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Optimization of steel castellated and cellular beams using finite element method and genetic algorithms

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Abstract. The quest to consume resources more consciously and effectively encourages the use of optimization processes. In this sense, the present study aims to employ computational optimization techniques to determine the maximum strength of open-web steel beams, for two groups of different cut lines, one generating beams with holes in the hexagons-shape and another in ellipse format. From these two models of cut lines, a set of parameters is defined that establish different configurations for the cut line in an analyzed I-shaped profile, for example: distance between holes; position along the web height; hole dimensions; among others. A computational routine is implemented to generate, from the analyzed I-shaped profile and for certain parameter values, a finite element mesh. The load capacity of the castellated and cellular beam is defined through a nonlinear analysis using the finite element program FEMOOP and a three-node triangular finite element in plane stress state. This element was chosen due to the need for a very refined mesh in the discretization process of the different possible configurations of cut lines, therefore, the finite elements are small and do not require high degree interpolating functions. The optimization process consists of defining, for a given I-shaped profile, which configuration of the cut line produces a castellated or cellular beam with greater load capacity. The implemented routines are validated from numerical and experimental models found in the literature, and it is expected, from an analysis of the beams found in the populations obtained by the evolution of the genetic algorithm, an increase in the load capacity of the analyzed I-shaped profile.

Keywords: open-web beams, optimization, genetic algorithm

1 Introduction

With the constant development of technologies and, with the automation of the cutting and welding processes, the open-web steel beams configure a constructive solution that adds strength gain and reduction of the structure's own weight. The open-web I-shaped profile has a greater moment of inertia and flexural stiffness for the cross section, and therefore a greater flexural strength. In addition to the efficiency and economy of steel, open-web beams also offer architectural benefits, as they allow better interaction with the building's installations.

Several studies on the structural behavior of open-web beams have been developed, most of them using standardized cutting lines: cellular beams, with circular openings, and castellated beams, in the Peiner, Litzka and Anglo-Saxon patterns. Among the studies, we can cite Alves and Lubke [1], Dias [2], Sonck and Belis [3], Silveira [4].

In order to evaluate the existence of other patterns of cut lines that bring a better structural performance, or to reaffirm the effectiveness of the classic models already widely used, a computational formulation is presented to determine a cut line in I-shaped profile. Two shapes are considered for the cut lines, one that generates hexagonal holes and another that generates elliptical holes. The use of these two types of shear line allows the validation of the numerical model proposed through the widely studied standard geometries and the evaluation of the influence of the use of narrower or wider holes on the load capacity of the beam.

The load carrying capacity of the beam is evaluated through structural analysis using the finite element method. For this, a plane triangular finite element with three nodes is used, implemented considering plane stress states for both linear and non-linear analysis of the material. The three-node triangular element for the numerical analysis of structures in plane stress states was chosen because it requires a very detailed discretization of the structure, thus allowing to define the geometry of the holes with greater refinement.

Optimization techniques are used for automated generation of the cutting line shape, ensuring the simplification of the design process and enabling a better understanding of the structural behavior of the elements. In addition, the consideration of the non-linear behavior of the material allows a better use of the dimensioned structure.

2 Formulation of the optimization problem

In this paper, the objective is to determine dimensions of the cut line in steel profiles for the formation of open-web beams, so that these beams have the highest possible load capacity. Once the type of load acting on the beam is defined, the objective function is to maximize this load, or minimize its inverse.

Despite the simple form of the objective function shown in eq. (1) it presents significant difficulties in the optimization process used, as it does not explicitly present its relationship with the design variables.

$$f(x) = q \tag{1}$$

Having defined the properties of the materials according to technical specifications, the load capacity of the open-web beam must be determined, verifying the ultimate and service limit conditions.

The service limit condition depends on the type of use to which the beam will be subjected, in this paper the limit deflection (δ_{ser}) of L/250, where L is the beam span, is considered as a parameter to verify the service limit state. The ultimate load and the service load are determined through the load-displacement curve obtained from a physical nonlinear analysis of the open-web beam using the finite element method using a finite element for plane stress analysis.

