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Optimized analysis of T, Circular and Rectangular cross-sections of reinforced concrete under biaxial bending

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Abstract. He analysis of polygonal reinforced concrete sections subjected to biaxial bending is of great interest in civil construction. Generally, its dimensioning is done through design charts, generating a section that is often uneconomical. Therefore, this work aims to implement an algorithm capable of optimally distributing a number of steel bars in a T, rectangular or circular cross section in such a way that it is safe and efficient for a combined loading of moment and axial force. it is also the objective of this work, the development of a graphical interface that stores the data provided by the user and performs the optimization and detailing of a cross section. This algorithm uses the sequential linear programming method, where a nonlinear problem of determination of the resistant efforts of the section is linearized, having its optimal point defined at each step using the Simplex method. The development of this graphical interface took place through the Visual Studio Community software, using the C# programming language. The implemented algorithm also allows considering the dimensions of the T, circular and rectangular section as a design variable, defining an optimized section through an objective function given by the cost of materials. Using numerical examples available in the literature, one can prove the efficiency of the algorithm presented in this work. The results obtained were also evaluated through the axial force-moment interaction curves, being able to observe that the optimized sections did not present slack in relation to the applied efforts.

Keywords: optimization, graphic interface, C#, biaxial bending.

1 Introduction

With population growth and a reduced space to allocate new inhabitants, many cities have presented a verticalization process, in which there is the need to construct multi-story buildings. The calculation of these buildings is complex and time consuming, especially when it comes to structural elements subject to composite bending, where their design is done, in most cases, through abacuses, generating, in most cases, an uneconomical element.

Over the years several softwares have been developed that are capable of quickly dimensioning a building, respecting not only the structure's resistive capacity, but also its functionality. However, despite the agility, some elements still remain oversized, due to the difficulty of implementing an algorithm capable of optimizing them.

Aiming to solve these difficulties, Faria [1] developed an algorithm that defines the minimum amount of steel and its position inside any concrete polygonal section, in a way that this amount can resist, safely, subject to composite bending, so that the amount of steel is the minimum necessary to resist the internal forces and guaranteeing the structural safety.

Using the optimization routines developed by Faria [1], the present work aims to develop a graphical interface that receives the data provided by the user of the program so that it is possible to apply the optimization algorithm. In addition, to implement a routine capable of generating the iteration curves of moments for any given section and a routine for cost optimization. Through numerical examples that the literature makes available, the efficiency of the algorithm can be proven.

2 Literature review

Using the sequential linear programming method together with the simplex method of optimization of nonlinear problems, Faria [1] developed an algorithm that defines the quantity, position and diameter of the steel bars that compose a polygonal reinforced concrete section capable of resisting, safely and efficiently, to the composite flexural forces.

To understand how the software developed in this work works, the formulations developed by Faria [1] will be briefly presented: Cross section, materials, section deformation, resistant efforts, objective function, design variables and constraints.

2.1 Cross section and materials

Figure 1 shows a section that the program implemented by Faria [1] can evaluate. The steel bars are defined punctually inside the section. The user can define a section with any shape as well as define holes in its interior.

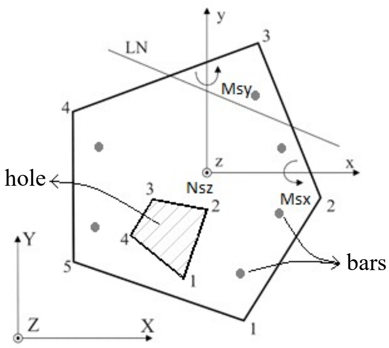

Figure 1. cross section shape.

Figure 2 shows the general form of the stress-strain curve that can be represented in the program implemented by Faria [1]. In this figure, Fi are the strain ranges, Li are the limit strains of these ranges, and for each range are defined the coefficients that will characterize the polynomial curve to be considered, which can be represented by a polynomial of up to degree 3. These stress-strain curves are used to define the material properties of concrete and reinforcing steel.

