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Optimal power flow considering non-smooth generation cost function and emissions using AMPL and Knitro solver

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Abstract. The Optimal Power Flow (OPF) problem aims to analyze the planning and operation of electrical power systems and has been widely used due to the benefits that can be obtained. This work analyzed the application of language AMPL (Modeling Language for Mathematical Programming) and the Knitro commercial solver, to solve the Economic and Environmental Dispatch (EED) problem, modeled as an OPF. In the proposed OPF modeling were considered equality constraints such as active and reactive power balance and inequality constraints such as generator, transformer and Shunt VAR compensator constraints that represent the operational and physical limits of the system. The problem was implemented as single and combined objective functions aiming minimize the generation cost with and without valve point effect and emissions. These characteristics make the problem more complex, nonlinear, and non-convex. Additionally, a simple heuristic was proposed to deal with the discrete characteristic related to the value of transformer taps. The IEEE 30-bus test system was presented to illustrate the application of the proposed problem. Finally, the obtained data were compared with the literature and the superiority of the approach was demonstrated.

Keywords: Optimal Power Flow, Valve-Point Effect, Emissions, AMPL, Knitro.

1 Introduction

When analyzing electric power systems, economic aspects are usually considered one of the most important concern aspects by the system planners and operators. Economic Dispatch (ED) and Optimal Power Flow (OPF) are tools widely used to resolve these issues. However, the OPF when compared to DE, does not only consider the economic aspects, but also the operational and technical constraints [1].

An OPF adjusts control variables in the electric power system to optimize an objective function while satisfying a set of physical, operational and environmental constraints [2]. The OPF problem consists in a large-scale nonlinear, non-convex and highly constrained optimization problem [3].

The OPF has been extensively researched since the first studies proposed by Carpentier in 1962 [4]. Since then, many classical optimization and heuristics-based methods have been applied to solve OPF problem [1]. Among classical optimization methods are: Linear Programming (LP), Newton Methods (NM), Quadratic Programming (QP) Nonlinear Programming (NLP), Integer Programming (IP) and Interior Point Method (IPM) and some of the most recent heuristic-based methods studied are: modified Shuffle Frog Leaping Algorithm (MSFLA)[5], Genetic Algorithm with a new multi-parent crossover [3], Backtracking Search Optimization Algorithm (BSA) [6], Bat Algorithm (BA) [7] and Modified Crow Search Optimizer (MCSO) [8]. A comprehensive survey of various optimization methods used to solve OPF problems can be found in Pandya and Joshi [9] and Naderi et al. [10].

In nowadays, in the literature, almost all works study the application of heuristic-based methods to solve OPF problems that considering practical constraints and combined objectives, understanding that classical methods and commercial solvers cannot adequately handle with non-convexity and highly constrained characteristics imposed by this kind of problem [1]. Some of their shortcomings mentioned are: they do not guarantee finding the global optimum and classical methods involve complex calculations with long time [6]. However, the new generation of commercial solvers can provide, if properly modeled, a new possibility to deal with these problems.

This paper uses the AMPL language for modeling the OPF problem studied, an algebraic modeling language to describe high-complexity mathematical programming problems, developed by Robert Fourer, David Gay, and

Brian Kernighan in 1985. It supports a large amount of free and commercial solvers [11]. The solver that will be used with AMPL in this work is Knitro, a commercial solver for nonlinear optimization developed by Zienna Optimization LLC. It is designed for high level complexity problems, achieving great efficiency in solving linear, smooth quadratic optimization and nonlinear (convex and non-convex) problems. It has three problem solving methods: interior point algorithm with conjugated gradient, direct interior point algorithm, active-set methods [12].

Commercial solvers like Knitro use classic techniques to solve optimization problems. During the research for the development of this work, it was noticed that the application of commercial solvers to solve the OPF problem is still underexplored. A relevant publication that uses this approach is the paper written by Pourakbari-Kasmaei et al. in [1].

Therefore, the main contribution of this work can be considered the application of the AMPL language and the adoption of the commercial solver, both available in the laboratories of the Federal University of ABC, for solving a multi-objective OPF, considering the optimization of generation costs with valve-point effect, that raises the degree of nonlinearity to the problem making it significantly more challenging [13], and pollutant emissions that are increasingly taken into account in this kind of problem [14].

