Topology optimization of 3D truss structures considering stress, displacement and buckling constraints

Verônica C. H. Pazda[1], Eduardo L. Cardoso[1]

[1]*Mechanical Engineering Department, UDESC*
Rua Paulo Malschitzki, 200, Joinville, SC 89.219-710, Brazil
veronica.pazda18@edu.udesc.br, eduardo.cardoso@udesc.br

Abstract. This work addresses the development of 3D optimal trusses using the topology optimization method. The aim is to minimize the total volume with local stress and displacement constraints as well as buckling constraints. The augmented Lagrangian method is also implemented to solve the optimization problem. The formulation and the implementation are assesed by means of some benchmark problems found in the literature. Results also show the importance of properly selecting the material parametrization.

Keywords: Topology Optimization, Stress, 3D Truss, Displacement, Constraint.

1 Introduction

In engineering, a structural design must satisfy the design requirements. Thus, the structure is able to fulfill a set of functions during its useful life. In this environment, structural optimization is highlighted, aiming at structural reliability and economic viability. In deterministic structural optimization problems, the objective function is usually the volume or weight of the structure, and constraints are stress and displacement, Verzenhassi [1].

In the midst of structural optimization, one can mention the topology optimization. According Bendsøe and Sigmund [2], the purpose of topology optimization is to find the optimal lay-out of a structure within a specified region. For that, the only known quantities in the problem are the applied loads, the possible support conditions and some additional design constraints.The optimization of truss structures has been part of the study of several authors over the years, since the pioneering work of Dorn et al. [3], Fleron [4], to current works such as Guilherme [5], where buckling and flexibility constraints were adopted.

For problems involving stress constraints, qp relaxation is commonly used. On the report of Bruggi [6], stress constraints imposition has always been one of the more challenging topics in structural optimization. The formulation from this kind of problem may end up producing feasible sets of equations that are, in general, nonconvex and may include a degenerate subdomain with zero measure. An efficient way manage with the stress singularity problem is to act on the stress constraints, relaxing the degenerate zones. Other characteristics of topology optimization are the high number of elements, and the nonlinearity of constraints.

Local buckling constraints were addressed in works such as Zhou [7] and Rozvany [8]. For truss optimization problems, some inconveniences are still found. In the present work, local buckling is given by Euler's critical load, where it is defined based on the length of the bars. The topology optimization may remove some bars of the ground structure. This effect can result in the algorithm miscalculating the true Euler buckling load, resulting in a poor optimum solution, Torii et al. [9]. Therefore, a relaxation was used for the buckling constraints.

Thus, topological optimization is used in order to minimize the volume of three-dimensional truss structures, subject to local stress, displacement and buckling constraints. The Finite Element Method (FEM) is used for the discretization of the problem. Furthermore, the optimization is written in terms of the Augmented Lagrangian method, grouping the objective function and the local stress, displacement and buckling constraints into a single functional. Algorithm validation will be performed by analyzing two classical optimization problems.

2 Topology optimization

Topology optimization allows the simultaneous determination of topology and shape of a structure contained in a given region of space, the Design Domain. The optimization problem is usually composed of three elements:

objective function, design variables and functional constraints. The effective Young's module in each element is assumed as $E_j = x_j^p E_j^0$, where x_j is the design variable associated to element j and E_j^0 is the base value for the element. The problem is set as

$$P \begin{cases} \underset{\mathbf{x}}{\text{Min}} & V(\mathbf{x}) = \sum_{j=1}^{n_e} x_j A_j L_j H_j(\mathbf{x}) & \\ \text{S.t.} & \mathbf{K}(\mathbf{x})\mathbf{U}(\mathbf{x}) = \mathbf{F} & \\ & g_j^\sigma(\mathbf{x}) = \frac{\sigma_{eq_j}(\mathbf{x})}{s^\sigma \overline{\sigma}} - 1 \leq 0 & j = 1..ne \\ & g_j^b(\mathbf{x}) = -\frac{A_j \sigma_j(\mathbf{x}) s^b}{P_j(\mathbf{x})} - 1 \leq 0 & j = 1..ne \\ & g_j^u(\mathbf{x}) = \left(\frac{u_j(\mathbf{x})}{\overline{u}_j}\right)^2 - 1 \leq 0 & j = 1..nrd \\ & \underline{x}_i \leq x_i \leq \overline{x}_i & i = 1..ne \end{cases}, \tag{1}$$