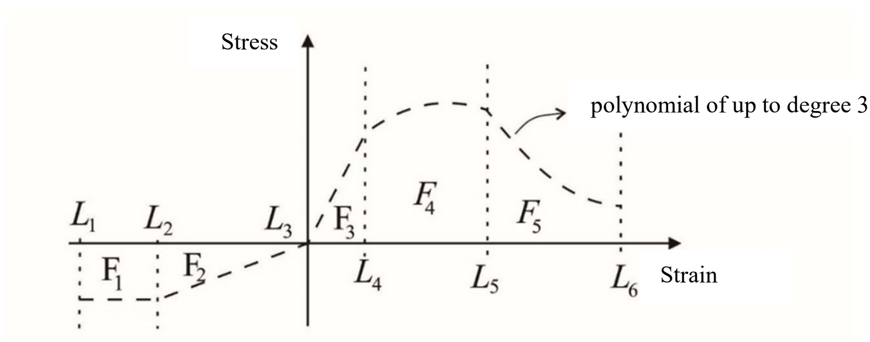

Figure 2. Generalization of the stress-strain curve analyzed by the algorithm.

2.2 Section Deflection and Resistant Efforts

Considering the Euler-Bernoulli beam model, where the plane section remains plane, and the total adherence between the concrete and the steel bars of the reinforcement, there is a single plane strain of the reinforced concrete cross section subjected to composite bending, as shown in Equation 1, where: ε_0 is the axial strain at the origin of the reference system; k_x and k_y are the curvatures in relation to the yz and xz planes, respectively.

$$\varepsilon(x, y) = \varepsilon_0 + k_x y - k_y x \tag{1}$$

From the deformation of the section (equation 1) and the stress-strain relationships of the materials, it is possible to determine the value of the efforts that a given section can resist. These efforts are the resistant moment around the "x" and "y" axis (M_{Rx} and M_{Ry}) and the normal force (N_{Rz}), being calculated as shown in equations 2 to 4. In these equations, n is the number of steel bars present in the cross section, A_{si} is the area of these n bars, x_{si} and y_{si} are the coordinates of the bar center, σ_{si} is the normal stress to the cross section at the point coincident with the bar center and σ_c is the normal stress to the cross section at any point of the section.

$$M_{Rx} = \iint_A \sigma_c y dA + \sum_{i=1}^{n} A_{si}\sigma_{si}\, y_{si} \tag{2}$$

$$M_{Ry} = \iint_A \sigma_c x dA + \sum_{i=1}^{n} A_{si}\sigma_{si} x_{si} \tag{3}$$

$$N_{Rz} = \iint_A \sigma_c dA + \sum_{i=1}^{n} A_{si}\sigma_{si} \tag{4}$$

2.3 Objective Function and Design Variables

In linear sequential programming, an objective function is one that you want to determine its maximum or minimum, depending on the type of the problem at hand. In the routines implemented by Faria [1], the objective function is given by the equation (5), where the optimization is given by the minimum of this function. In this equation, the term n is the number of bars; A_{si} are the areas of the n bars; e **x** is the set of design variables.

$$f(x) = \sum_{i=1}^{n} A_{si} \tag{5}$$

The design variables are all continuous, being given by the bar areas and the parameters that define the deformation of the section, as shown in Equation (6).

$$x = (A_{\mathrm{s1}} \quad A_{\mathrm{s2}} \quad \dots \quad A_{\mathrm{sn}} \quad \varepsilon_0 \quad k_x \quad k_y)^T \tag{6}$$

As in this work new formulations were added, the objective function and the vector of variables defined in Faria [1] were changed, as will be seen in the methodology.

2.4 Constraints

For the design and detailing of a reinforced concrete cross section under composite bending the following constraints are defined: limit deformation in the concrete, limit deformation in the steel, resistant forces in the section greater than or equal to the internal forces. Other restrictions that must be considered are the lateral restrictions for bar diameters and for the step size in the linear approximation used by Faria [1].

Limit deformations of concrete and steel

Figure 3 shows the strain limits for a cross section of a linear reinforced concrete element under action of composite bending according to Eurocode 2 [2] and ABNT NBR 6118 [3]. According to the figure, the most compressed fiber of the section should present specific linear strain in absolute value less than or equal to εcu. In the case of fully compressed sections, the linear specific strain in absolute value in the fiber distant ((εcu-εc2) /εcu)h from the most compressed fiber should be less than εc2. On the other hand, the limiting linear specific strain at the application points of the bars should be, at most, 1%.