Defining δ_{ser}=L/250, the ultimate deflection is obtained as the product between a positive factor β, greater than unity, and the service arrow, ie, $\delta_{ult}= \beta.\delta_{ser}$. The value of β is defined by the user, the analyzed models suggest that $\beta > 5$. With the definition of the deflection and the load-displacement curve of the beam, q_{ult} and q_{ser} are obtained. The load carrying capacity of the beam will be the smallest value between q_{ult} and $\alpha.q_{ser}$. The value of α is defined by the user being suggested by the authors by the models analyzed $1.7 \leq \alpha \leq 2$.

The design variables, indicated in Fig. 1 and Fig. 2 are parameters that will define the dimensions and positions of the cuts that will form the open-web beam, these variables are continuous with positive values, since they represent dimensions. Due to limitations such as profile height and practical conditions related to the cutting process, they have lower (l_i) and upper (u_i) limit values that will be inserted in the project as lateral constraints eq. (2) to eq. (5).

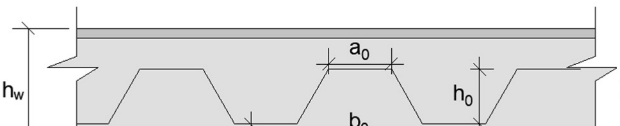

Figure 1 - Design variables hexagonal holes

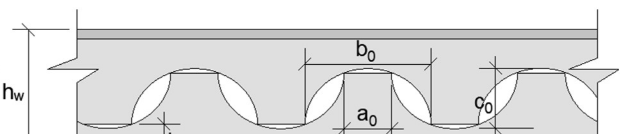

Figure 2 - Design variables elliptical holes

$$l_1 \leq a_0 \leq u_1 \tag{2}$$

$$l_2 \leq b_0 \leq u_2 \tag{3}$$

$$l_3 \leq h_0 \leq u_3 \text{ (or } l_3 \leq c_0 \leq u_3 \text{ for ellipse)} \tag{4}$$

$$l_4 \leq h_i \leq u_4 \tag{5}$$

Boundary constraints are defined when defining the search space for variables by specifying their minimum and maximum values. In addition to the boundary constraints, it is necessary to respect the constraints generated by the relationships between the variables. These constraints are defined as shown in eq. (6) and eq. (7) for the problem with hexagonal holes and eq. (6), eq. (8) and eq. (9) for elliptical holes. When a set of values for the variables does not meet these constraints, a user-defined penalty value is assigned to the objective function.

$$a_0 \leq b_0 \Rightarrow b_0 - a_0 \geq 0 \tag{6}$$

$$h_0 + h_i \leq h_w - l_4 \Rightarrow h_w - l_4 - h_0 - h_i \geq 0 \tag{7}$$

$$h_i - \left(\frac{1}{2}c_0 - h_0\right) \geq l_4 \Rightarrow h_i - \left(\frac{1}{2}c_0 - h_0\right) - l_4 \geq 0 \tag{8}$$

$$h_0 + h_i + \left(\frac{1}{2}c_0 - h_0\right) \leq h_w - l_4 \Rightarrow h_w - l_4 - h_i - \frac{1}{2}c_0 \geq 0 \tag{9}$$

The optimization process was carried out using the method of genetic algorithms, an evolutionary process that defines a set of values for the variables of the problem based on the model of natural evolution.

These methods consist of defining within the analysis space, a finite set of values for the variables, which is initially defined randomly within this space and then a criterion is adopted for defining the next sets of values. The difference between the criteria used to define the next sets of values for the variables, define different types of evolutionary methods to search for the optimal point.