The remainder of the paper is organized as follows. First, the OPF formulation is presented in brief in section 2. Then, the main features of the computing implementation are presented in section 3. Next, the results after solving different cases of OPF problem using Discrete Optimal Power Flow (DOPF) and Continuous Optimal Power Flow (COPF) are discussed in section 4. Finally, conclusions are drawn in the last section of this paper.

2 Problem formulation

The OPF solves the power flow problem by providing the optimal adjustment of the control variables for a given load configuration, minimizing an objective function, such as the cost of active power generation, transmission losses and/or pollutant emissions. The FPO considers the operating limits of the system and in this work, it is formulated as a restricted, non-convex and multi-objective nonlinear optimization problem, according to the formulation as follows [15]:

Minimize:

$$f(x,u) \tag{1}$$

Subject to:

$$g(x,u) = 0 \tag{2}$$

$$h(x,u) \leq 0 \tag{3}$$

where $f(x,u)$ is the objective function to be minimized, $g(x,u)$ is the set of equality constraints, $h(x,u)$ is the set of inequality constraints, x represents the vector of dependent variables or state variables and u represents the vector of independent variables or control variables.

2.1 Control and state variables

Control variables are the set of variables that can be modified to satisfy the load flow equations, this set of variables in formulating an OPF problem include active power generation at PV buses except at the slack bus, voltage magnitude at the PV buses, tap settings of transformer and shunt VAR compensation. State variables are the set of variables which describe any unique system state and include injected active power at slack bus, voltage magnitude at load buses (PQ), reactive power generation of all generation units and line flow [6].

2.2 Equality constrains

The equality constraints correspond to the active and reactive power balance equations and are represented respectively according to eq. (4) and (5) [16]:

$$P_{G_k} - P_{L_k} = V_k \sum_{m \in K} V_m (G_{km} \cos\theta_{km} + B_{km} \operatorname{sen}\theta_{km}) \tag{4}$$

$$Q_{G_k} - Q_{L_k} + Q_k^{sh} = V_k \sum_{m \in K} V_m (G_{km} \operatorname{sen}\theta_{km} - B_{km} \cos\theta_{km}) \tag{5}$$

where P_{G_k} and Q_{G_k} are the injected active and reactive power at k bus by generators; P_{L_k} and Q_{L_k} are the active and reactive load demand at k bus; Q_k^{sh} are the shunt VAR compensation related with k bus, which can be fixed or variable; V_k is the voltage magnitude at k bus; V_m is the voltage magnitude at m bus; G_{km} , B_{km}: represents respectively the real and imaginary parts of element of the admittance matrix ($Y = G + jB$); θ_{km} : angular difference between k and m buses.

2.3 Inequality constrains

The set of system physical and operational limits that must be respected in the optimization process are represented through the inequality constraints [17] and it is given in this work as follows:

Generator constraints:

$$P_{G_k}^{min} \leq P_{G_k} \leq P_{G_k}^{max} \tag{6}$$

$$Q_{G_k}^{min} \leq Q_{G_k} \leq Q_{G_k}^{max} \tag{7}$$

Voltage magnitude constraints:

$$V_k^{min} \leq V_k \leq V_k^{max} \tag{8}$$

Transformer constraints:

$$t_{km}^{min} \leq t_{km} \leq t_{km}^{max} \tag{9}$$

Shunt VAR compensator constraints:

$$Q_{C_k}^{min} \leq Q_{C_k} \leq Q_{C_k}^{max} \tag{10}$$

where $P_{G_k}^{min}$, $P_{G_k}^{max}$, $Q_{G_k}^{min}$ and $Q_{G_k}^{max}$ are respectively the upper and lower limits of active and reactive power injected at k bus by the generators; V_k^{min} and V_k^{max} are respectively the upper and lower limits of voltage magnitude at k bus; t_{km} represents the regulating transformer tap setting in the km branch; t_{km}^{min} and t_{km}^{max} are respectively the upper and lower limits of regulating transformer tap settings in the km branch; $Q_{C_k}^{min}$ and $Q_{C_k}^{max}$ are respectively the upper and lower limits of reactive power injected by VAR compensators at k bus.

