# Implementation of a non-intrusive approach using a global-local strategy for the Generalized Finite Element Method

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**Abstract.** In general, the standard finite element method available in most commercial software faces difficulties solving complex structural problems. The Generalized/eXtended Finite Elements Method (G/XFEM) is a powerful tool for the solution of the class of problems involving discontinuities and singularities such as crack propagation analysis. In recent years, several investigations have been proposed to make G/XFEM available in the industry routine. A trend is the complementary bundling of commercial FEM codes with in-house G/XFEM codes, requiring non-intrusive coupling between these two types of software. This work presents the implementation of a non-intrusive coupling of multiscale iterative analysis by G/XFEM using global-local enrichment functions (IGL-GFEM$^{gl}$). The problem is decomposed into three scales. Commercial FEM software may perform the global simulation, but here, the same in-house software is used to evaluate these results. G/XFEM global-local code handles meso and local scale in the INteractive Structural ANalysis Environment (INSANE), an open-source software developed at the Federal University of Minas Gerais. The meshes of the two bigger scales are conform and the coupling between them is straightforward established. The Iterative Global-Local (IGL) strategy accounts for displacement compatibility and force balance at the interface between global and mesoscale. Numerical examples are presented to evaluate the performance of this strategy.

**Keywords:** Generalized Finite Element Method; Global-Local Analysis; Non-intrusive coupling.

## 1 Introduction

Problems involving singularities, discontinuities, complex geometries and localized deformations are commonly faced in the engineering applications. These problems often challenge the conventional formulation of the Finite Element Methods (FEM). Modeling this class of problems can be burdensome or even not possible when using FEM, and significant errors may be incorporated in the approximate solution. The Generalized Finite Element Method (GFEM) was proposed aiming to avoid this limitation. As proposed ([1], [2] and [3]), GFEM formulation combines characteristics of Meshless Method (MM) with the conventional finite element mesh.

In the GFEM approach, the MM's concept of Partition of Unity is applied using conventional FEM shape functions along with local approximation functions. The latter, known as enrichment functions, should be defined properly to be able to capture local behaviors consistently, which usually request a-priori knowledge about the solution of the problem [4]. The GFEM formulation reduces mesh dependency, i.e., the h-refinement plays a less important role in the process of problem modeling [5]. It is noteworthy that, as stated by Belytschko et al [5], GFEM formulation is considered equivalent to the eXtended Finite Element Method (XFEM). Hereon, the G/XFEM notation will be hold.

There is a natural interest in the exploration of strategies that avoid a-prior knowledge of the solution itself. The so-called Generalized Finite Element Method with global-local enrichment functions (GFEM$^{gl}$) consists in a solution strategy that provides the enrichment function of the problem (global domain) based on the solution of local problems (sub-domains) [6]. The most interesting feature of this strategy relies on the possibility of the solution of complex problems, whose local behavior is not known a-priori.

While the G/XFEM has been widely accepted and applied with success in the solution of complex problems in the past decades, there are few implementations available in commercial software [7]. The GFEM$^{gl}$, on the other hand, has been implemented only in research software so far and it is not available in program suites with industrial appeal, such as Abaqus and ANSYS. Strategies for the use of G/XFEM and GFEM$^{gl}$ along with the available commercial software has been investigated in the past few years [8]. Those strategies are here called coupling methods.

A variety of coupling methods has been developed recently, resulting in interesting alternatives for the solution of engineering problems by a different combined strategies [8]. The non-intrusive coupling is a class of coupling methods that has drawn special attention due to its features. In this context, the intrusiveness may be understood as the level of code modification of the software involved and quantity of information exchanged between software necessary for a certain coupling method. Therefore, a non-intrusive coupling method requires few or no modification in the source code of FE solvers, exchanging quantities usually available on FE software, such as nodal displacements, nodal forces and stiffness matrix [9]. The less code modification and information exchange, the more non-intrusive the method.

Non-intrusive approaches have been applied for the solution of a variety of problems, such as 2D global and 3D local models coupling [10], dynamic transient problems [11], crack propagation [12] and thermal gradients [13]. It is also worthy to mention the hierarchical non-intrusive algorithm proposed in Fillmore and Duarte's work [14].