The individuals that make up the population in each generation of the evolutionary method are formed by the values of these design variables. The form for storing these data adopted in this paper is the binary representation of each variable and the individual is the binary representation in sequence of all the variables (a_0, b_0, h_0 or c_0 and h_i). The number of bits that will be used to represent each variable is defined by the user, not being necessary to define the same number of bits for all variables, which allows a greater discretization of the most important variables in the analysis. The binary representation of the variables was chosen because the codes implemented in this work were done in a low-level programming language, which generally makes the codes faster.

The number of individuals in the initial population is defined by the user and these individuals are randomly defined within the search space for the set of variables in order to have 100% of the initial population meeting the set of restrictions for the problem under analysis.

The next generation is defined proportionally to the aptitude values of the individuals of the previous generation. Individuals with a higher fitness value are more likely to be chosen to form the next generation. Thus, pairs of individuals are drawn from the previous generation, considering in this draw the highest probability of individuals with the highest fitness, which will form pairs of individuals from the next generation, which may or may not undergo the crossover process, according to a rate defined by the user, and by the mutation process. It is also possible to apply elitism and scaling processes.

For the optimization process, the linear triangular element, the CST element (Constant Strain Triangle), a three-node flat triangular finite element with two degrees of freedom per node, was used. This element is used under plane stresses in numerical simulation using the Finite Element Method. The degrees of freedom are given by the displacements of translations in the directions of the x and y axes that form the plane of the triangular element. The triangular element used for physical nonlinear analysis of plane structures that differs from the linear triangular element only in the definition of the constitutive matrix and in the calculation of internal forces and their derivatives. A similar formulation is presented in Ladeira [5].

In order to use the triangular finite element in the structural evaluation of hollow-core steel beams, it is necessary to discretize the beam into triangular elements, for which a mesh generator was implemented. The mesh generator allows the use of beam symmetry when possible. There are specific functions for meshing at the beginning and end of the beam, for the step region and for the region formed when the symmetry axis passes through the center of the hole. The size of finite elements is defined as a function of a user-defined limit leg (cl). The smaller this parameter, the more discretized the mesh and the longer the analysis time of the finite element method.

The implementation was carried out through the Finite Element Program FEMOOP, Finite Element Method Object Oriented Program, (Guimarães [6]), elaborated in C++ language and developed in a way that new elements and analysis algorithms are implemented without the need for detailed knowledge of its structure, according to the concepts of object-oriented programming.

To validate the model used in the numerical analyzes necessary to determine the load capacity of the open-web beams, the experimental results of Toprac and Cooke [7] and the respective numerical model Silveira [4] were used. In the present paper, the same geometry and material properties of those papers were used and consistent results for the load displacement curve were found, allowing the validation of the model.

3 Examples and Results

A beam with a span of 4 meters simply supported subjected to a uniformly distributed load was analyzed. The cross section of the beam is of type I doubly symmetrical with 200mm flanges and 300mm web. The thickness of the tables is 8mm and the web is 5mm. The properties of steel are; elastic modulus (E) of 205GPa, Poisson coefficient (ν) equal to 0.3 and steel yield stress (f_y) of 350MPa.

The finite element mesh is defined using the methodology presented in this paper for each variable assignment and considering a limit leg of 1.5 cm, taking advantage of the symmetry of the beam. For the four design variables referring to the problem, 14 bits were admitted for the binary size of the variable and the limit values from 0.05m to 0.4m for a_0, 0.05m to 0.4m for b_0, 0.05m to 0.27m for h_0 or c_0 and 0.05m to 0.235m for h_i.

The load capacity of the beam is considered to be equal to the uniformly distributed load that generates a maximum deflection equal to the limit value adopted for the deflection, which in this example was considered L/250, that is, 1.6 cm. In this way, the genetic algorithm seeks to provide the data for the generation of holes in order to maximize the behavior of the beam regarding the limit state of excessive deformation.