2.4 Objective function

An objective function may incorporate economic, safety or environmental aspects and are solved using appropriate optimization techniques. Some of the objective functions employed in the OPF problem are generation cost reduction (with and without multiple fuel options or valve point effect), optimization of active and reactive power transmission losses, reduction of pollutant emissions from generating units and voltage profile improvement [17]. Furthermore, in many cases, as seen in Bouchekara et al. [3], Chaib et al. [6], Shaheen et al. [8], Ghasemi et al. [18], and Elattar and ElSayed [19] the OPF problem can be studied with the intention of optimizing more than one objective simultaneously. In this situation, the problem is formulated as multi-objective and can also be called multi-criteria, multi-performance, or vector optimizations [20]. The formulation of the single and combined objective functions discussed in this work are presented below.

Generation cost reduction: the generation cost minimization objective function can be written as the sum of the generation costs of each generating unit i, as shown in eq. (12):

$$F_C = \sum_{i=1}^{NG} F_{G_i}\left(P_{G_i}\right) \tag{12}$$

where F_C represents the total operating cost function; F_{G_i} and P_{G_i} are respectively the cost function and active power output of the generating unit i; NG is the total of system generating units. In this work, for generation cost modeling of each generating unit i it was considered the widely used quadratic function, shown in eq. (13) [21]:

$$F_{G_i}\left(P_{G_i}\right) = a + b_i P_{G_i} + c_i P_{G_i}^2 \tag{13}$$

where a_i, b_i e c_i are operating cost coefficients of the generating units i. The constant a represents the fixed portion of generating units operating costs as labor and maintenance. The coefficients b and c represent the costs that are directly linked to the use of fuel.

Valve-point effect: to model the generation cost function more assertively, it is also necessary to consider real operating characteristics of the generating units, such as the valve-point effect [22]. These effects directly affect the fuel cost on thermal units. In practice, it occurs during the process of internal temperature control of the boiler and turbine during the opening of adjustment valves of these components, which causes the loss of steam and temperature that needs to be compensated by the equipment. During this cooling and heating process a ripple effect is produced in the curve. This effect can be represented mathematically as a sinusoidal function and can be added to the quadratic cost function, resulting in eq. (14).

$$F_{G_i}(P_{G_i}) = a_i + b_i P_{G_i} + c_i P_{G_i}^2 + \left| d_i \; sin(e_i(P_{G_i}^{min} - P_{G_i})) \right| \tag{14}$$

where d_i and e_i are coefficients associated with the valve-point effect of the generating unit i.

Emissions: in the equation adopted by this work and based on Chaib et al. [6], the total emission in ton/h of atmospheric pollutants such as sulfur oxides SO_x and nitrogen oxides NO_x caused by fossil fuel thermal units can be expressed as in eq. (15):

$$E = \left(\sum_{i=1}^{NG} 10^{-2} \left(\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2 \right) \right) + \left(\omega_i e^{(\mu_i P_{G_i})} \right) \tag{15}$$

where α_i, β_i, γ_i, ω_i e μ_i are coefficients of the emission characteristics of the generating units i.

In this work, the weighted sum strategy is used for the case that the generation cost function and emission function are combined. The equation that represents the combination of the two functions is presented in eq. (16).

$$F_T = \; F_C + \lambda_E E \tag{16}$$

where λ_E is weighting factor related to pollutant emissions, the value adopted was 1000 as in Chaib et al. [6].

3 Computing implementation

As previously mentioned, the proposed modeling was implemented in the programming language AMPL, and the optimization is achieved using the solver Knitro. The problem was formulated considering continuous values of the variables, which means that the variables can assume any value in the range defined by the limits, this methodology was named as COPF. However, in practice, the transformer tap settings assumes a discrete value contained in the limits. With the objective of evaluating the results, considering the discrete values of the taps, a heuristic was elaborated that allows to adjust the values of the taps in discrete values in a simple way, this methodology was denominated as DOPF. The methodology used is described in the flowchart in Fig. 1.

