

# Degradation prediction of in-service railway bridges supported by Semi-Markov process

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**Abstract.** This article describes a model for predicting the degradation of in-service railway bridges based on a semi-Markov continuous time process. This model relies on the history of inspections of 588 bridges located on a heavy-haul railway line in Brazil, between 2016 and 2020. A dedicated computational tool developed in Matlab allows the automated data processing. A parametric study is performed to understand which factors derived from the bridge structural characteristics, as well as operational and environmental factors, most influence the deterioration model. The type of material proves to be a decisive factor and therefore two specific prediction models are stablished, one for concrete bridges and other to steel bridges. The prediction models have an efficiency equal to 93.7%, for concrete bridges, and 95.1% for steel bridges.

**Keywords:** bridge management system; heavy-haul railways; deterioration models; semi-Markov continuous time process; parametric study; validation; forecast analysis.

## **1** Introduction

Over the last few decades Bridge Management Systems (BMS) have been increasingly used by infrastructure managers to support decisions concerning the maintenance or replacement of railway bridges. The decision of repair or replace is closely related to the extent of the structural deterioration, which in most situations are irreversible, despite the use of advanced construction methods, dedicated code-design specifications, and high-performance materials [1].

BMS have been developed using different approaches depending on the countries. Some infrastructure managers resort to commercial solutions, others use modified commercial solutions according to their own specificities [2]. A detailed comparison of various BMS is carried out in several works [3-11]. The current condition evaluation is normally supported by periodic visual or remote inspections using dedicated condition ratings (CR). Otherwise, the prediction of the future condition involves the use of accurate deterioration models [3].

Concerning the CR, the normal procedure adopted during the inspections is based on the assignment of a number to a specific bridge element property, according to a deterioration severity scale, representing the opinion of the qualified experts. This approach is somehow subjective and does not require calculations. From the practical point-of-view, the evaluation of the condition of a bridge element is derived from the rating of distinct properties, not only structural but also non-structural, which are latter aggregated into a single condition rating. This CR is used to decide the appropriate intervention strategy.

In what concerns the deterioration models, several studies have been carried out to understand the deterioration behavior of bridges. Bridge deterioration models can be classified as deterministic and stochastic models [12]. More recent approaches use Artificial-Intelligent (AI) models and hybrid models.

This study gives relevant contributes for the state-of-art since applies stochastic models to the deterioration prediction of in-service railway bridges specifically dedicated to freight traffic and based on an extended inspection archive composed by approximately 600 bridges, with distinct typologies, materials and ages. In relation to previous works, a relevant contribute is achieved, particularly with the assessment of the reliability of the prediction model based on the bridge's CR measured during recent inspections carried out in the year 2021. Thus, this work constitutes the first dedicated study applied to the Brazilian freight railway network, and therefore, will provide a relevant contribute for the decision-making process of the infrastructure managers concerning

#### 2 Finite-state Markov chain

The infrastructure managers need to closely follow-up the progression of the CR as well as predict the future condition. This procedure is often performed based on a Markov chain, which is able to consider the several processes involved in bridge's deterioration, and therefore, is traditionally more reliable than a simple deterministic extrapolation of the CR [3].

Markov processes are stochastic, i.e., random processes that characterize the system behavior over the time. Since it is possible to vary both the condition rating and the time, these processes can be discrete or continuous in state and in time. A finite-state Markov process describes the evolution of a finite number of state conditions for

$$P(X_{t+1} = i_{t+1} | X_t = i_t, X_{t-1} = i_{t-1}, \dots, X_1 = i_1, X_0 = i_0) = P(X_{t+1} = i_{t+1} | X_t = i_t)$$
(1)

which the Markov properties are adjusted. In this specific Markov process, given the current condition rating, the future condition rating is assumed to be independent from the previous states. This can be expressed by a discrete stochastic process X(t) with a discrete state-based model as:

where  $i_t$  is the condition rating at time t and P is the conditional probability of any future event given the present and past events. In a state-based model, considering a scale with 5 finite CR. i.e.,  $S_i$  (i = 1, 2, ..., 5), as stated in Section 3.2, the Markov P matrix will be:

