loads per floor:

Figure 5. Accumulated loads on the wall under analysis. Source: Illustration of the author.



The value of **50 KN/m of total accumulated load on the** analyzed wall was then established as reference.

The dimensioning of this wall was made based on the script established by Ramalho and Corrêa (2003) and is still developed in the calculation memory below, considering the isolated wall, as shown in Figure 6.

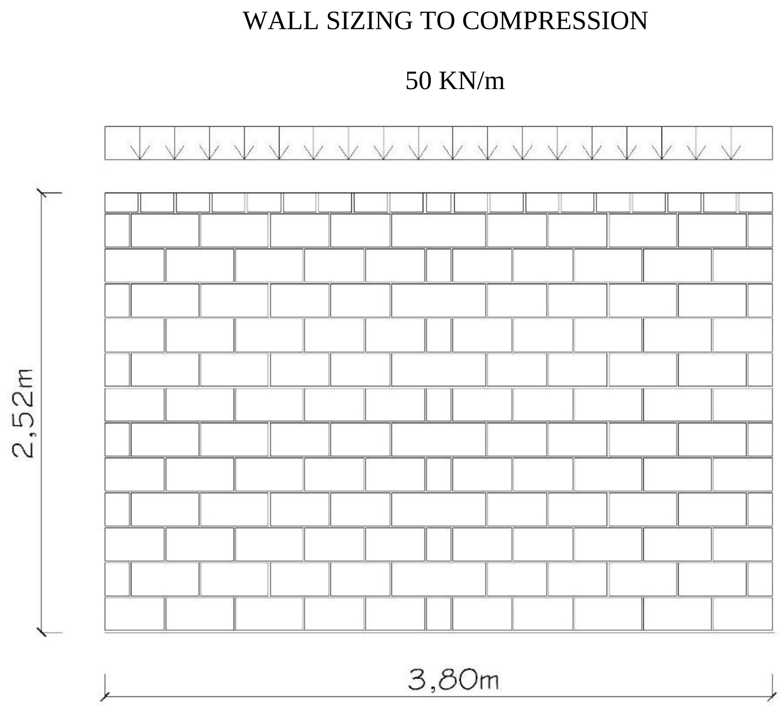


Figure 6. Static wall scheme under analysis submitted to compression. Source: Illustration of the author.

Slenderness index test (λf), according to NBR 10837:

$$\lambda f = \frac{h}{t} = \frac{2{,}52m}{0{,}14m} = 18 \leq 20 \tag{13}$$

Being: λf = slenderness index; h = wall height; t = wall thickness.

Calculation of the Actuating Voltage on the wall (): $falv,c$

$$falv,c = \frac{F}{A} = 357{,}14KN/m^{2} \tag{14}$$

Being: F = total force acting on the wall; A = sturdy wall area.

Calculation of Resistant Stress (): $\bar{f}alv,c$

$$\bar{f}alv,c = 0{,}20 \times fp \times R \tag{15a}$$

$$R = 1 - \left(\frac{h}{40 \times t}\right)^{3} \tag{15b}$$

Inasmuch

fp = wall resistance; R = resistance reduction factor associated with slenderness; h = wall height; t = wall thickness,

$$\bar{f}alv,c = 0{,}182\, fp \tag{16}$$

Easing resistances:

$$0{,}182´\ fp = 357{,}14KN/m^{2}\backslash$$

$$fp = 1.962{,}32KN/m^{2}$$

Converting the unit, the value of 1.96MPa is found.

It is known that the union of blocks by mortar results in loss of monolithicness and, for this, a safety factor known as prism efficiency should be used that should be arbitrated between 0.5 and 0.7.

