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COMPUTING DEFLECTIONS OF TOPOLOGICALLY OPTIMIZED BEAMS

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Abstract. In civil engineering, computing deflections is a fundamental step in the structural design process. However, most structural optimization codes do not directly compute or evaluate displacements in the resulting optimized structures. In that context, this work presents the basis of an approach to compute and analyze deflections (vertical displacements) in optimized beams. Using Matlab®, we implemented an extension to the FEM-based 99 Line Topology Optimization Code (Sigmund, 2001), which is able to compute deflections and plot deformed shapes of optimized structures, allowing users to analyze deformation during the design process. We also compared maximum deflections of optimized versus nonoptimized beams. According to results, for constant boundary conditions, optimized beams present smaller maximum deflections.

Keywords: deflection, FEM, topology optimization, 99 lines, displacement.

1 Introduction

Structural designers must satisfy several constraints such as time and material saving and efficiency. Therefore, optimization methods are employed to assure quality and functionality, according to Vitorino [5]. The 99 line code written in Matlab® by Sigmund [4], aims at solving the beam optimization problem – for different structures and several types of loads – by minimizing compliance, employing the Solid Isotropic Material with Penalization (SIMP) approach; applying a FEM-based formulation, a given design domain is topologically optimized. However, no displacements at the optimized resulting beam are directly computed or analyzed. On the other hand, determining the maximum vertical displacement (deflection) is mandatory when designing beams, since limiting values are stablished in design codes.

In that context, we developed an extension to the FEM-based 99 Line Topology Optimization Code by Sigmund [4], to compute deflections and plot the deformed shape of the optimized structures. Thus, this works presents the deformed shape for optimized beams and the comparison of deflections of optimized versus nonoptimized beams with equal material, geometry and boundary conditions.

2 Code extension

2.1 Reference beams and simulation scheme

The code extension computes maximum deflections for both reference beams ($\delta_{máx}$) and optimized beams ($\Delta_{máx}$); it also plots their deformed shapes. Reference beams are nonoptimized, rectangular section structural members, with the same material, geometry, volume fraction and boundary conditions of the optimized beams designed by the original 99 Line Code. To verify displacement results of the reference beam numerical model, analytical elastic curve equations were employed. In the example we present in this paper, structures were simulated to compare maximum deflections $\delta_{máx}$ and $\Delta_{máx}$ for a given volume fraction range. Figure 1 illustrates the whole simulation scheme.