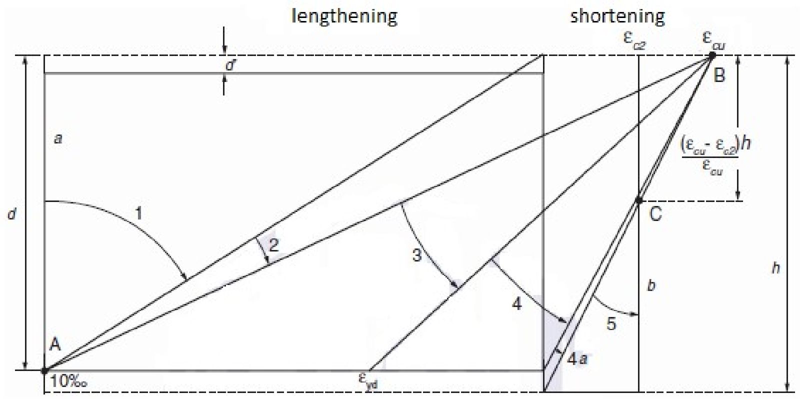

Figure 3. Deformation domains according to NBR 6118 (2014).

Solicitant forces

For a generic section to support the loads, the normal moments and forces obtained from equations 2, 3, and 4 must be greater than or equal to the forces acting on the section, which are: the requesting moment about the X axis (Msx), requesting moment about the Y axis (Msy), and requesting normal force (Ns).

Side constraints

Side constraints are practical limitations on design variables. For example, a design variable referring to a dimension cannot in practice be less than zero or greater than a certain limit, thus fixing lateral constraints or barriers for the design variable. Among the design variables presented previously, the variables referring to the area of the bars must have barriers, i.e., they must have diameters less than or equal to the user-defined diameter and greater than zero.

In the optimal point search method implemented by Faria [1], a starting point x_0 s defined and the next point that meets the design constraints and generates a reduction in the objective function is obtained from the iterative equation $x_{(k+1)}=x_k + d$, where d is the step size. Thus, the nonlinear problem analyzed becomes a sequence of linear problems with variables given by the vector **d**. For the linear approximation used in the method implemented in this work to be valid, barrier constraints must be imposed on the step size, i.e., $|d|\leq\Delta$.

3 Methodology

The methodology chosen for the construction of the graphical interface program that captures and analyzes all the data described in the previous chapter is the definition of a main form from which several other necessary forms are called while reading the necessary parameters for the routines implemented by Faria [1]. To develop this graphical interface, the Visual Studio Community software, made available free of charge by Microsoft, was used, and the programming language used was C#.

In Faria [1] a routine capable of providing the resistant forces of a reinforced concrete polygonal section is used. This routine was developed by Caldas [4] and has been successfully used in the development of other works. In this work, besides the development of the graphical interface, the possibility of representing a circular section was included in this routine.

In addition to the development of the graphical interface that receives the data to perform the optimization, the parameters that define the concrete cross section were included. Of the different types of cross sections used for reinforced concrete linear elements, the T-type, circular and rectangular sections were analyzed.

3.1 Main form

The program has a main form and that, through buttons, it is possible to access secondary forms for the insertion of data necessary for the algorithm to perform the optimization. As shown in Figure 4, the main form has sixteen buttons, nine of which give access to secondary forms and the rest are for performing functions such as: open and save the file, perform the optimization, move the section, and zoom in and out of the image. In addition, at the top of the form there is a menu strip containing functions and shortcuts to the other forms.