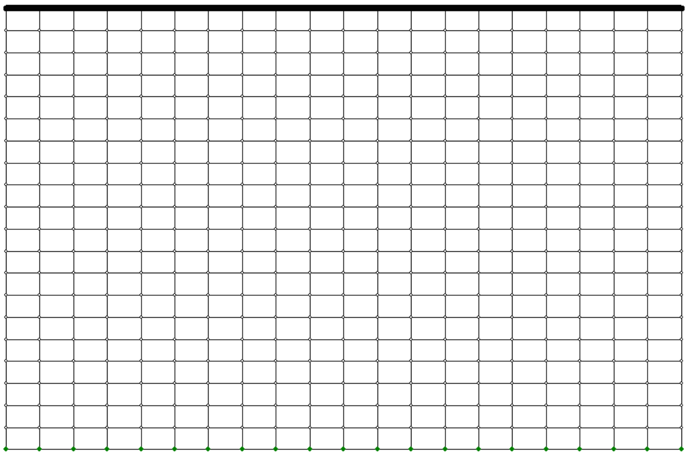
Using prism efficiency = 0.7 you get:

$$\frac{1{,}96MPa}{0{,}7} = \mathbf{2{,}8MPa}$$

As the lowest resistance allowed by standard for structural blocks is **4.5MPa**, this should be the value adopted for the wall blocks under analysis.

Then, to determine the maximum collapse load, the plastic analysis limit of four-node quadfinite elements was used (Buzar, 2004). Thus, the geometry of the analyzed wall was subdivided into 400 elements, configuring a total of 441 nodes. Figure 7 illustrates this subdivision:

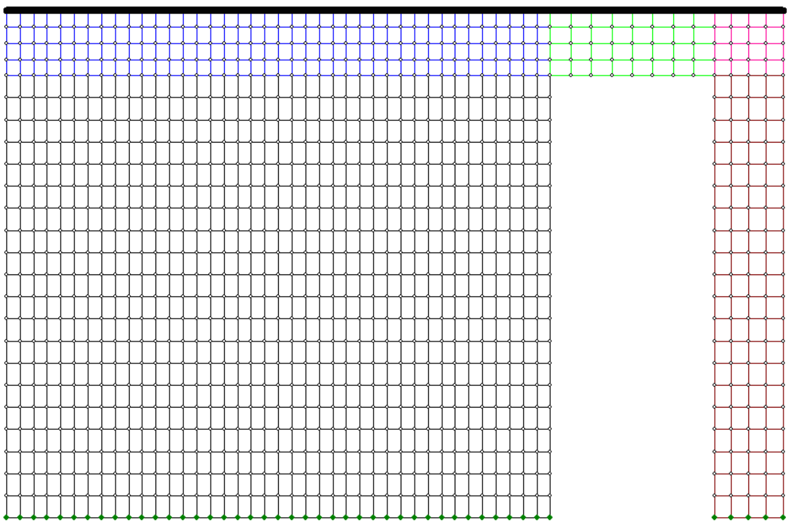
Figure 7. Subdivision of the mesh wall. Source: Illustration of the author.



These data were introduced in a finite element limit plastic analysis (APLEF) software with the intention of obtaining the collapse load factor (Buzar, 2004). The collapse load factor is the measure of how many times the load acting on the problem under analysis should be increased in order to lead the structure to collapse. The value found in this situation was **λ = 12.14**. This means that only a value 12.14 times higher than the actual applied load (**607KN/m**) would be able to bring this wall to ruin. For the 3.8m extension of this wall, distributed loading would set up a force of **2,306.60KN**.

The second situation simulates the wall with intervention and removal of material. This is the installation of a door of 80 x 210 cm, as shown in Figure 8. In this situation, the wall under analysis was subdivided into 1088 elements, configuring 1185 nodes.

Figure 8. Subdivision of the wall with mesh intervention. Source: Illustration of the author.



The collapse load factor obtained in this situation was **λ = 8.46.** This means that only a value 8.46 times higher than the actual applied load (**423.00KN/m**) would be able to bring this wall to collapse even after the removal of material equivalent to the span of a door. For the 3.8m extension of this wall, distributed loading would set up a force of **1,607.40KN**.

Using the stress formula derived from the Material Resistance and knowing that the blocks used are of resistance 4,5MPa, the value of the force necessary to occur crushing can be obtained simply in both cases:

1) Wall without opening:

$$\sigma = \frac{F}{A} \Rightarrow \qquad \mathbf{F = 2{,}394.00KN}$$

Being:

σ = Voltage; F = Total force applied; A = Area of the cross section of the wall.