Considering the interesting features of non-intrusive coupling, this paper presents a non-intrusive implementation of the GFEM$^{gl}$ as proposed by Li, O'Hara and Duarte [9]. In this approach, the problem is divided into three scales: global, meso and local. The global scale may be solved by conventional FE analysis, although in this paper the same in-house software was used throughout the analysis. The GFEM$^{gl}$ approach is used at meso and local scales. Coupling between FEM and GFEM$^{gl}$ results is performed through the non-intrusive method Iterative Global-Local (IGL) proposed by Whitcomb [15]. Therefore, this non-intrusive implementation of the GFEM$^{gl}$ is referred as IGL-GFEM$^{gl}$.

# 2 Iterative Global Local implementation of the Generalized Finite Element Method with global-local enrichment functions (IGL-GFEM$^{gl}$)

## 2.1 A non-intrusive coupling strategy: Iterative Global Local (IGL)

Whitcomb [15] proposed a highly non-intrusive method for the solution of FE analysis using a global-local approach. Although the method was originally designed for FE analysis, it has been successfully adapted for others numerical methods, such as the G/XFEM ([8], [14] and [16]).

Consider a static problem set at a global domain $\Omega$. Linear elastic behavior is assumed all over problem's domain. There is special interest in the solution in terms of strain and stress in a small and well-known region $\Omega_L$, called the local domain. Let $\Omega_C$ be its complementary domain with respect to $\Omega$. The region defined by $\Gamma = \Omega_L \cap \Omega_C$ is the interface. The problem is presented in Figure 1.

The global problem is defined on $\Omega$, which is coarsely discretized. There is a stiffness matrix $K_{GC}$ associated to the degrees of freedom in the $\Omega_C$ region and a stiffness matrix $K_{GL}$ associated to the $\Omega_L$ as well. The local problem, on the other hand, is defined on $\Omega_L$, which is now discretized with a refined mesh. The matrix $K_L$ represents the stiffness of the local model.

In the conventional global-local FE analysis, the solution obtained by the global model at the interface $\Gamma$ is used as boundary conditions for the local problem. Since usually $K_L \neq K_{GL}$, the local domain $\Omega_L$ is represented differently in the global and local models. As consequence, forces equilibrium at $\Gamma$ is not satisfied [17].

Whitcomb's method reaches force equilibrium at the interface $\Gamma$ by an iterative process of boundary conditions exchange between global and local models, which the method was named after. The IGL method can be described by the following steps:

1. Global analysis: the global model is solved and the solution $u_G$ is obtained. The solution of the global model in the degrees of freedom of the interface $\Gamma$ is denoted $u_G^{\Gamma}$.

2. Local analysis: the local model is solved using $u_G^\Gamma$ as boundary conditions on Γ. The solution $u_L$ is obtained.

3. Interface: the equilibrium at the interface is verified. This is done by computation of reaction at the interface of global ($f_C^\Gamma$) and the local ($f_L^\Gamma$) models. When the equilibrium is not satisfied, there are residual forces to be considered. The residual forces vector $f_R$ is computed as follows:

$$f_R = -(f_L^\Gamma + f_C^\Gamma) = -[(K_L \cdot u_L - f_L) + (K_{GC} \cdot u_{GC} - f_{GC})]|_\Gamma \tag{1}$$

4. Convergency: after computation of the residual forces, the convergency of analysis is checked. The convergency criterion proposed by Whitcomb [15] is the maximum absolute value of the residual forces vector $f_R$ and the recommended value for residual tolerance is $10^{-7}$. This value was adopted hereon.

5. Correction and iterations: if the convergency criterion is satisfied, the analysis is completed. Otherwise, the global model is updated by the addition of $f_R$ and the algorithm goes back to step 1.

Figure 1 schematically presents the process described. The final solution $u$ obtained is composed by the global solution in the domain $\Omega_C$ and the local solution in the domain $\Omega_L$. The governing equation of the global-local problem is:

$$(K_{GC} + K_L) \cdot u = f \tag{2}$$

where $f$ is the load vector of the problem and $u$ is the approximated solution for displacements. For reference and validation matters, in problems with matching mesh at interface, the same solution $u$ should be obtained by a unique model as indicated in the last third of Figure 1.