$$P = \begin{pmatrix} S_5 & S_4 & S_3 & S_2 & S_1 \\ S_5 & S_4 & S_5 & P_{55} & P_{53} & P_{52} & P_{51} \\ 0 & P_{44} & P_{43} & P_{42} & P_{41} \\ 0 & 0 & P_{33} & P_{32} & P_{31} \\ 0 & 0 & 0 & P_{22} & P_{21} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(2)$$

where  $P_{ij}$  is the probability of the current  $CR_i$  progress to  $CR_j$  after the time interval  $\Delta t$ , and  $P_{ii}$  is the probability of the current  $CR_i$  remain in the same rating for time interval  $\Delta t$ . Matrix *P* is subjected to the following conditions [13-14]:

- 1.  $0 \le P_{ij} \le 1$ , with *i* and j = 1, 2, ..., n, where *n* is the total number of CRs
- 2.  $\sum_{i}^{n} P_{ii} = 1$  with i = 1, 2, ..., n
- 3.  $P_{ij} = 0$  for i < j,

In predicting the future condition by a Markov deterioration model this indicates that a bridge at a certain CR deteriorates naturally without improving its CR and considering no maintenance operation within the time period  $\Delta t$ . Another aspect to be considered in Markov matrices is the fact that CR transitions are always step-by-step.

Markov's first-order matrices are the result of stochastic processes that have an important property denominated *memoryless*. This allows to predict the future CR of bridges with several decades of existence even if there are no inspections records performed periodically until the current CR [12, 14-18].

Given the probabilities vector  $p(t_i)$ , containing the various CR probabilities at the initial instant  $t_i$ , and considering Markov P matrix that reproduces the deterioration of the bridge on the time interval  $\Delta t$ , it is possible to predict the future CR for the final instant  $t_f$ , by means of the probabilities of CR  $p(t_f)$  [19].

A difference exists between a discrete-time Markov process and a semi-Markov process. A discrete-time Markov process performs transitions on discrete equally spaced time steps and the use of this method requires at least two consecutive CR records (without any maintenance interventions) for several bridge components to guarantee reliable transition probabilities [16].

A semi-Markov process is an extension of a discrete-time Markov process in which a random time is added between transitions [20]. In a continuous-time process, the optimization of the model involves estimating Qintensity matrix, which represents a matrix of transition rates (independent of  $\Delta t$ ) and directly related to P Markov matrix. The relation between P Markov matrix and the Q intensity matrix, both with the same dimensions, is given by Eq. (3) [20-21]:

$$P_{\Lambda t} = e^{(Q \times \Delta t)} \tag{3}$$

The solution of this first order differential equation, called Chapman – Kolmogorov equation, is given by Eq. (4) [20]:

$$\frac{\partial}{\partial t}P_{\Delta t} = P_{\Delta t}Q \tag{4}$$

In case of 5 different CRs, the Q intensity matrix is of dimension 5 and expressed by:

$$Q = \begin{bmatrix} -\theta_5 & \theta_5 & 0 & 0 & 0\\ 0 & -\theta_4 & \theta_4 & 0 & 0\\ 0 & 0 & -\theta_3 & \theta_3 & 0\\ 0 & 0 & 0 & -\theta_2 & \theta_2\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \begin{bmatrix} \theta_5\\ \theta_4\\ \theta_3\\ \theta_2 \end{bmatrix} = \begin{bmatrix} Q_{54}\\ Q_{43}\\ Q_{32}\\ Q_{21} \end{bmatrix}$$
(5)

the Q intensity matrix should satisfy the following conditions [21]:

- 1.  $Q_{ij} \ge 0$  for  $i \ne j$ , where  $Q_{ij}$  represent the probability rate of transition between consecutive CRs *i* and *j* 2.  $Q_{ii} = -\sum_{j \ne i} Q_{ij}$ , i = 1, ..., k where *k* is the number of CRs
- 3.  $Q_{ii} = 0$  for i < j