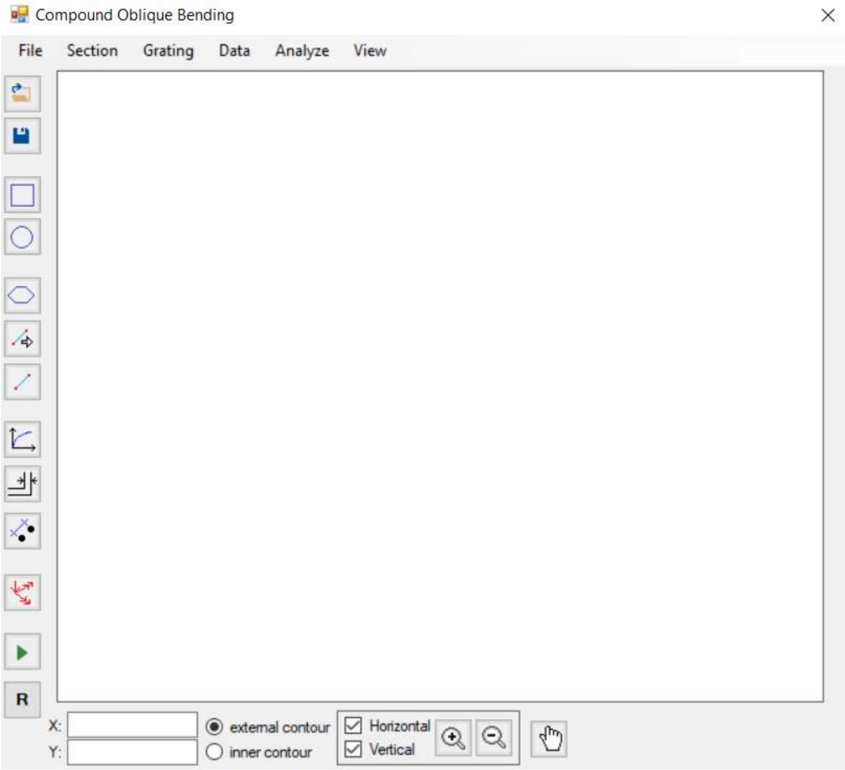

Figure 4. main form.

3.2 Secondary forms

As already mentioned, the buttons on the main form trigger secondary forms for data entry. These are: spacing, loading, polygonal section, bar spacing, stress-strain curve, interaction curve data, minimum reinforcement, rectangular section, circular section, initial section, cover, cost, and limit values.

3.3 Additional Formulations

In order to improve Faria [1] algorithm, new functions were added regarding the optimization of the reinforcement of a circular section, the cost optimization of a concrete section (T, circular or rectangular) and the definition of the moment interaction curve.

Formulations for the circular section

For a given cross section, it is possible to obtain the resistant forces by integrating the normal stress function acting on this section, as shown in the following equations:

$$M_{Rx} = \iint_A \sigma_c y dA \tag{7}$$

$$M_{Ry} = \iint_A \sigma_c x dA \tag{8}$$

$$N_{Rz} = \iint_A \sigma_c dA \tag{9}$$

Where σ_c is the stress normal to the section at any point of the section.

To develop this integral, a change of the infinitesimal element dA was made. Furthermore, the term x^2 can be replaced, leaving it as a function of y, as shown in equations (10) and (11):

$$R^2 = x^2 + y^2 \tag{10}$$

$$x = \sqrt{R^2 + y^2} \tag{11}$$

As mentioned in section 2.1 of this paper, the program analyzes a stress-strain curve that presents up to degree 3. Thus, σ_c can assume the following function formats demonstrated in equations (12), (13), (14) and (15):

$$\sigma_c = a \tag{12}$$

$$\sigma_c = ax + b \tag{13}$$

$$\sigma_c = ax^2 + bx + c \tag{14}$$

$$\sigma_c = ax^3 + bx^2 + cx + d \tag{15}$$

Therefore, it is enough to substitute the value of σc in equations (7), (8) and (9) to find the resistant forces of the section.

As a circular section presents symmetry in any axis passing through the center, it is possible, to facilitate the calculations, to find the resulting moment $M_{y'}$ in an auxiliary axis X'Y'. After that, it is just a matter of returning the moment to the original axes of the section.

To find the location of the X'Y' axis, it is necessary to use the section deformation equation, ε(x,y) , where ε is the axial strain at the origin of the reference system; k_x and k_y are the rotations of the section about the x and y axes, respectively.

$$\varepsilon(x,y) = \varepsilon + k_x y + k_y x \tag{16}$$

Since α is the angle between the X and X' axes, the equation for the deformation of the section can be parameterized as a function of the new axes:

$$x' = \cos(\alpha) * x - sen(\alpha) * y \tag{17}$$

$$y' = \mathrm{sen}(\alpha) * x - cos(\alpha) * y \tag{18}$$

$$\varepsilon(x', y') = \varepsilon + \left(-k_x * sen(\alpha) - k_y * \cos(\alpha)\right)x' + \left(k_x * cos(\alpha) - k_y * \mathrm{sen}(\alpha)\right)y' \tag{19}$$

Where the terms k_x*sen(α)-k_y*cos(α) and k_x*cos(α)-k_y*sen(α) are the rotations of the Y' and X' axes, respectively, represented by k_x' and $k_{y'}$. As the rotation $k_{y'}$ is equal to zero, it is possible to obtain, through equations (20), (21) and (22), the angle α of rotation of the axes, which will be a function of k_x and k_y.

$$k_x * cos(\alpha) - k_y * \mathrm{sen}(\alpha) = 0 \tag{20}$$

$$Tg(\alpha) = \frac{k_y}{k_x} \tag{21}$$

$$\alpha = tg^{-1}\left(\frac{k_y}{k_x}\right) \tag{22}$$

Finally, it is necessary to locate where in the stress-strain curve the analyzed section is located. Then, when the user defines the equations, the strain limits and the internal forces, the algorithm calculates the maximum and minimum deformation of the circular section, showing in which of the strain intervals that the user defined the section is located.