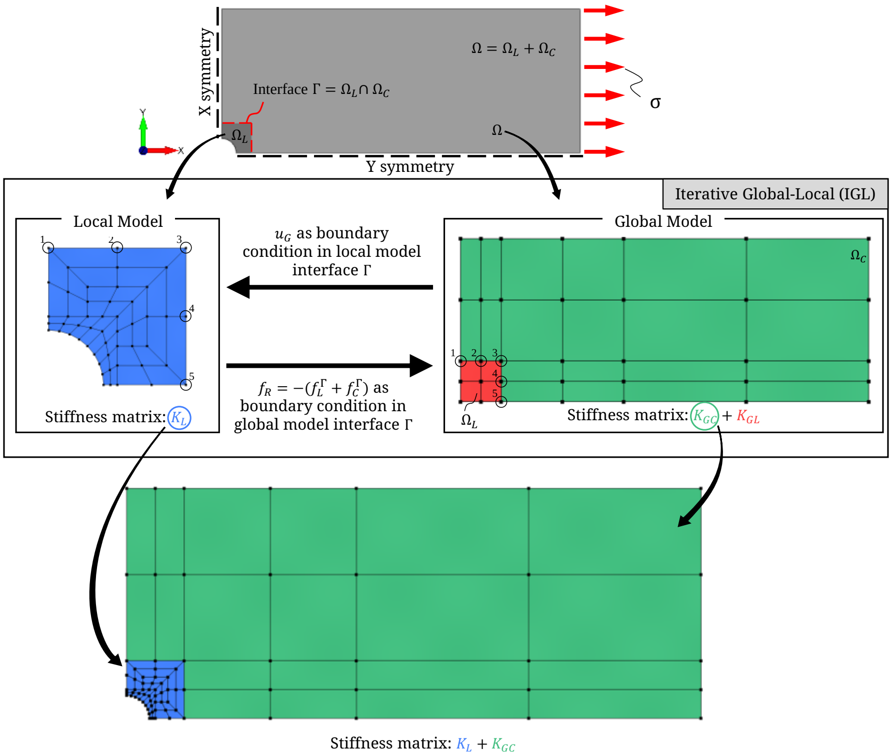



Figure 1. Iterative Global Local (IGL) approach.

## 2.2 Generalized Finite Element Method with global-local enrichment functions (GFEM$^{gl}$)

In this section, the main concepts of the GFEM$^{gl}$ are briefly presented. As mentioned before, GFEM$^{gl}$ key feature is the dismissal of prior knowledge of the local solution to define G/XFEM enrichment functions. Instead, the approximated solution of the local model is used to provide the enrichment functions in the global model. Figure 2 schematizes the analysis proceedings, which are detailed as follows:

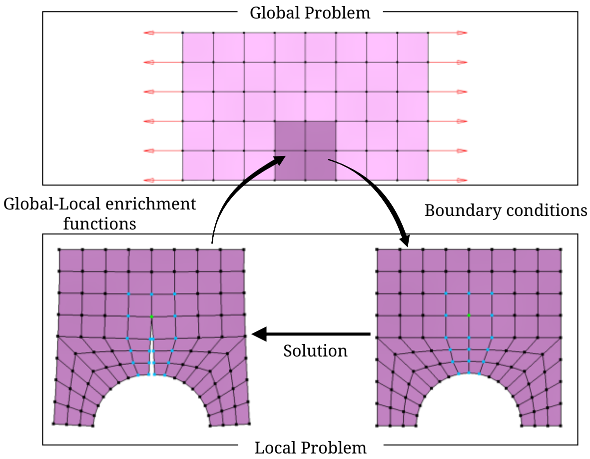



Figure 2. GFEM$^{gl}$: Global-local strategy for the G/XFEM.

1. Global model: the whole problem is coarsely modeled in a global scale. All local features are neglected, such as singularities, discontinuities, complex geometries and stress concentrators.
2. Local model: regions with local behaviors are modeled providing for all its features. The models mesh should be consistent, i.e., the local model is usually built on a refined mesh.
3. Global analysis: the global model is solved and the solution $u_G$ is obtained. The solution of the global model in the degrees of freedom of the interface Γ is denoted $u_G^\Gamma$. This is a FE analysis.
4. Local analysis: the local model is solved using $u_G^\Gamma$ as boundary conditions on Γ. The solution $u_L$ is obtained. This is a G/XFEM analysis.
5. Enrichment functions: the solution $u_L$ provides the enrichment functions for the global problem.
6. Global analysis: the global model is revisited, but now the solution is reach by the G/XFEM solver. The enrichment functions defined in the previous step is applied on a set of nodes of this model. This is the final analysis.