4. 
$$\theta_i = O_{ii}$$

An approach for the calculation of  $\theta_i$  is the following [22-23]:

$$\theta_i = Q_{ij} = \frac{N_{ij}}{\sum \Delta t_i} \tag{6}$$

where  $\theta_i$  and  $Q_{ij}$  represent the probability of transition between consecutives condition ratings *i* and *j*,  $N_{ij}$  is the number of bridges that progress from  $CR_i$  to  $CR_j$ , and  $\sum \Delta t_i$  is the sum of the time intervals between the observations beginning from the initial CR. The values of  $\theta_i$  that form the matrix Q ( $k \times k$ ) are essential to define the efficiency of the prediction model. This efficiency is measured calculating the likelihood value, which represents a distance between the real conditions ratings and the ratings estimated by the model. The most efficient models are the ones that present lower values of the likelihood distance. Thus, the likelihood distance (*L*) is given by Eq. (7) and can be rearranged considered as a sum of logarithmic terms[21]:

$$L = \prod_{i}^{N} \prod_{i}^{D} P_{ij} = \sum_{i}^{N} \sum_{i}^{D} \ln P_{ij}$$
<sup>(7)</sup>

where N is the total number of bridges, D is the total number of time intervals per bridge and  $P_{ij}$  the probability of the structure transition from condition state *i* to *j*.

The likelihood distance can be minimized by means of a constrained nonlinear optimization problem, where the Q matrix that best adapts to the target condition ratings is estimated. The maximum likelihood was obtained using a Matlab<sup>®</sup> function called *fmincon* and considering as restrictions  $\theta_i \ge 0$  and |L| being as small as possible.

Finally, sojourn time  $T_i$  that the bridges remain in a certain CR before progressing to the next CR can be estimated by Eq. (8) [23]:

$$T_i = \frac{1}{\theta_i} \tag{8}$$

where i = 1, ..., k and k = number of CR – 1.

## 3 Case study - Brazilian freight railway network

The case study involves the bridges of the freight railway line connecting Rondonópolis terminal (Mato Grosso state) to the largest seaport in Latin America, port of Santos (São Paulo state), in Brazil. This railway line is operated by RUMO and includes a total of 594 in-service concrete or steel bridges, however, only 588 bridges were considered for this study corresponding to the ones with 2 or more inspection reports. The study was based on an extended database containing the history of inspections carried out on the bridges, which are performed on an annual basis, according to the Brazilian standard NBR-9452 requirements [24] which considers five discrete CR, from 1 to 5, being 1 the worst condition and 5 the best condition.

However, this standard presents some limitations since it was designed specifically for concrete structures and do not consider some specificities related to freight railway bridges, where, for example, the serviceability is related to the staff safety on sidewalks (and not to passengers' comfort), and are typically located in regions with difficult accessibility for maintenance actions, unlike the roadway bridges.

A computational tool was developed in Matlab® software and allows to perform quick analyzes based on information derived from the bridges' inspections, as the Markov probabilistic matrix, obtained from the data collected from all bridges, or specific probabilistic matrices considering restricted sets of bridges according to the material, bridge typology, age, number of spans, transported weight per year (MGT – Millions of Gross Tons) and environmental aggressiveness. The tool works from an Excel® database containing the CRs and the corresponding inspection dates, as well as several structural and operational information. The data can be filtered according to previously selected parameters, particularly, type of material, age, total length, number of spans, MGT and class of environmental aggressiveness. Then, several Q intensity matrices are calculated by the maximum likelihood approach and visualized in a dedicated graphical user interface, which is presented in Fig. 1.



Fig. 1 – Graphical user interface of the computational tool developed in Matlab®. i) the average deterioration curves, considering the specific user's definitions, and considering or not the maintenance, ii) the sojourn time of the bridges in a specific CR, and iii) the probability of the bridge network remain in a specific CR.