For the genetic algorithm, proportional selection with elitism was considered, guaranteeing the two best individuals in the following generations. For the crossover process, a probability rate of 0.6 was considered. A mutation with a probability rate of 0.01 was considered. The scaling of aptitude values was not allowed and the penalty value equal to zero was adopted for individuals who do not meet the restrictions defined in this paper. For population size, 20 individuals were analyzed in a total of 60 generations. Due to the computational cost of finite element analysis in evaluating the fitness value of each individual in the population, mainly in non-linear problems, no analysis was carried out with a larger population number, it is intended in future works to evaluate different population sizes.

3.1 Linear analysis with hexagonal holes

In the first analysis, the hexagon hole is considered. After 60 generations, the genetic algorithm defines the values of 5.0 cm, 35.7 cm, 18.7 cm and 5.1 cm for the variables a_0, b_0, h_0 and h_i, respectively, which generate a distributed load of 124 kN/m for a maximum deflection of 1.6cm.

Since generally the design load considering the last combination is around twice as large as the design load considering the service combination, a stress analysis was performed for a load value with twice the value obtained for the service load. , the results obtained are shown in the Fig. 3.

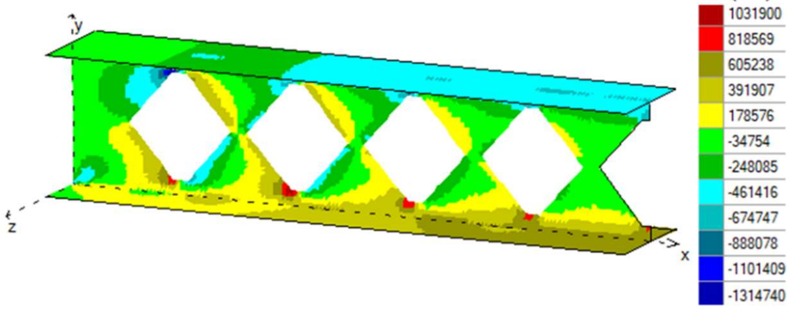

Figure 3- Stress distribution in beam with hexagonal hole for linear analysis

Since the yield stress of the steel is given by 350000kPa for traction and compression, it is verified that several regions of the table and web have stresses greater than the yield stress of the steel. It can be concluded that the loading relative to the ultimate load capacity of the beam is much smaller than the loading analyzed.

3.2 Linear analysis with elliptical holes

In the second analysis, the ellipse-type hole is considered. After 60 generations, the genetic algorithm defines the values of 13.1 cm, 31.4 cm, 24.0 cm and 7.7 cm for the variables a_0, b_0, c_0 and h_i, respectively, which generate a distributed load of 103 kN/m for a maximum deflection of 1.6cm. Figure 4 presents the stress analysis for a load value with twice the value obtained for the service load found for the beam.

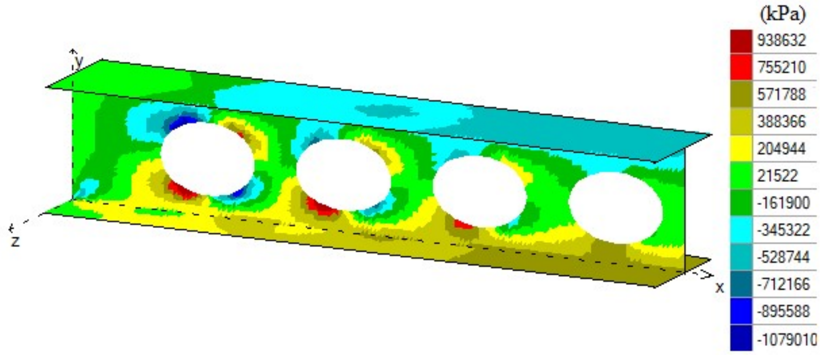

Figure 4 - Stress distribution in the elliptical hole beam for linear analysis

In a similar way, several regions of the table and web are also observed with stresses greater than the yield stress of the steel. It can be concluded that the loading relative to the ultimate load capacity of the beam is much smaller than the loading analyzed.