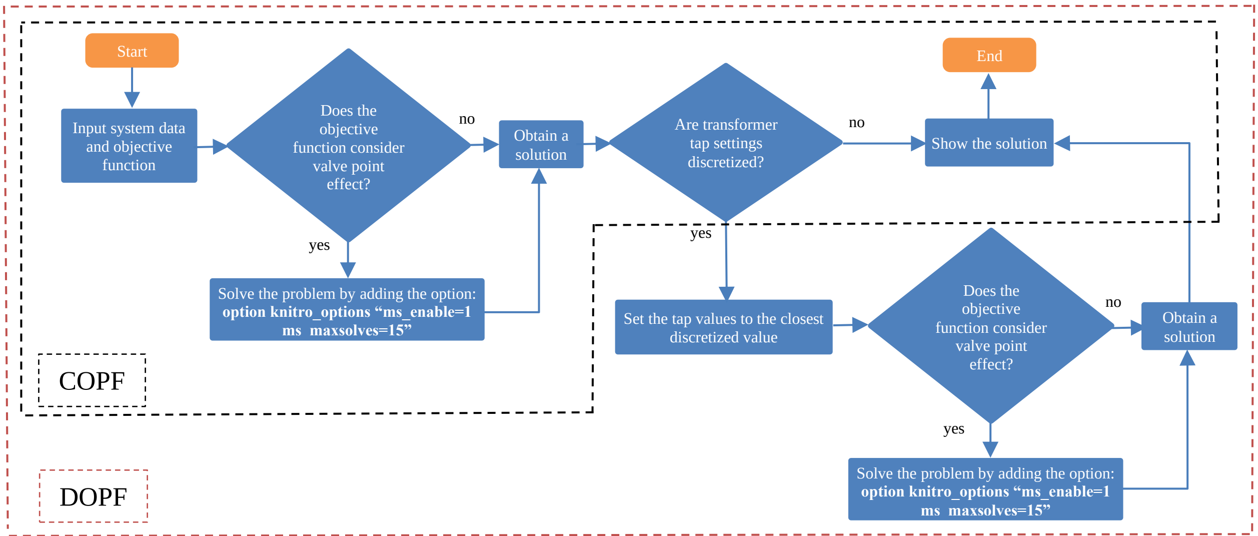

Figure 1: Flowchart of the methodology

To solve the problem, when considering the valve-point effect, the *ms_maxsolves* command was used to escape of local minimums. The above command makes the solution algorithm start at different points, with the best solution found being the final solution. After running a large number of tests, an adequate value of 15 entry points was reached for this parameter.

In addition, through the *alg* parameter it is possible to manually select which of the available techniques the solver will try to solve the problem, interior point algorithm with conjugated gradient, direct interior point algorithm or active-set methods. It is also possible to choose run all the techniques in parallel or use the automatic mode in which the solver automatically selects the technique according to the characteristics of the problem [23]. This parameter was set to automatic mode in this work.

These three algorithms are available in the Knitro package to offer different options for solving nonlinear programming problems, also allowing the possibility of interacting these methods during the solution (crossover), which provides greater flexibility in their use [23], as seen in Fig. 2.

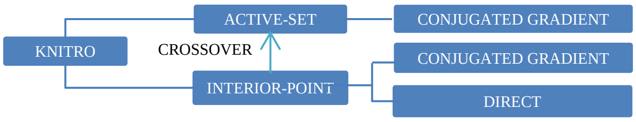

Figure 2: Knitro solution options

In the following chapter, the application and the results obtained by the proposed methods are presented.

4 Application and results

This chapter presents the results obtained from the application of the proposed modeling to the IEEE 30-bus test system in 3 different case studies that consider as objective function: cost (Case 1), cost with valve-point effect (Case 2) and cost with emission (Case 3). Simulations are performed on a computer at the Federal University of ABC with an Intel® Core™ i7-8700 processor, 2.93 GHz and 8.00 GB of RAM.

The IEEE 30-bus system is commonly used in the literature to perform this kind of analysis and represents a portion of the American Electric System, in the Midwest of the USA, in December 1961. Its data were provided by Iraj Dabbagchi of AEP and entered in the IEEE Common Data Format in 1993 by Rich Christie at the University of Washington [16]. The main characteristic and diagram of the test system are seen respectively in the Tab. 1:

Table 1. The main characteristics of IEEE 30-bus test system

System characteristics	Value	Details
Buses	30	-
Branches	41	-
Generators	6	Buses: 1, 2, 5, 8, 11 and 13
Shunts	9	Buses: 10,12, 15, 17, 20, 21, 23, 24 and 29
Transformers	4	Branches: 6-9, 6-10, 4-12 and 28-27

Cost and emission coefficients as well as active and reactive power limits of generators used in this paper are given in Tab. 2. For transformer discrete tap settings have been considered a step of 0.0125 following the methodology presented in Chaib et al. [6]. The other limits and characteristics of the system can also be obtained in Chaib et al. [6].