### 4 Parametric study

The parametric study allows to understand which factors derived from the bridge structural characteristics, as well as operational and environmental factors, most influence the deterioration prediction model. Seven factors were analyzed, particularly, the material, age, total length, number of spans, bridge typology, transported cargo weight (in MGT) and the environmental conditions. The analyzes were performed for three different time moments, the current time (simply considered 08.01.2021), the time of the end of concession of the railway operator (01.01.2058) and the time of 100 years (01.01.2121) corresponding to the typical lifespan for railway bridges. Fig. 2 presents the average deterioration curves, in terms of condition rating, derived from the intensity

matrices of the 433 concrete bridges ( $Q_c$ ) and 155 steel bridges ( $Q_s$ ). It is possible to observe, that the average degradation model of the concrete bridges, which condition ratings varies between CR3 and CR1, is distinct from the average degradation model of the steel bridges, which condition ratings varies between CR3 and CR2.

Additionally, the initial condition state of the steel bridges (2.69) is worst in comparison to the concrete bridges (3.29), however, over the time, the degradation of the concrete bridges is more pronounced in comparison to the steel bridges, which presents a horizontal trend over the time. Anyway, as expected, regardless the material type of the bridge, the CR condition decreases over the years. Another interesting aspect to be highlighted is related to the achieved long-term CR, which reveals that the steel bridges will keep on CR2, while the concrete bridges can achieve the CR1. Based on these results, it is quite evident that for further analysis the use of two distinct deterioration models, one for concrete bridges and another for steel bridges, is required.



Fig. 2 – Deterioration curves depending on the type of bridge superstructure material (Concrete or Steel)

#### 5 Model validation

The Semi-Markov prediction model based on continuous time was validated based on the comparison between predicted and real verified deterioration ratings. For this purpose, the history of inspections carried out between 2016 and 2020 were used to develop the prediction model, while the model performance was evaluated for the period between January and October 2021. During these 9 months period, the condition ratings values derived from inspections on 588 bridges (433 in concrete and 155 in steel) were compared with the predictions of the deterioration model, to verify its efficiency and reliability. Table 1 presents an excerpt of the summary of inspections performed between January and October 2021.

| Bridge<br>Code | Last<br>Inspection<br>Date | Material<br>(Bridge<br>superstructure) | Age     | Total<br>Length | Nr. of<br>spans | Bridge Type              | MG<br>T<br>year | Envir<br>. Agg | Real<br>CR (given by<br>inspector) |
|----------------|----------------------------|--|---------|-----------------|-----------------|--------------------------|-----------------|----------------|------------------------------------|
| A3             | 05/01/2021                 | Steel                                  | ]25-50] | > 100           | $\geq$ 4        | A – Box/twin girders     | $\leq$ 30       | IV             | 2.00                               |
| A6             | 02/01/2021                 | Concrete                               | > 50    | ]50-100]        | 3               | T – Continuous<br>girder | > 30            | II             | 4.00                               |
|                |                            |  |         |                 |                 |                          |                 |                |                                    |
| A55            | 03/01/2021                 | Concrete                               | > 50    | $\leq$ 50       | 3               | T – Continuous<br>girder | > 30            | Ι              | 4.00                               |
| A56            | 03/01/2021                 | Concrete                               | > 50    | ]50-100]        | 3               | R - Arch                 | > 30            | Ι              | 3.00                               |
|                |                            |  |         |                 |                 |                          |                 |                |                                    |
| A225           | 08/01/2021                 | Concrete                               | ]25-50] | $\leq 50$       | 1               | S – Solid deck           | > 30            | Ι              | 4.00                               |
| A234           | 08/01/2021                 | Concrete                               | ]25-50] | $\leq 50$       | 2               | U – Portal frame         | > 30            | Π              | 4.00                               |
|                |                            |  |         |                 |                 |                          |                 |                |                                    |
| A584           | 05/01/2021                 | Steel                                  | ]25-50] | $\leq 50$       | 1               | A - Box/twin girders     | $\leq 30$       | Ι              | 3.00                               |
| A585           | 05/01/2021                 | Steel                                  | ]25-50] | $\leq 50$       | 1               | $A-Box/twin \ girders$   | $\leq$ 30       | Π              | 4.00                               |
|                |                            |  |         |                 |                 |                          |                 |                |                                    |

Table 1 – Excerpt of the summary of inspections performed between January and October 2021.