Formulations for varying the concrete section

In the computational routines of Faria [1] the design variables are the positions, diameter and number of bars to be distributed within the polygonal concrete section. In this work the variables that define the concrete section are included, among the different possible sections the T, circular and rectangular sections are analyzed

In the optimization of the fixed concrete section subjected to composite bending the objective function was related to the steel area and the optimization process sought to minimize this area. Considering now that the concrete area also varies, the objective function should minimize these areas giving preference to one over the other according to a coefficient, which will be defined as the material cost. Considering that the production of the linear concrete element requires a form and that this also has a cost, the cost of the form is inserted in the objective function. This portion depends on the external perimeter of the concrete element cross section. The following equation defines the analyzed objective function.

$$f(x) = cc * A_c + ca * \sum_{i=1}^{n} A_{si} + cf * p_c \quad (23)$$

In equation (23), cc is the cost of concrete per cubic meter, ca is the cost of steel per cubic meter, cf is the cost of form per square meter, Ac is the area of concrete, Asi is the area of the i-th bar of the section, n is the number of bars in the section, and pc is the perimeter of the concrete section. f(x) presents the cost of the reinforced concrete element per unit length.

Moment interaction curve

Using the computational routine developed in Faria [1] that determines the deformation of the section for any polygonal section submitted to bending and axial loading, it was developed in this project a routine to define the iteration curve of moments in sections submitted to composite bending. For this the user must define the cross section, the material properties of steel and concrete, the normal stress applied to the section, and the necessary parameters for the construction of the moment interaction curve. These parameters are the number of points along the axis $M_y = 0$ (nM_x), or $M_x = 0$ (nM_y), that will be used to construct the curve, step size to determine a specific M_x (dM_x), step size to determine a specific M_y (dM_y), and define if the sweep of points will be made along the axis $M_y = 0$ or $M_x = 0$.

In the moment iteration curve construction routine first the maximum moments are determined. For example, if the moment sweep is along the My = 0 axis, then the maximum positive and negative moments about this axis (Mxp and Mxn) are determined.

4 Application examples

In order to verify the effectiveness of the work developed, some examples were tested to verify not only the functionality of the graphical interface developed, but also the various functions implemented.

4.1 Reinforcement optimization

In this example a rectangular section of 25x40cm was used. The distance considered from the center of the reinforcement to the external face of the section is 2.5cm. The section will be detailed with 10mm bars, the horizontal and vertical spacing between bars, as well as the cover, are defined according to what is established in table 7.2 of ABNT NBR 6118 (2014). The stress-strain curves of steel and concrete are defined according to item 8.2.10 of ABNT NBR 6118 (2014). For this example, the characteristic strength of the concrete was considered to be 25MPa and the type of steel used was CA-50.

It was admitted that the section is subject to compound oblique bending, given by the following efforts: Nd = -500kN, Mxd = 30kNm and Myd = 50kNm. The section defined after parameter input is shown in Figure 5.