Table 2. Cost and emission coefficients and generators active and reactive power limits for IEEE 30-bus system

Gen.	a_i	b_i	c_i	d_i	e_i	α_i	β_i	γ_i	ω_i	μ_i	$P_{G_k}^{max}$ (MW)	$P_{G_k}^{min}$ (MW)	$Q_{G_k}^{max}$ (MVAr)	$Q_{G_k}^{min}$ (MVAr)
G_1	0	2.00	0.00375	18	0.037	0.04091	−0.0005554	0.000006490	0.000200	0.02857	250	50	0	0
G_2	0	1.75	0.01750	16	0.038	0.02543	−0.0006047	0.000005638	0.000500	0.03333	80	20	50	-40
G_5	0	1.00	0.06250	14	0.040	0.04258	−0.0005094	0.000004586	0.000001	0.08000	50	15	40	-40
G_8	0	3.25	0.00830	12	0.045	0.05326	−0.0003550	0.000003380	0.002000	0.02000	35	10	40	-10
G_{11}	0	3.00	0.02500	13	0.042	0.04258	−0.0005094	0.000004586	0.000001	0.08000	30	10	24	-6
G_{13}	0	3.00	0.02500	13,5	0.041	0.06131	−0.0005555	0.000005151	0.000010	0.06667	40	12	24	-6

The DOPF and COPF has been run for indicated cases and obtained results are presented in the Tab. 3. Table 3 also presents a comparative analysis of DOPF and COPF with the Backtracking Search Optimization Algorithm (BSA) meta-heuristic comparing the results obtained for cases one, two, three of this paper with respectively cases one, seven, ten of the paper written by Chaib et al. [6].

Table 3. Comparison of DOPF and COPF with BSA for solving the three different cases

VAR	CASE 1			CASE 2			CASE 3		
	BSA	**COPF**	**DOPF**	**BSA**	**COPF**	**DOPF**	**BSA**	**COPF**	**DOPF**
$\boldsymbol{P_{G_1}}$	177.3838	177.1108	177.1076	198.7273	200.0000	200.0000	112.9189	112.9431	112.9422
$\boldsymbol{P_{G_2}}$	48.8335	48.6898	48.6895	44.3031	43.0308	43.0323	59.3719	58.9625	58.9631
$\boldsymbol{P_{G_5}}$	21.2907	21.3030	21.3933	18.5637	18.5951	18.5942	27.6576	27.6178	23.4476
$\boldsymbol{P_{G_8}}$	21.0186	21.0245	21.0269	10.0000	10.0000	10.0000	34.9989	35.0000	35.0000
$\boldsymbol{P_{G_{11}}}$	11.4675	11.8569	11.8579	10.1017	10.0000	10.0000	27.0652	27.2429	27.2429
$\boldsymbol{P_{G_{13}}}$	12.0602	12.0000	12.0000	12.0000	12.0000	12.0000	26.4502	26.6407	26.6436
$\boldsymbol{V_{G_1}}$	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000	1.1000
$\boldsymbol{V_{G_2}}$	1.0806	1.0877	1.0877	1.0778	1.0863	1.0864	1.0855	1.0919	1.0920
$\boldsymbol{V_{G_5}}$	1.0545	1.0613	1.0613	1.0520	1.0590	1.0592	1.0606	1.0685	1.0687
$\boldsymbol{V_{G_8}}$	1.0633	1.0691	1.0691	1.0574	1.0652	1.0653	1.0757	1.0797	1.0799
$\boldsymbol{V_{G_{11}}}$	1.0946	1.1000	1.1000	1.0802	1.1000	1.1000	1.1000	1.1000	1.1000
$\boldsymbol{V_{G_{13}}}$	1.1000	1.1000	1.1000	1.0803	1.1000	1.1000	1.1000	1.1000	1.1000
$\boldsymbol{t_{6-9}}$	1.0250	1.0399	1.0375	1.0000	1.0430	1.0500	1.0000	1.0470	1.0500
$\boldsymbol{t_{6-10}}$	0.9000	0.9000	0.9000	1.0125	0.9000	0.9000	0.9500	0.9000	0.9000
$\boldsymbol{t_{4-12}}$	0.9625	0.9782	0.9750	1.0250	0.9819	0.9875	1.0000	0.9810	0.9875
$\boldsymbol{t_{28-27}}$	0.9625	0.9614	0.9625	1.0000	0.9616	0.9625	0.9625	0.9661	0.9625
$\boldsymbol{Q_{C_{10}}}$	4.2998	5.0000	5.0000	4.3411	4.9999	5.0000	3.4844	5.0000	5.0000
$\boldsymbol{Q_{C_{12}}}$	4.6378	5.0000	5.0000	4.9527	4.9999	5.0000	4.5129	5.0000	5.0000
$\boldsymbol{Q_{C_{15}}}$	4.9106	5.0000	4.9904	4.2358	5.0000	4.9504	4.7990	4.6854	4.9944
$\boldsymbol{Q_{C_{17}}}$	5.0000	5.0000	5.0000	4.7605	4.9999	5.0000	4.9965	5.0000	5.0000
$\boldsymbol{Q_{C_{20}}}$	4.0889	4.2874	4.3019	4.0597	4.3171	4.3053	3.9809	3.9850	4.0488
$\boldsymbol{Q_{C_{21}}}$	5.0000	5.0000	5.0000	4.5901	5.0000	5.0000	4.7684	5.0000	5.0000
$\boldsymbol{Q_{C_{23}}}$	3.1843	2.6955	2.7225	4.1971	2.7595	2.7372	3.8535	2.5117	2.5197
$\boldsymbol{Q_{C_{24}}}$	4.8423	5.0000	5.0000	5.0000	5.0000	5.0000	4.2332	5.0000	5.0000
$\boldsymbol{Q_{C_{29}}}$	2.5810	2.3079	2.4231	4.1450	2.3295	2.2462	1.6339	2.1516	1.8879
$\boldsymbol{F_{G_i}}$($/h)	799.0760	**798.9152**	**798.9160**	830.7779	**829.6830**	**829.6850**	835.0199	**834.9510**	**834.9623**
$\boldsymbol{VD}$ (p.u.)	1.9129	2.4778	2.4901	1.2050	2.4016	2.3657	1.9214	2.5931	2.5889
Losses (MW)	8.6543	8.5851	8.5850	10.2908	10.2260	10.2265	5.0626	5.0071	5.0095
$\boldsymbol{E}$ (ton/h)	0.3671	0.3663	0.3663	0.4377	0.4425	0.4425	0.2425	**0.2425**	**0.2423**
CPU time (sec)	-	0.007	0.024	-	0.012	0.016	-	0.014	0.031
Iterations	-	13	12	-	24	24	-	12	12