### 2.3 IGL-GFEM$^{gl}$ for the solution of multi-scale problems

In a recent work, Li, O'Hara and Duarte [9] present a solution strategy combining IGL and GFEM$^{gl}$, hereon called IGL-GFEM$^{gl}$. Figure 3 presents the workflow of the method proposed.

The problem is divided into three scales. The global scale covers the whole problem. There is no concern with the modeling of local features. These characteristics usually lead to coarse meshes. Regions where local behaviors are expected define the local scale. At this scale, the problem modeling incorporates all these relevant behaviors. Finally, the mesoscale is also defined. It is a region contained in the global scale. Its main function is to ensure a matching mesh at its interface with the global scale.

Information exchange between global and meso scales are provided by IGL. As those scales are supposed to have matching meshes, this process is performed exactly as described in section 2.1. On the other hand, meso and local scale are solved using GFEM$^{gl}$ strategy, i.e., the mesoscale problem is solved by G/XFEM using enrichment functions built using the local scale solution.

## 3 Numerical Examples

Three numerical examples of IGL implementation are presented in this section. First, the IGL application is demonstrated in the solution of a simple bar axially loaded. Then, a 2D problem is solved using this strategy. Finally, a numerical example of the IGL-GFEM$^{gl}$ is presented.

Figure 4 presents the Problem 1, which was solved for the cross-section area $A = 0{,}5\ m^2$. The elasticity modulus of the global model elements is $E = 2{,}0\ GPa$. For the local elements, two scenarios were evaluated. In the case 1, the elasticity modulus of the local elements 2 and 3 is $E^* = 1{,}0\ GPa$. In the case 2, these elements are defined with $E^* = 4{,}0\ GPa$. In both cases, local elements 1 and 4 were defined with the same elasticity modulus of the global elements.

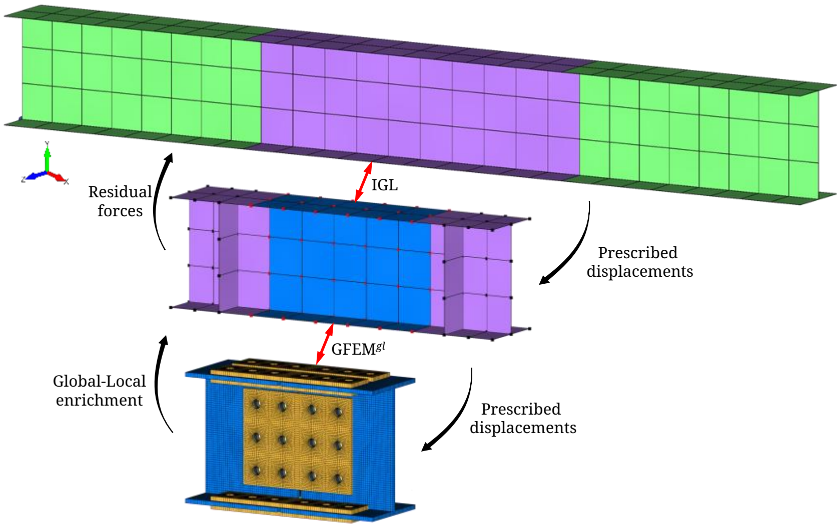



Figure 3. IGL-GFEM$^{gl}$: Iterative Global Local with global-local strategy for the G/XFEM.

The results are presented in Table 1. Both cases took the same number of iterations. The final reaction force at the local model is equal to the load applied at the bar end, as expected. The results obtained by the IGL converges to the ones evaluated by hand calculations. It is worthy to mention that in the case were the local model was stiffer than the global one, the residual forces oscillated between positive and negative values.