where ne is the number of elements, nrd is the number of displacement constraints, g_j^σ are the stress constraints, g_j^b are the buckling constraints, g_j^u the displacement constraints and $\underline{\mathbf{x}}$ and $\overline{\mathbf{x}}$ are the side constraints. A_j is the area of element j, L_j is the length of element j. Equilibrium equation $\mathbf{K}(\mathbf{x})\mathbf{U}(\mathbf{x}) = \mathbf{F}$ is solved by using the Finite Element Method, where $\mathbf{K}$ is the stiffness matrix, $\mathbf{U}$ is the displacement vector and $\mathbf{F}$ is the force vector. Furthermore, σ_{eq_j} is the equivalent stress, s^σ is the safety factor with respect to stress, $\overline{\sigma}$ is the yield stress, σ_j is the normal stress, s^σ is the safety factor for buckling, P_j is the Euler's critical load and $\overline{u}_j$ is the upper limit for the displacement constraint.

Function $H_j(\mathbf{x}) = 1 - e^{-bx_j} + x_j e^{-b}$ is a Heaviside like function used to avoid intermediate results (bars with intermediate values of x) in the final design, where b is a "sharpness" parameter.

2.1 Stress relaxation

Normal stress at element j is $\sigma_j(\mathbf{x}) = \Psi_j(\mathbf{x})\sigma_j^0$ where Ψ_j is the relaxation proposed by Bruggi [6], where

$$\sigma_j^0 = E_j^0 \mathbf{B}_j \mathbf{R}_j \mathbf{H}_j \mathbf{U} \tag{2}$$

is the nominal stress, $\boldsymbol{B}_j$ is the displacement strain matrix, $\boldsymbol{R}_j$ is the rotation matrix and $\boldsymbol{H_j}$ is the local-global localization matrix of element j. The equivalent stress σ_{eq_j} is given by

$$\sigma_{eq_j}(\mathbf{x}) = \sqrt{\sigma_j^2(\mathbf{x}) + \epsilon^2}, \tag{3}$$

where the term ϵ is used to avoid singular values in the sensibility analysis ($\epsilon = 1 \times 10^{-6}$ is used in this work).

2.2 Critical load Relaxation

Local buckling constraints depend on local stresses and the critical load of each element. It is assumed that the stress is relaxed as discussed in the previous subsection. Nonetheless, it is also important to update the critical load as it is also a function of the effective Young's modulus (design variables). The critical load P_j is defined as

$$P_j(\mathbf{x}) = \Phi_j(\mathbf{x}) P_j^0, \tag{4}$$

where $\Phi_j = \mathbf{x}_j^r$ is the proposed relaxation function for critical loads and

$$P_j^0 = \frac{E_j^0 I_{z_j} \pi^2}{L_j^2}, \tag{5}$$

where I_{z_j} is the moment of inertia of element j.

2.3 Augmented Lagrangian method

The augmented Lagrangian function is described in Birgin and Martínez [10]. The function consists of transforming the constrained problem into an equivalent unconstrained problem P^k, where k is an outer iteration, Heiden [11]. Thus, the new objective function is given by a functional that contains the original objective, the constraints, the Lagrange multipliers $\boldsymbol{\mu}^\sigma$, $\boldsymbol{\mu}^b$, $\boldsymbol{\mu}^u$ and a penalty parameter c.

$$P^k \begin{cases} \underset{\mathbf{x}}{\text{Min}} \quad \mathcal{L}_A^k = V(\mathbf{x}) + \frac{c^k}{2}\left\{ \sum_{j=1}^{ne} \langle \frac{\mu_j^{\sigma\,k}}{c^k} + g_j^\sigma \rangle^2 + \sum_{j=1}^{ne} \langle \frac{\mu_j^{b\,k}}{c^k} + g_j^b \rangle^2 + \sum_{j=1}^{nrd} \langle \frac{\mu_j^{u\,k}}{c^k} + g_j^u \rangle^2 \right\} \\ \text{S.t.} \qquad \underline{\mathbf{x}} \le \mathbf{x} \le \overline{\mathbf{x}} \end{cases}, \tag{6}$$