From the bridges inspected in 2021 the CR values were 3.69 for concrete bridges and 3.30 for steel bridges. However, 484 bridges maintained the CR values equal to the previous period while 26 bridges decreased the CR values. The remaining 78 bridges increased the CR values due to maintenance interventions or due to a misjudgment of the inspector, and therefore were removed for validation purposes. Fig. 3 details a comparison between the predicted and real known CR values for the 372 concrete bridges (Fig. 3(a)) and 138 steel bridges (Fig. 3(b)) that maintained or decreased the CR values in 2021. This validation strategy is the most appropriate since one of the conditions for the application of the semi-Markov process is that only bridges that maintained or deteriorated the condition ratings between consecutive inspections should be considered. The success of the condition rating assignment, evaluated based on the ratio between the predicted average CR value and the real average CR value, varied between 93.7% for concrete bridges and 95.1% for steel bridges. For concrete bridges, the predicted average CR value was equal to 3.29 and the corresponding real average CR value was equal to 2.69 and the corresponding real average CR value was equal to 2.83.



Fig. 3 - Comparison between the real and predicted CR values: (a) Concrete bridges, (b) Steel bridges.

#### 6 Conclusions

The model was developed based on the information derived from the history of inspections of 588 bridges located on the heavy-haul railway line that connects Rondonópolis to Santos, both in Brazil, in the period between 2016 and 2020. The inspection history was only focused on structural criteria since the condition rating values presented a higher variability between inspections, and therefore a more reliable estimation of the deterioration curves is obtained. A parametric study was performed to evaluate the influence of the bridge structural characteristics, as well as operational and environmental factors, on the deterioration prediction model. This study analyzed the influence of the type of material, age, total length, number of spans, type of structural system, annual transported weight (in MGT) and environmental aggressiveness. The main result is that the type of material is the main factor for this bridge network. Therefore, the analyzes were always carried out considering two degradation matrices, one for concrete bridges and another for steel bridges.

The model validation involved the comparison between predicted and real verified deterioration ratings based on the history of inspections carried out between January and October 2021. The results highlighted the success of the condition rating assignment, evaluated based on the ratio between the predicted and real average condition rating. For concrete bridges 93.7% of agreement and for steel bridges 95.1% of agreement.

Due to the large number of bridges managed by the concessionaire, it is not financially viable to carry out maintenance on all of them in a short period of time, so the objective of this work was to analyze the bridge network over the concession period (next 27 years) until the date of 01.01.2058. It is clear the importance of using reliable bridge deterioration forecasting models to support the long-term maintenance plans carried out by bridge managers. Study preventive maintenance scenarios combined with corrective maintenance scenarios is extremely important in order to support decision-making and strategies in the maintenance of the bridge network. It also enhances the fact that a hasty decision-making can generate excessive costs in the future.

As future developments of this work, it should be referred the development of specific methodologies of inspectors training to reduce the subjectivity in the condition ratings assessments. Another topic of interest, already initiated, is the development of prediction models based on artificial intelligence and the comparison of their results with the method proposed in this study. Finally, the authors are also focused on the development of advanced drive-by inspection methodologies, where a dedicated set of sensors are installed on-board of inspection vehicles envisaging the damage detection on railway bridges.

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#### References

[1] Abed-Al-Rahim I, Johnston W. Bridge Element Deterioration Rates. Transportation Research Record, 1490, National Academy Press, Washington, D.C, USA, 9-18, 1995.

[2] Mirzaei A, Adey B, Klatter L, Kong J. Overview of Existing Bridge Management Systems - Report by the IABMAS Bridge Management Committee, 6th International Conference on Bridge Maintenance, Safety and Management, 2012.

[3] Ryall M. Bridge Management, 2nd Edition, Elsevier, 2010.

[4] Godart B, Vassie P. Bridge Management Systems: Extended Review of Existing Systems and Outline framework for a European System, BRIME - Deliverable D13, 2001.