2) Wall with opening (deceiting 0.80m from door span):

$$\sigma = \frac{F}{A} \Rightarrow \qquad \mathbf{F = 1{,}890.00KN}$$

It is observed that the values obtained by the material strength and numerical example are very close.

In the figure, the following study analyzes the results obtained from the force necessary to occur the collapse through the resistance of the material and through the calculation of flat state of stresses, considering the limit plastic analysis, and comparades with the existing requesting force.

SITUATION 1 - WALL WITHOUT OPENING:

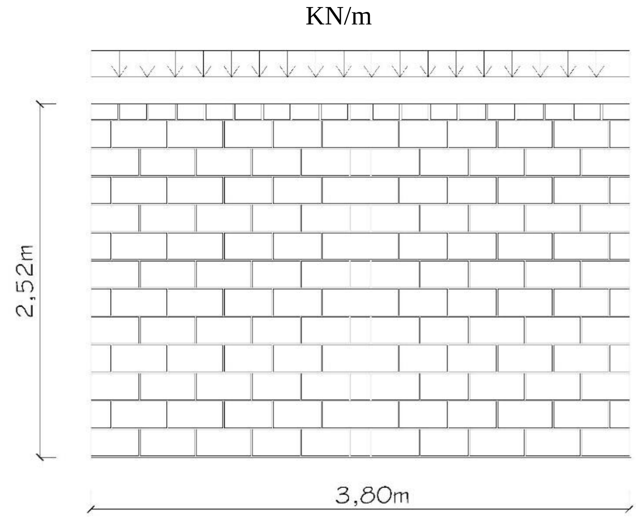


Figure 9. Structural masonry wall without material removal.

BLOCK RESISTANCE → **F = 2,394.00KN**
BY PLASTIC ANALYSIS LIMIT → **F = 2,306.60KN**
PER EXISTING → **REQUEST F =** 50KN/m x 3.8m = **190.00KN**

It is observed in Figure 10 that the requesting force on the wall is about 8% of the value needed to achieve the collapse.

SITUATION 2 - WALL WITH OPENING:

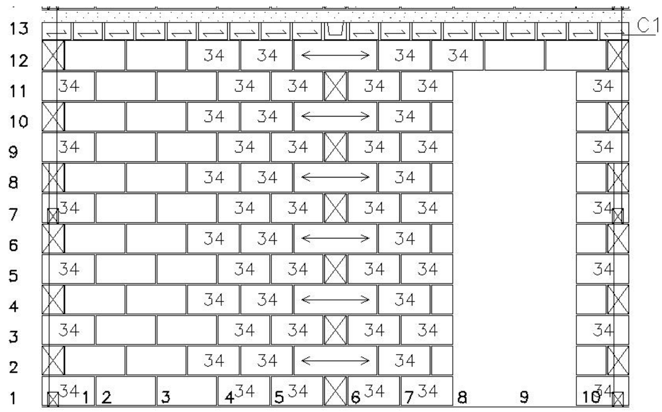


Figure 10. Structural masonry wall with material removal - door placement.

BLOCK F RESISTANCE → **= 1,890.00KN**

BY PLASTIC ANALYSIS LIMIT → **F = 1,607.40KN**

PER EXISTING → **REQUEST F =** 50kgf/m x 3.8m = **190.00KN**

In this case, the requesting force on the wall is of the order of 11% of the value required to achieve the collapse.

The final result indicates that, for this case (four-floor building), even using the minimum resistance established by the norm (4.5MPa), the possibility of collapse of the structure due to this intervention is remote.

Figure 11 and 12 show the actions of the maximum burden of ruin in both situations in order to analyze the rupture mechanism obtained.