where $\langle a \rangle = \max(0.0, a)$. Initial outer iteration $k = 0$ starts with a given penalty parameter $c^0 > 0$, null Lagrange multipliers and an user-provided initial point $\mathbf{x}^0$. Each sub problem Eq. (6) is then solved, returning $\mathbf{x}^k$. Multipliers are updated according to

$$\begin{aligned} \mu_j^{\sigma\,k+1} &= \langle \mu_j^{\sigma\,k} + c^k g_j^\sigma(\mathbf{x}^k) \rangle, \;\; j = 1..ne \\ \mu_j^{b\,k+1} &= \langle \mu_j^{b\,k} + c^k g_j^b(\mathbf{x}^k) \rangle, \;\; j = 1..ne \\ \mu_j^{u\,k+1} &= \langle \mu_j^{u\,k} + c^k g_j^u(\mathbf{x}^k) \rangle, \;\; j = 1..nrd \end{aligned} \tag{7}$$

and penalty parameter is updated as $c^{k+1} = \gamma x^k$, where $\gamma = 1.1$ is used in this work. Sensitivities of $\mathcal{L}_A$ with respect to $\mathbf{x}$ were obtained analytically (adjoint method) and a simple Steepest descent with side constraints is used to solve the inner problem.

Parameter b in the objective function is updated at each outer iteration (k) as

$$b = \begin{cases} 7.0, & \text{if } k \le 12 \\ 12.0, & \text{if } 12 < k \le 15 \\ b = b + 5.0, & \text{if } k > 15 \end{cases}, \tag{8}$$

for the problems studied herein.

3 Results

Two benchmark problems found in the literature were adapted and used to assess the proposed formulation Weldeyesus et al. [12]. Both examples use the same data: $E^0 = 210 \times 10^9$ Pa, $A = 7.8 \times 10^{-2}$ m^2, $\overline{\sigma} = 100 \times 10^6$ Pa and $s^\sigma = s^b = 1.0$. The same initial point $\mathbf{x}^0 = \mathbf{1}$ is used in all examples. Design variables $\boldsymbol{x}$ are constrained to be within the range $[1 \times 10^{-3}, 1]$, where the lower value corresponds to "void" and the upper value to base material.

Example 1: A "bracket" with dimensions of $2 \times 3 \times 6$ m, as shown in Fig. 1a, with initial volume 1.657m^3. A point load with magnitude $100kN$ is applied at $(1, 3, 3)m$. A displacement constraint is set to node located at $(1, 3, 3)$m, with limit value $\overline{u} = 1 \times 10^{-3}$m in the z direction, along with local stress and buckling constraints. Exponent $r = 0.25$ was used for relaxation of critical loads, $p = 3.0$ for stiffness and $q = 1.5$ for stress relaxation.

Example 2: A "tower" with dimensions of $2 \times 2 \times 4$ m, as shown in Fig. 1b, with initial volume 0.780m^3. A point load with magnitude 1000kN is applied at $(1, 1, 4)$m. Displacement constraint is set to the node located at $(1, 1, 0)$m in the z direction, limit value $\overline{u} = 1 \times 10^{-2}$m, along with local stress and buckling constraints. Exponent $r = 0.25$ was used for relaxation of critical loads, $p = 3.0$ for stiffness and $q = 2.5$ for stress relaxation.

Figure 2a and Fig. 2b show the optimized structures. Bars with $x = 1$ are shown as solid elements and bars with $x = 1 \times 10^{-3}$ are not relevant to the final design. An important aspect of the present formulation is the fact that all design variables are constrained at the optimized solution. Figure 3a and Fig3b, show the normal forces of the optimized solutions.

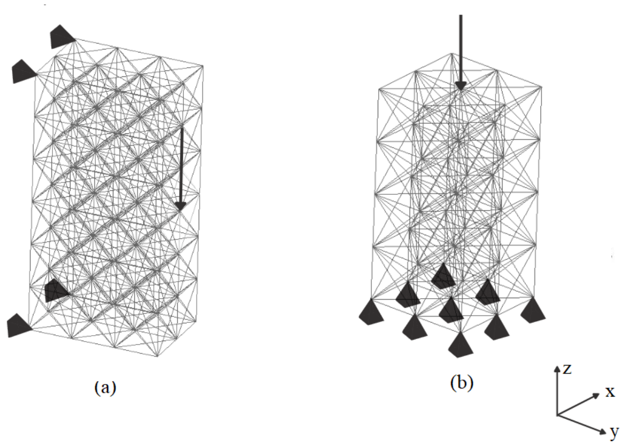

Figure 1. (a) Bracket problem (example 1); (b) tower problem (example 2).