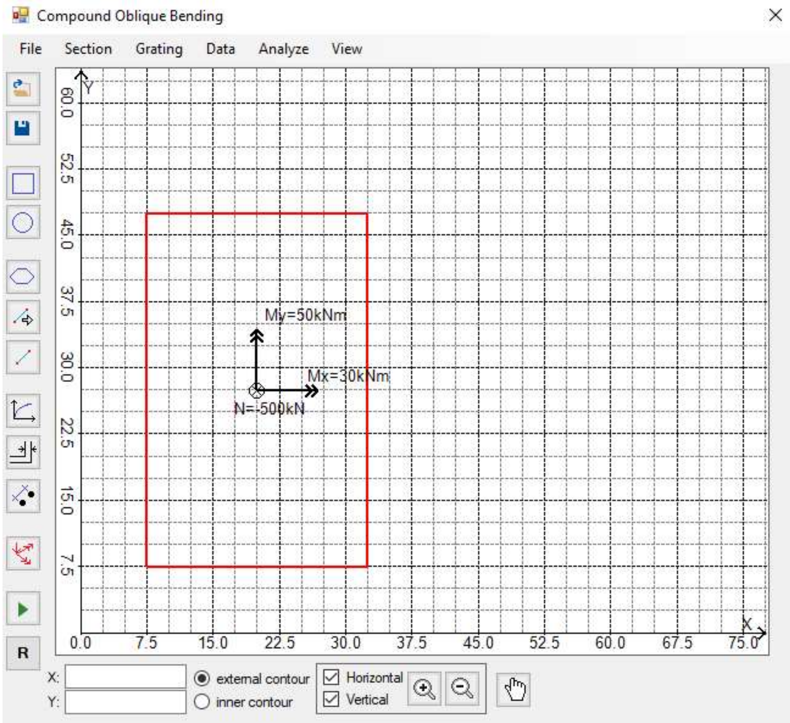

Figure 5. Initial user-defined section.

After the analysis, the algorithm defined that two 10mm steel bars would be needed at the most tensioned end (upper left corner) and one bar at the most compressed end (lower right corner). The final section is shown in figure 6. In addition, the program opens a form to show the location and diameter of the bars, as shown in figure 7.

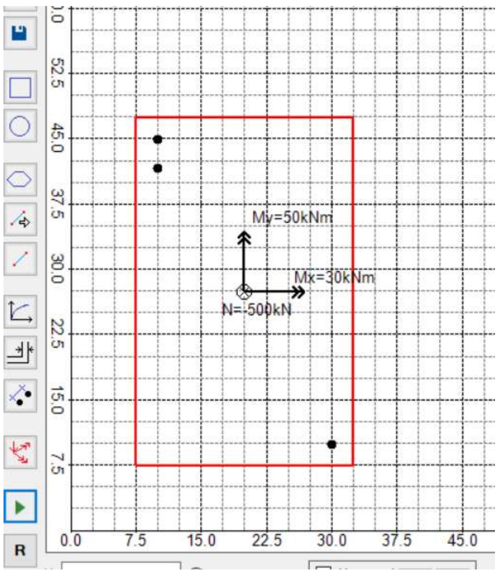

Figure 6. Final section generated by the program.

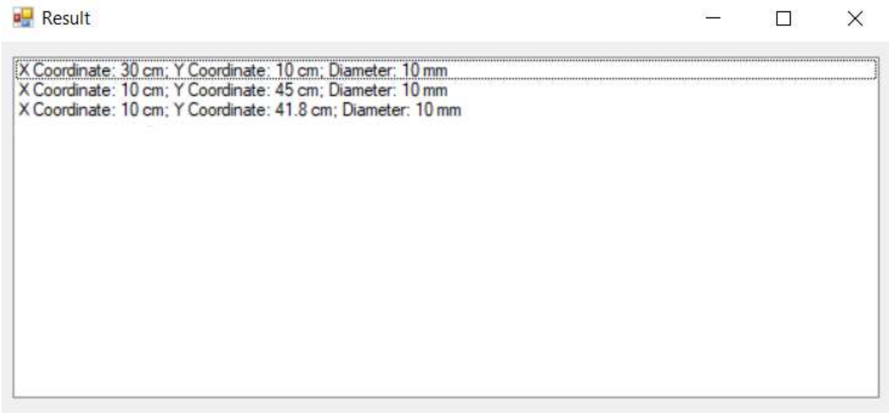

Figure 7. Bar location and diameter.

4.2 Moment Interaction Curve

As presented in the methodology of this work, two important computational routines used by the program developed in Araújo [5], are the routines that given the characteristics of the cross section and material properties, one returns the forces acting on the section when the strain of the section is given, and the other does the opposite, returning the strain of the section when the internal forces are given. In this work, these routines were used to determine the moments interaction curve of any cross section.

For this example, the section shown in Figure 8 below was used. The section is rectangular with base equal to 26cm, height equal to 30cm and with four bars of 10 mm CA50 steel. A cover of 20 mm, stirrups of 5 mm, and reinforcement diameter of 10 mm will be considered, i.e., the center of the reinforcement to the external face of the section presents a distance of 30 mm. All parameters required for the detailing of the section are defined according to item 8.2.10 of NBR 6118 (2014). For the concrete the value of 20 Mpa was adopted as characteristic strength and for the steel the CA-50 steel was adopted.