Observing the Tab. 3, it is possible to verify that for case 1 the COPF and DOPF obtained values for the generation cost of 798.9152 $/h and 798.9152 $/h respectively, for case 2 it obtained values for the generation cost with valve-point effect of 829.6830 $/h and 829.6850 $/h respectively and for case 3 it obtained for the generation cost and pollutant emissions the values of 834.9510 $/h, 0.2425 ton/h , 834.9623 $/h and 0.2423 ton/h respectively. While the BSA presented by Chaib et al. [6] reached 799.0760 $/h for case 1, 830.7779 $/h for case 2, 835.0199 $/h and 0.2425 ton/h for case 3. Demonstrating that the COPF and DOPF performed better than BSA for all cases. Another point to be highlighted is the implementation processing performance, which for all cases was less than 1 second and with a maximum of 24 interactions, indicating a great performance. Therefore, the techniques studied by this paper, based on commercial solver, showed promising results when compared to a state-of-art meta-heuristic.

5 Conclusions

The obtained results show that the technique studied in this paper using AMPL and the commercial Knitro solver, both for CFPO and DFPO modeling, achieved promising results for solving the OPF problem, considering generation cost optimization, generation cost with valve-point effect and generation cost combined with emissions when compared to a state-of-the-art meta-heuristic such as the BSA.

We can also conclude that the successful application of a technique based on mathematical programming and a commercial solver to solve single and multi-objective OPF problem, as presented by this work, comes as a highly promising alternative for the operation and planning of the SEPs. Another point to be highlighted is the contribution of this work in expanding the field of study related to solving simple and multi-objective OPF problems, which, as it was possible to verify in the bibliographic review of this work, has in recent times focused on the development and application meta-heuristic methods.

It is important to emphasize that this article is based on the studies developed for the elaboration of the dissertation of the master's program in Electrical Engineering at the Federal University of ABC.

As suggestions for future work, a study related to the use of AMPL in conjunction with Knitro solver or other commercial solver to solve the OPF problem can be considered, including in its modeling realistic restrictions of multiple fuels and prohibited zones of operation. And the implementation of a hybrid algorithm considering the association of a meta-heuristic with a commercial solver to improve the search process.

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