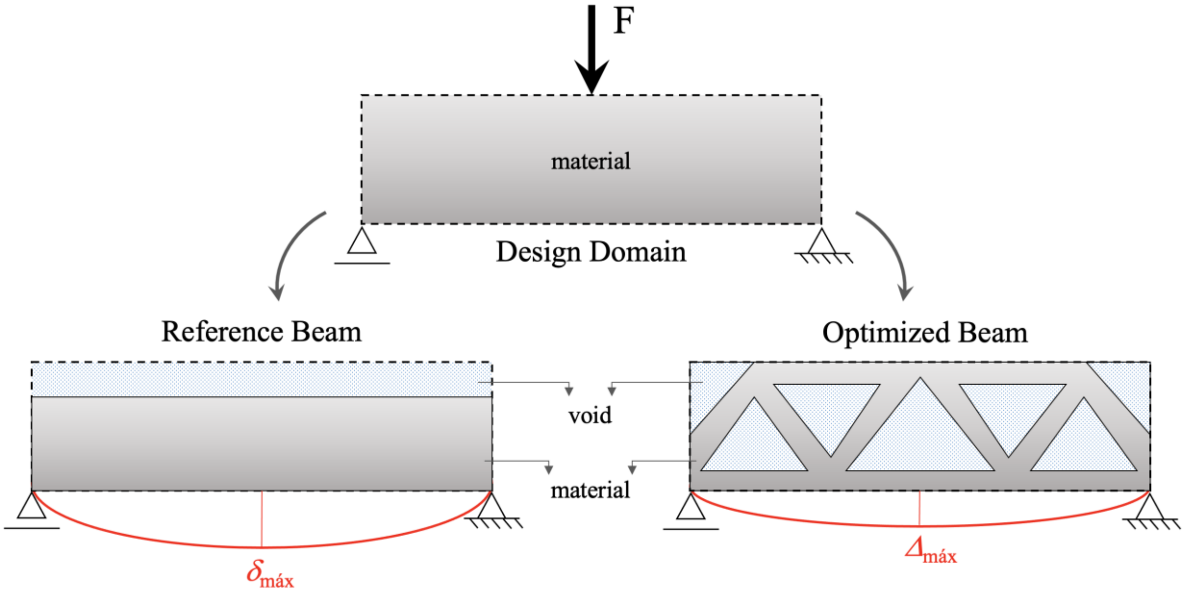

Figure 1. Simulation scheme

2.2 Post-processing

For post-processing, the code extension plots the deformed shape of the beams using Matlab® and outputs a file compatible with Paraview®, as shown in Figure 2 for an optimized beam.

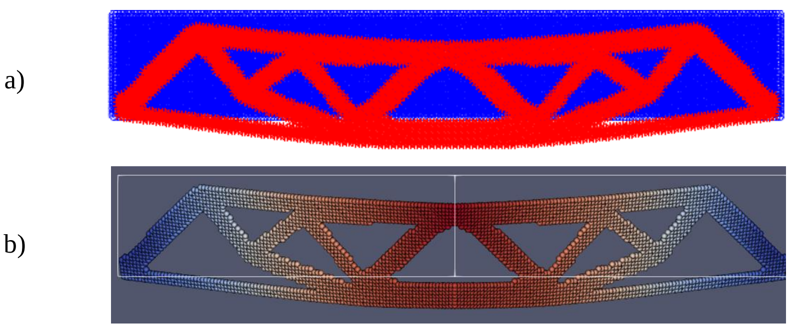

Figure 2. – Optimized beam's deformed shape post-processing: a) Matlab® plot; b) Paraview© plot

2.3 Illustration of a volume fraction range

An illustration of a volume fraction range for a cantilever beam is shown in Figure 3.