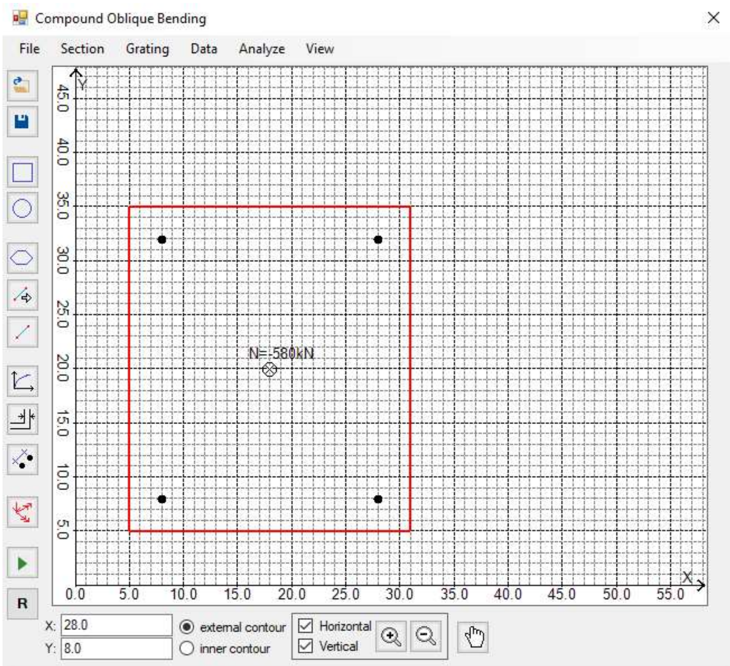

Figure 8. Rectangular cross section defined for the analysis of the moment interaction curve.

To obtain the moments iteration curve, in this example the Mx axis was considered, i.e., My = 0, and 50 points for the determination of the curve. Figure 9 shows the moment interaction curve for the section analyzed in this example.

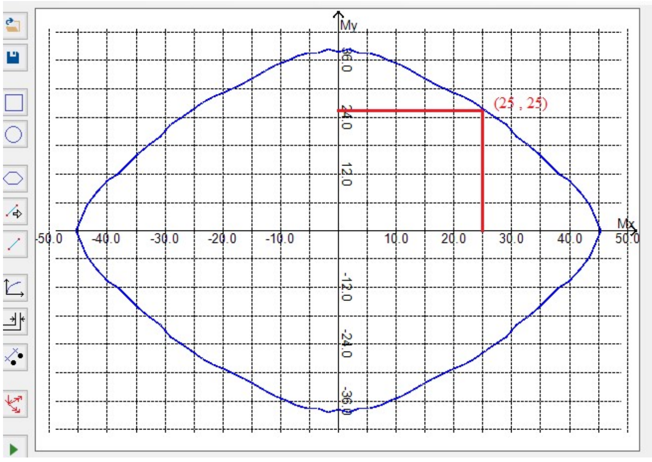

Figure 9. Moment iteration curve obtained by the program.

The moments iteration curve shown in Figure 9 defines the boundary between the safe and unsafe regions for the analyzed section submitted to a normal compressive force of 580 kN. That is, the ordered pair (M_x, M_y) is a safe force for the section if it lies on the curve shown in Figure 9 or in the internal region bounded by this curve.

For example, the point referring to the ordered pair M_x=25kNm and M_y=25kNm is under the curve, that is, the section is optimized for this combination of efforts.

4.3 Cost optmization

ABNT NBR6118 (2014) prescribes that sections of reinforced concrete linear elements under flexural and axial compressive load should have a minimum diameter of 21.5 cm. The minimum reinforcement for circular sections is 6 bars of 10 mm equally distributed along the perimeter of the section.

For this example, were considered d_{min} = 22 cm, d_{max} = 50 cm, var = 2 cm and A_{min} = 360 cm2. Furthermore, the following forces were defined, all calculation forces acting at the section centroid: N_d = -580kN, M_{xd} = 25kNm and M_{yd} = 25kNm. A cover of 20 mm, stirrups of 5 mm, and reinforcement diameter of 10 mm will be considered, i.e., the center of the reinforcement to the external face of the section presents a distance of 30 mm. All parameters required for the detailing of the section are defined according to item 8.2.10 of NBR 6118 (2014). For the concrete the value of 20 Mpa was adopted as characteristic strength and for the steel the CA-50 steel was adopted.