The rupture of the wall submitted to the maximum load in situation 1 (without intervention) is illustrated in Figure 11:

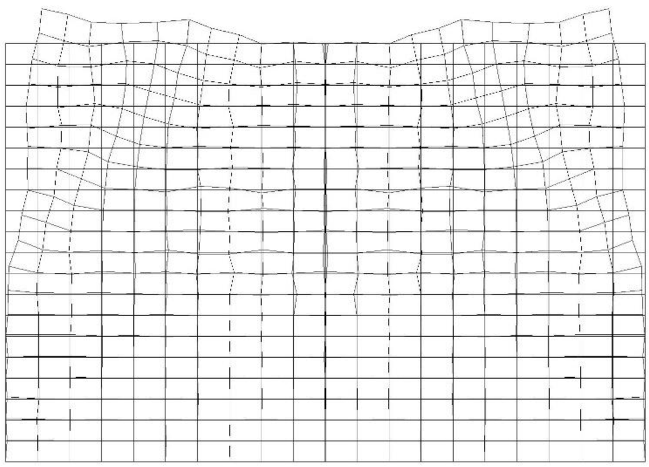


Figure 11. Wall rupture mechanism subjected to maximum load in situation 1 (without intervention). Source: Illustration of the author.

It is notepoint that this simulation did not take into account the existence of the resistance graute rebar equal to 9MPa armed with 10mm rebars that exists in this project. The existence of this verga certainly helps to combat eventual ruin shown in Figure 11.

In situation 2 (wall with removal of material for door installation), the wall rupture mechanism subjected to maximum load is illustrated in Figure 12:

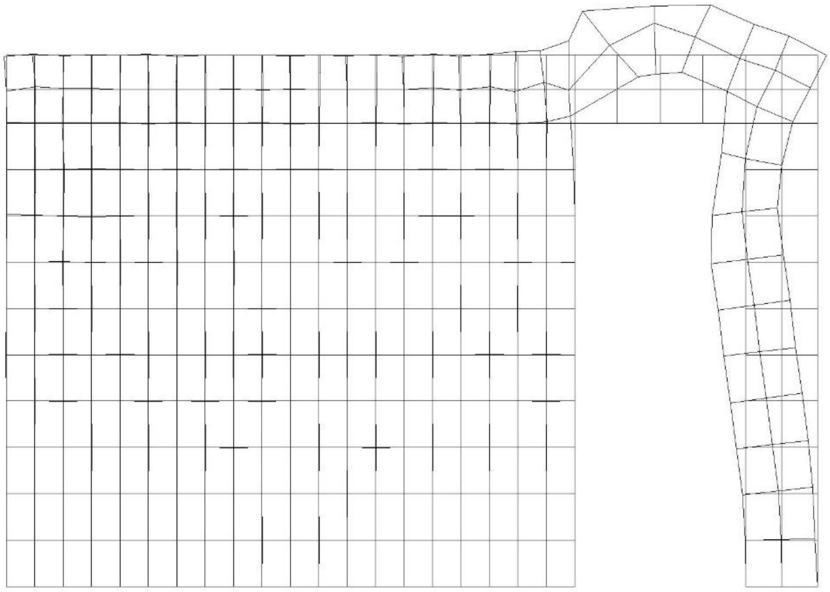


Figure 12. Wall rupture mechanism subjected to maximum load in situation 2 (removal of material). Source: Illustration of the author.

It can be infered from the figure that the breakout occurs in the two fragile points of the wall: the trim for installation of the landmark and smooth (doll) and above the door span.

These two points of ruin can be easily stiffened with rebar and graute in order to better combat this eventual collapse.

Another possibility would be to increase the size of the garrison, which would thus gain more robustness and would no longer function as a slender pillar.

Conclusions

Generally speaking, it can be concluded that, in this punctual intervention of the wall under analysis, there was no danger to the overall stability of the structure. It is verified, therefore, that there is a resistance reserve in the blocks large enough to keep the structure intact after the installation of the illustrated door. It is indicated, therefore, that the existing fear of making reforms in structural masonry buildings is something that deserves to be faced and studied, since surprising results can be obtained. However, the importance of using blocks with proper technological control and within the current normist is emphasized.

The adoption of the hybrid finite element models in the present study was motivated by the fact that these models present in their formulation the equations of balance and compatibility independently, which allows the assembly of static and kinematic PLs to perform the limit plastic analysis through mathematical programming. An additional motivation is that the four-knot quadrilateral hybrid finite element is probably the most accurate bilinear element for a wide range of stress problems and flat deformations (Zienkiewicz & Taylor, 1995).

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