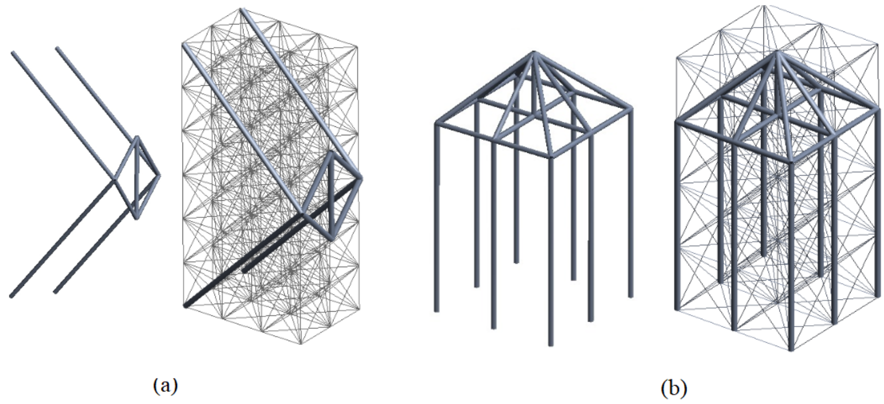

Figure 2. Optimized topologies for example 1 (a); example 2 (b).

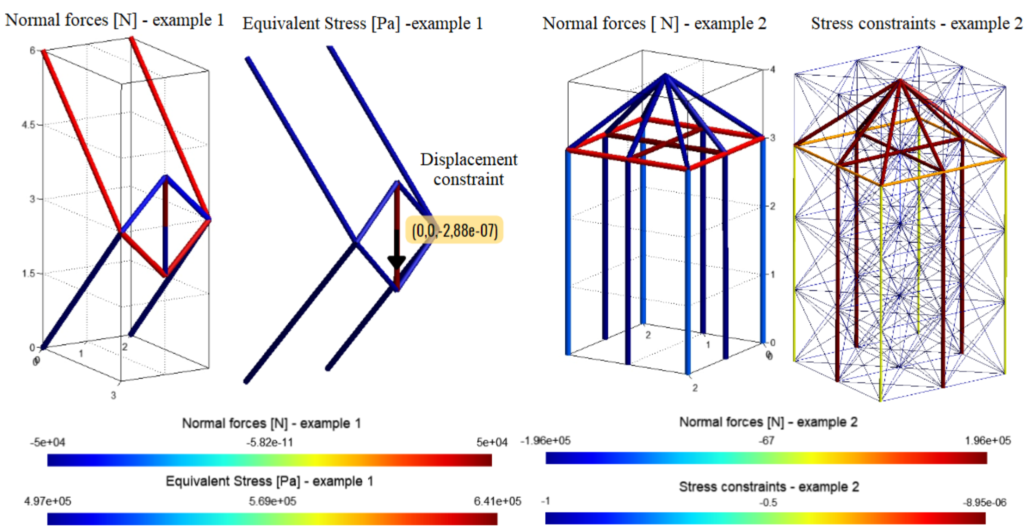

Figure 3. Additional information for example 1 (a); example 2 (b).

Both results are feasible. The "bracket" has an optimized volume of $0.040\mathrm{m}^3$ and the displacement constraint is active at the optimized solution. Figure 3a shows the displacement constraint and the equivalent stress of this structure.

The "tower" resulted in a volume of $0.167\mathrm{m}^3$. The stress constraints are active at the optimized solution, according Fig. 3b. The buckling constraints in some of the upper bars are also active. It was found that for $r \in [0.1 - 0.4]$ the optimization result has the behavior denoted in Fig. 2b. For values greater than this interval, the optimized structure tends to incorporate intermediate bars in its central position and in the upper portion of the structure.

4 Conclusions

Volume minimization of 3D truss structures subjected to stress, buckling and displacement constraints is discussed in this work. Two benchmark problems found in the literature were studied. The results are feasible with no presence of intermediate design variables, showing the effectiveness of the formulation. The initial parameters of the problem and the values of the relaxation constants influenced the final topology of both problems.

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