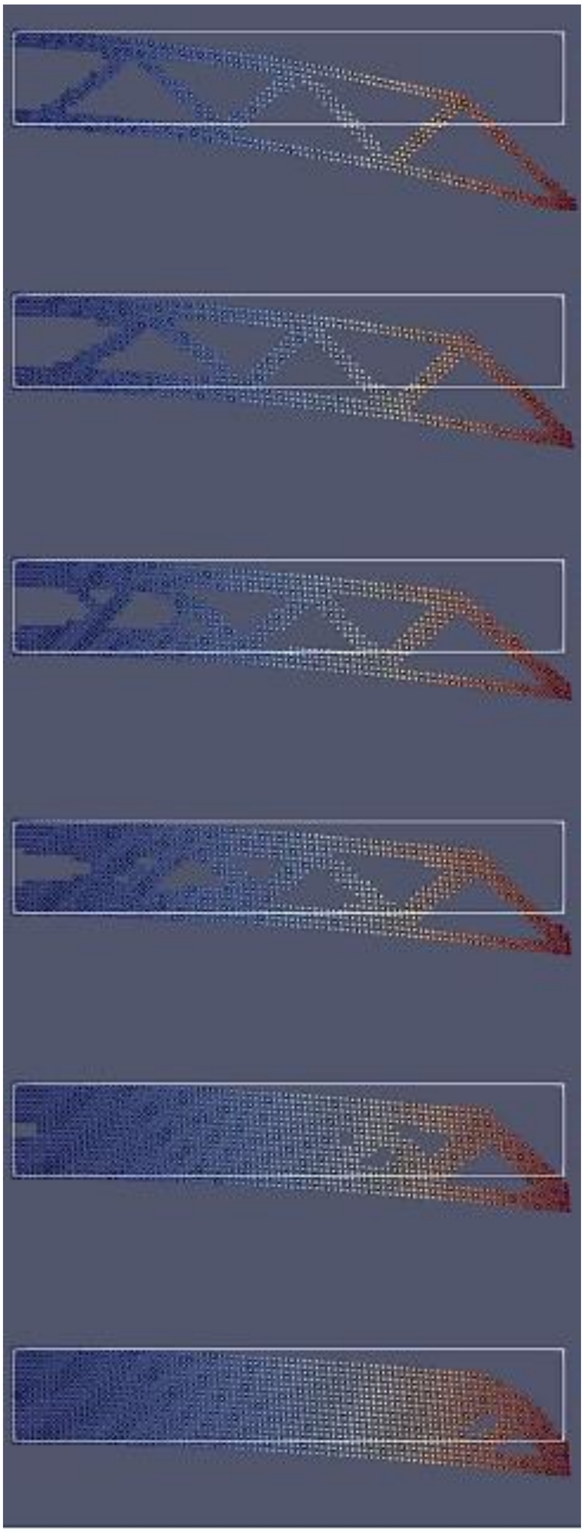

Figure 3. – Volume fraction variation for a cantilever beam

3 Results and discussion

In the simulations of both reference and optimized beams, we adopted the same rectangular design domain - with a ratio of 12 between height and length. The adopted input also included: point force load at the center F = 10^5 kN; degrees of freedom equivalent to a simply supported beam; Elastic Modulus E = $2x10^6$ kN/m^2; Poisson's ratio ν = 0.28; volume fraction ranging from f = 0,3 to f = 0,9. For the optimized beam, we also adopted minimum radius r = 1.5 and penalty p = 3.

Results are shown in Table 1, from which we can notice that, for a volume fraction of 30%, $\Delta_{máx}$ is about 12 times smaller than $\delta_{máx}$. Besides, as the volume fraction increases, $\delta_{máx}$ and $\Delta_{máx}$ tend to reach the same magnitude (they are theoretically identical for a 100% volume fraction).

Table 1 First simulation procedure results

f (%)	$\delta_{máx}$ (m)	$\Delta_{máx}$	$\delta_{máx}$ / $\Delta_{máx}$
30	2,47e-03	2,05e-04	12,08
50	3,37e-04	8,21e-05	4,11
70	1,02e-04	5,28e-05	1,94
90	6,50e-05	4,50e-05	1,45

Deformed shape and deflection magnitudes of reference and optimized beams for a volume fraction f = 30% are shown in Figure 4

a)

b)

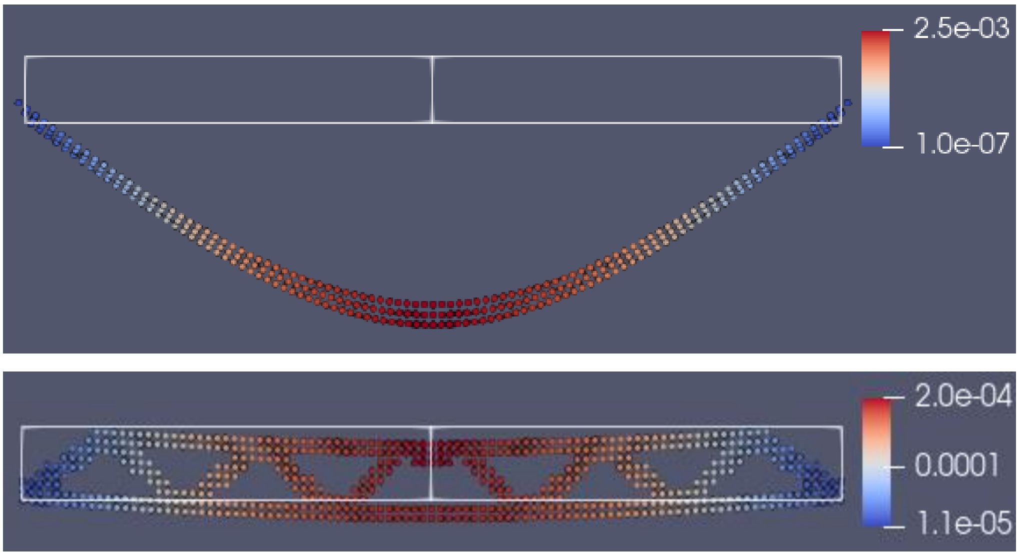

Figure 4. – Deformed shape and deflection color map for f = 30%.: a) reference beam; b) optimized beam

4 Conclusions

We found that, in relation to reference beams, optimized beams present smaller maximum deflections when we admit equal material, geometry and boundary conditions. We also found that differences in maximum deflection magnitudes are inversely proportional to the volume fraction. Finally, computing maximum deflection and plotting the deformed shape of optimized beams helps structural designers to improve decision-making and save material, since it is fundamental to analyze deformations of the resulting structures during the design process.

5 References

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