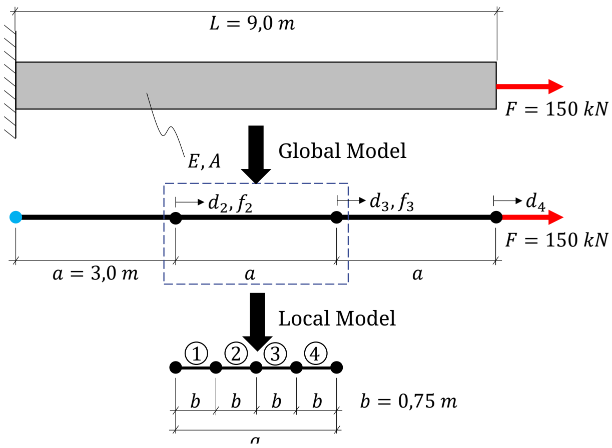


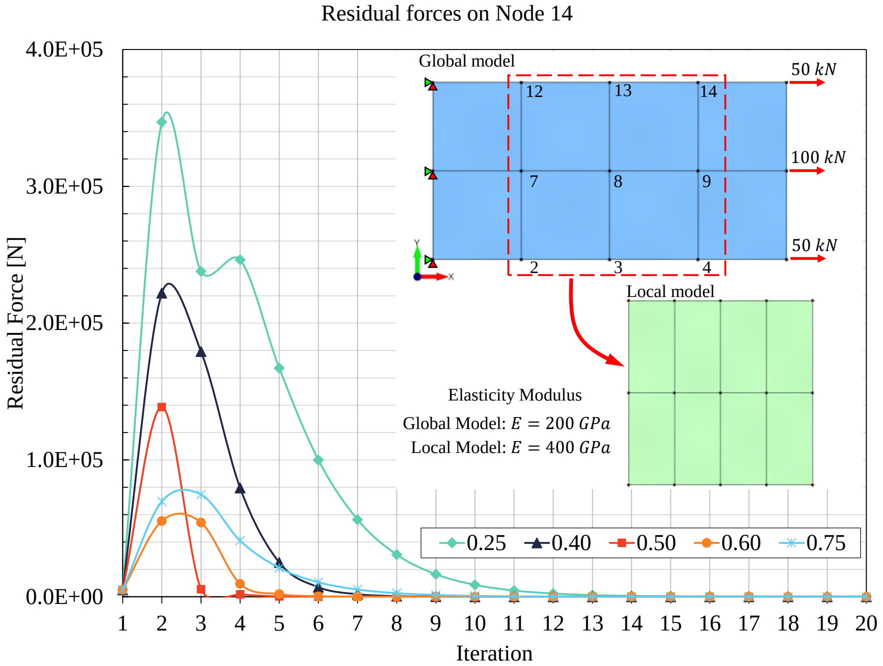

Figure 4. IGL Problem 1: bar under axial load.

Table 1. Results of the solution of the Problem 1 by IGL.

| Iteration | Residual forces f3 [N] | |
|---|---|---|
| | Case 1 | Case2 |
| 1 | -5.00E+04 | +5.00E+04 |
| 2 | -1.67E+04 | -1.67E+04 |
| 3 | -5.56E+03 | +5.56E+03 |
| 4 | -1.85E+03 | -1.85E+03 |
| 5 | -6.17E+02 | +6.17E+02 |
| 6 | -2.06E+02 | -2.06E+02 |
| 7 | -6.86E+01 | +6.87E+01 |
| 8 | -2.29E+01 | -2.29E+01 |
| 9 | -7.62E+00 | +7.63E+00 |
| 10 | -2.54E+00 | -2.54E+00 |
| 11 | -8.47E-01 | +8.48E-01 |
| 12 | -2.82E-01 | -2.83E-01 |

For the second numerical example, Figure 5 presents the problem simulated. Various relaxation factors were applied in order to investigate its influence on solution convergence. The number of iterations necessary for convergence is presented in the table of the Figure 5. The evolution of the residual force on the interface node 14 along the iterations also presented in Figure 5. The results indicate that the relaxation played an important role at the convergence rate in this numerical example, in which the local model is stiffer than the global model. It is worthy to mention that the solution diverged when no relaxation factor was considered.

Finally, Figure 6 presents a problem solved by IGL-GFEM$^{gl}$ approach. A plate with a central hole is loaded in tension. The domain was divided into three scales, according to the schemes of Figure 6. The solution was obtained by the IGL-GFEM$^{gl}$ strategy and the results in terms of displacements in y-direction are presented for each of the three scales. The maximum absolute value for this output is $2.781 \cdot 10^{-5}$ and occurs at the midspan of the plate. For reference, the problem was also solved by conventional FEM using the NASTRAN solver. The model with 15680 degrees of freedom lead to maximum absolute value for y-displacements of $2.831 \cdot 10^{-5}$. On the other hand, the IGL-GFEM$^{gl}$ solution was evaluated by 690 degrees of freedom and were completely performed in a in-house software.




| Relaxation Factor | Number of iterations |
|---|---|
| 0.10 | 183 |
| 0.15 | 99 |
| 0.20 | 67 |
| 0.25 | 88 |
| 0.30 | 42 |
| 0.35 | 44 |
| 0.40 | 56 |
| 0.45 | 24 |
| 0.50 | 17 |
| 0.55 | 43 |
| 0.60 | 34 |
| 0.65 | 48 |
| 0.70 | 67 |
| 0.75 | 77 |

Figure 5. IGL Problem 2: plane stress – problem description and residual forces convergence for different relaxation factors.