From the user-defined section, the program defines new sections with diameter equal to 22, 24, 26, ..., 50 cm. For each section, the program optimizes the reinforcement that should be inserted in the section so that the strains in the section are less than or equal to the limit strains prescribed by ABNT NBR 6118 (2014). For each section analyzed, the program calculates its cost per linear meter, printing on the results screen the section that presented the lowest cost. With the data mentioned above, the program generated the section shown in Figure 10.

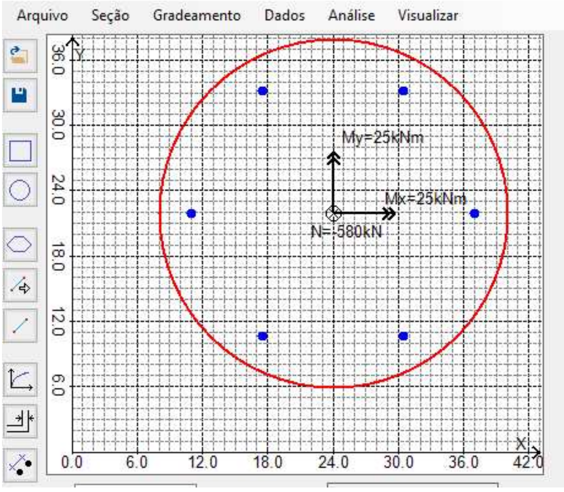

Figure 10. Final optimized section obtained by the program.

The section provided by the program has a diameter of 32 cm for which there is no need to include reinforcement beyond the 6 user-defined 10-mm bars.

Figure 11 shows the variation of the cost per linear meter in relation to the section diameter. The X marks on the curve for the diameters of 22 and 24 cm refer to the sections that were not possible to insert reinforcement so that they would meet the prescribed deformations. From Figure 11 it can be observed that the minimum cost (R$41.11/m) was obtained for the section with a diameter equal to 32 cm.

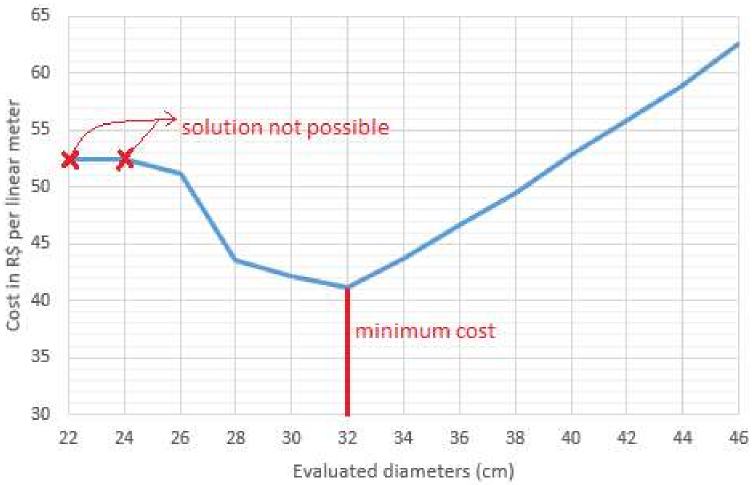

Figure 11. Cost versus diameter variation curve.

5 Conclusions

In this work, a computer program with a graphical interface was developed to streamline the data entry of a program developed by Faria [1] in which, in the latter, there was a series of routines that optimized and defined the amount of steel and its position within any polygonal concrete section subjected to composite bending. The data entries of this program are made through secondary forms, where it was sought to create an intuitive interface and easy to understand to the user.

Besides this, additional variables were inserted in the computational routines of Faria [1] to consider a possible variation of the concrete cross section in the optimization process of reinforced concrete sections subjected to composite bending. With these new implementations it is possible to define, for example, the number of bars and also the dimensions of a rectangular section subjected to composite bending minimizing the total cost of the materials concrete, steel and wood (or other form material) spent in the execution of the linear reinforced concrete element.

Within this project a computational routine was also developed to define the moment iteraction curve of sections subjected to composite bending.

Thus, through the methodology explained and the examples run, it can be concluded that the program developed in this work fulfilled its functions. It is also worth mentioning that the program has several help buttons and all the data input fields are controlled through codes, preventing the user from giving wrong data to the program.

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