Comparing the responses obtained for the two geometric shapes considered for the holes, it can be concluded that the hexagonal hole generates a better response considering the flexibility of the beam. In terms of limiting stresses, it is observed that the stress levels in the elliptical bore beam are lower than the stress levels in the hexagonal bore beam, but the applied loads are different, 124 and 103 kN/m. If we multiply the stress levels of the elliptical hole beam by the ratio 124/103, we get the same stress levels of the hexagonal hole beam. It is not possible to conclude which would be better in the ultimate limit state verification, which will only be possible by doing a non-linear analysis.

3.3 Nonlinear Analysis with Elliptical Holes

In the third analysis, the ellipse-type hole is considered, considering the non-linearity of the material. After 60 generations, the genetic algorithm defines the values of 13.3 cm, 32.4 cm, 23.2 cm, and 6.3 cm for the variables a_0, b_0, c_0 and h_i, respectively, which generate a distributed load of 76 kN/m for a maximum deflection of 1.6 cm.

The load-displacement curve of the open-web beam obtained in this example is shown in Fig. 5. It can be seen from the beam that for a uniformly distributed load of 77 kN/m, the beam has the service limit deflection, therefore, this load is defined as the service limit load. It is considered in this paper that the ultimate load is twice the service load, or the load that generates a maximum deformation equal to 5 times the service limit deformation. In the configuration defined by the genetic algorithm, the limit deflection of 8 cm (5 times the service deflection) was reached before the last load given by twice the service load, thus, the ultimate load is that obtained for the deflection value of 8 cm, that is, 151 kN/m.

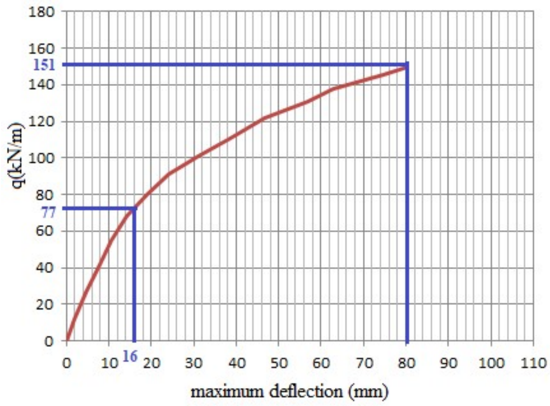

Figure 5 - Load-displacement curve of the open-web beam for non-linear analysis

Figure 6 shows the normal stress distribution along the beam axis along the half of the open-web beam obtained from the analysis of this example. The stresses in this figure were obtained for the last step of the incremental displacement process that generated the load-displacement curve shown in Fig.5.

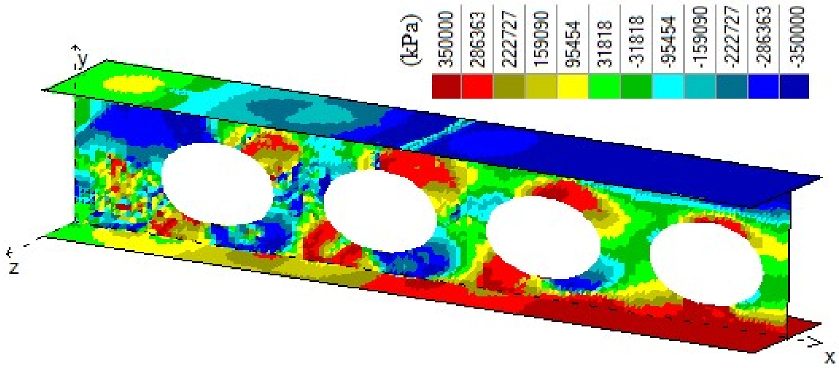

Figure 6 -Stress distribution in beam with elliptical hole for non-linear analysis

Comparing the answer of this example with the answer for the elliptical hole with linear analysis, it is observed that the service limit load of 103 kN/m found for a linear analysis is much higher than the limit load of 77 kN/m found for analysis not linear. This is due to the fact that, as can be seen in the load-displacement curve, from 30kN/m onwards, part of the material already presents non-linear behavior, which cannot be observed by linear analysis.