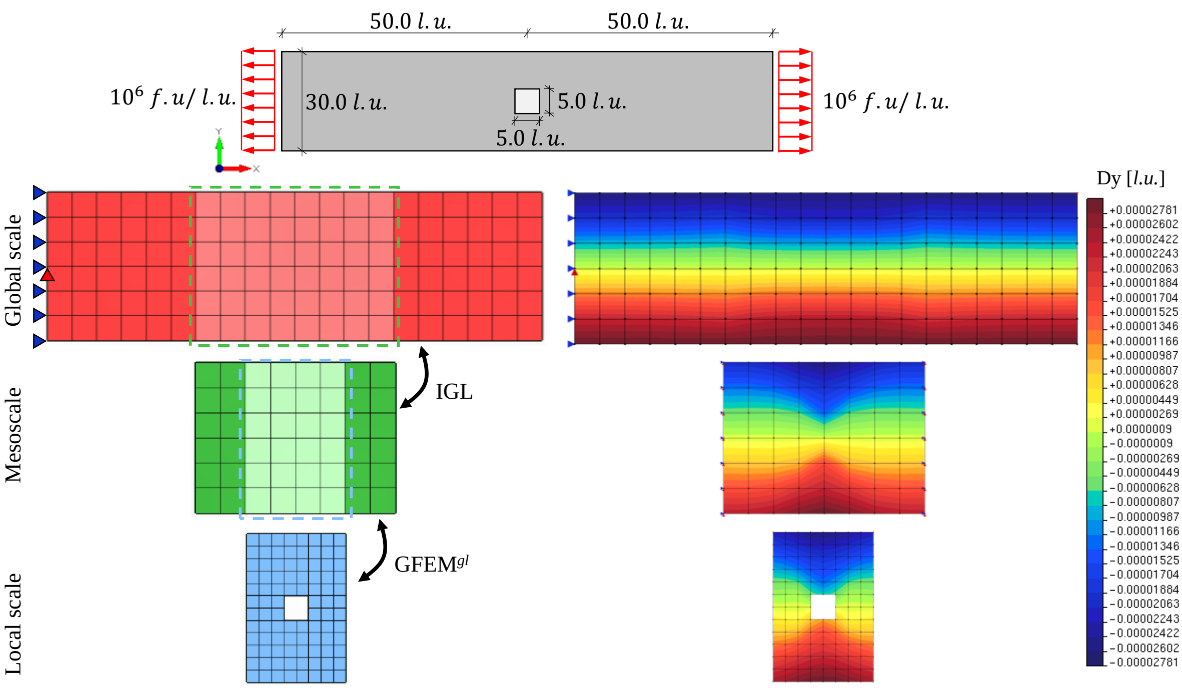



Figure 6. IGL-GFEM$^{gl}$ Problem 3: plate with hole – problem description and displacements in y direction.

## 4 Conclusions

The use of the IGL method and the IGL-GFEM$^{gl}$ were successfully implemented in INSANE (INteractive Structural ANalysis Environment), a research structural analysis software developed at the *Universidade Federal* *de Minas Gerais*. The implementation was validated by reference solutions obtained by conventional FE analysis and by hand calculations. The method accuracy and robustness were investigated through a set of problems. Considering the convergence criterion proposed by Whitcomb [15], the number of iterations varies according to the magnitude of the forces acting on the problem. A more convenient convergence criteria can be investigated.

Although highly non-intrusive, the IGL- GFEM$^{gl}$ implementation presented on this paper was tested only in the INSANE platform. The extension of this implementation to commercial software is still on going and the related results are going to be discussed in a future paper. Once the implementations are completed, new investigations will be carried out aiming to evaluate the method robustness and application. The convergence variables, such as relaxation and convergence criteria, are subject of further investigation as well.

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