[5] Woodward R, Cullington D, Daly A, Vassie P, Haardt P, Kashner R, Astudillo R, Velando C, Godart B, Cremona C, Mahut A, Raharinaivo L, Markey I, Bevc L, Peruš I. BRIME - Final Report. Deliverable D14, 2001.

[6] Austroads. Bridge Management Systems: State of the Art, Austroads Publication No. AP-R198/02, 2002.

[7] Farrera F. Optimización conjunta de las políticas de mantenimiento y rehabilitación en Puentes mediante Algoritmos Genéticos. Aplicación al Sistema de Gestión de Puentes del Estado de Chiapas (México), Tesis Doctoral, Universitat Politécnica de Catalunya, 2006. (In Spanish)

[8] Almeida J, Teixeira P, Delgado R. Life cycle cost optimization in highway concrete bridges management, Structure and Infrastructure Engineering, 11:10, 1263-1276, 2015.

[9] Powers N, Frangopol D, Al-Mahaidi R, Caprani C. Maintenance, Safety, Risk, Management and Life-Cycle Performance of Bridges, Proceedings of the 9th International Conference on Bridge Maintenance, Safety and Management (IABMAS 2018), 2018.

[10] Isailović D, Petronijević M, Hajdin R. The future of BIM and Bridge Management Systems, IABSE Symposium, Guimarães, 2019.

[11] Oliveira C, Greco M, Bittencourt T. Analysis of the Brazilian federal bridge inventory. Revista IBRACON de Estruturas e Materiais, 12(1), 1-3, 2019.

[12] Srikanth I, Arockiasamy M. Deterioration models for prediction of remaining useful life of timber and concrete bridges: A review, Journal of Traffic and Transportation Engineering, 7(2), 152-173, 2020.

[13] Madanat S, Mishalani R, Ibrahim W. Estimation of infrastructure transition probabilities from condition rating data, Journal of Infrastructure Systems, 1(2), 120–125, 1995.

[14] Morcous G, Lounis Z, Cho Y. An integrated system for bridge management using probabilistic and mechanistic deterioration models: application to bridge decks, KSCE Journal of Civil Engineering, 14(4), 527–537, 2010.

[15] Cesare M, Santamarina C, Turkstra C, Vanmarcke E. Modeling bridge deterioration with Markov chains. Journal of Transportation Engineering, 118(6), 820-833, 1992.

[16] Morcous G. Performance prediction of bridge deck systems using Markov chains, Journal of Performance of Constructed Facilities, 20(2), 146–155, 2006.

[17] Zambon I, Vidovic A, Strauss A, Matos J, Amado J. Comparison of stochastic prediction models based on visual inspections of bridge decks, Journal of Civil Engineering and Management, 23, 553-561, 2017.

[18] Zhang Y, Chouinard L, Conciatori D. Markov chain-based stochastic modeling of chloride ion transport in concrete bridges, Frontiers in Built Environment, 4, 1-12, 2018.

[19] Scherer W, Glagola D. Markovian models for bridge maintenance management, Journal of Transportation Engineering, 120(1), 37–51, 1994.

[20] Kallen M, Noortwijk J. Statistical inference for Markov deterioration models of bridge conditions in the Netherlands, 3rd International Conference on Bridge Maintenance, Safety and Management (IABMAS 2006), 535–536, 2006.

[21] Kalbfleisch J, Lawless J. The analysis of panel data under a Markov assumption, Journal of the American Statistical Association, 80(392), 863–871, 1985.

[22] Kallen M. A comparison of statistical models for visual inspection data, 10th International Conference on structural safety and reliability (ICOSSAR), Osaka, Japan, 3235–3242, 2009.

[23] Jackson C, Sharples L, Thompson S, Duffy S, Couto E. Multistate Markov models for disease progression with classification error, Journal of the Royal Statistical Society: Series D (The Statistician), 52(2), 193–209, 2003.

[24] ABNT – Associação Brasileira de Normas Técnicas. NBR 9542: Inspection of concrete bridges and footbridges - Procedures. Rio de Janeiro, Brazil, 2019. (In Portuguese)

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