3.4 Nonlinear analysis full web beam

A stress analysis was also carried out for the normal stress in the direction of the beam axis for the full web beam with dimensions equal to the profile used to generate the open-web beam analyzed in the examples of this paper, that is, 200 mm thick tables 8mm and 300mm core with 5mm thickness. Figure 7 shows the formation of a plastic hinge in the middle of the span due to the maximum moment in this section. Comparing with the analysis of stresses in the open-web beam, the non-linear analysis verifies that, in addition to the region of maximum moment, the open-web beam also presents stresses at yield levels around some holes.

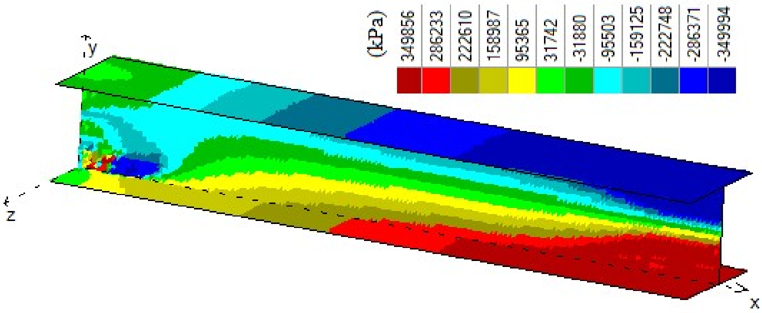

Figure 7 - Stress distribution in the full web beam

The load-displacement curve for the full web beam analyzed in Fig 8 is also presented.. This curve was obtained using the triangular finite element with a formulation considering the non-linearity of the material presented in this paper. This curve was also found using a bar element (one-dimensional) with formulation considering Timoshenko's beam theory. It is observed from the analysis of the two elements that the ultimate load of the beam is the same for the two analyses, the same happening for the initial behavior of the curve. However, it is observed that the analysis curve in plane stresses is able to verify the influence of the non-linearity of the material in a more comprehensive way along the length of the beam and the cross section, while the bar element as soon as it starts to present the influence of the nonlinearity of the material the section of maximum moment transforms in a plastic hinge.

Checking the limit deflection for the full web beam, it is observed that the service load obtained is 53 kN. Considering that the ultimate load is twice the service load or the load for the ultimate deflection is 5 times the service limit deflection, it is verified that the ultimate load for the full web beam is 106 kN. Comparing these values with those obtained by the open-web beam defined by the optimization algorithm presented in this paper, a significant increase in the load capacity of the hollow beam in relation to the original full web profile is observed.

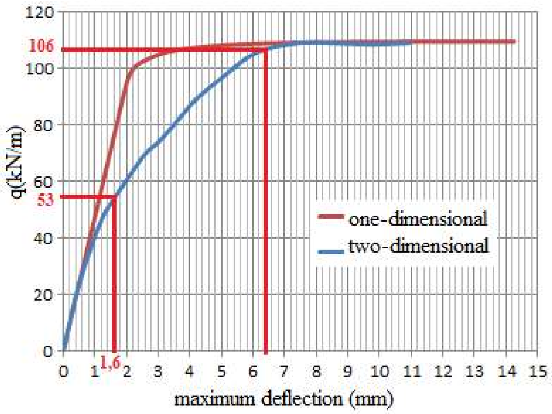

Figure 8 - Load displacement curve of the full web beam for non-linear analysis

4 Conclusion

A three-node triangular finite element model was implemented, as well as a topological optimization routine using the method of genetic algorithms for beams with hexagonal and elliptical openings. The implementation also allows linear and non-linear analysis for the optimization of the analyzed structures.

The implemented program was validated using numerical and experimental models, showing that the results are consistent for both circular and hexagonal openings.

Some examples were tested demonstrating that the optimization using the method of genetic algorithms has the potential to be developed and widely used.

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