

# Numerical analysis of punching shear in concrete reinforced flat slab using damaged plasticity model

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**Abstract.** Due to high shear stresses in the slab-column connection, reinforced concrete flat slabs may be vulnerable to punching shear failure. In this paper, nonlinear finite element analyses of a reinforced concrete slab-column connection under static loading were conducted using ABAQUS software and the concrete damage plasticity (CDP) model. The material parameters of the CDP model were calibrated based on experimental results of punching shear strength of a reinforced concrete flat slab presented in the literature. Failure mode and cracking pattern at ultimate load were evaluated, indicating that the calibrated model adequately predicts the punching shear response of the slabs.

**Keywords:** Slab, punching shear, reinforced concrete, damaged plasticity, cracking.

## 1 Introduction

Flat slabs are reinforced concrete elements composed of concrete plates supported directly by the columns, without beams. The main advantage of flat slabs, compared to slabs supported by beams, is the reduction of floor-to-floor height, resulting in lower total building height. Although flat slabs show higher ductility when bending, it is verified the possibility of a rupture by punching shear of the plate due to its reduced thickness in relation to the magnitude of the localized reaction and to the reduced dimensions of the area on which the reaction itself is distributed [1]. Due to its limited strain capacity, shear failure in concrete elements without transverse reinforcement is brittle and occurs abruptly after shear crack formation, motivating recent research aiming to identify the shear strength mechanisms [2] [3] [4].

According to Milligan [5], finite element analysis (FEA) is a low-cost and time way to predict the punching capacity, failure mode, crack patterns, and overall behavior of structural elements, by using a well calibrated finite element model based on existing experimental databases. Nonlinear finite element analyses can study the structural behavior of reinforced concrete slabs and supplement the existing experimental database by performing parametric studies.

The accuracy FEA model depends on the information of the constitutive model of the material. The concrete damage plasticity (CDP) model is widely recognized as a precise and practical constitutive model to simulate concrete behavior [6] and assumes that the concrete material is continuous, isotropic, and uniform, reducing the elastic modulus of concrete with inelastic strain. The CDP model considers the inelastic behavior of concrete by defining damage factors in both compression and tension [7].

Some input parameters of the CDP model are the key to improving the accuracy of numerical simulation, like the dilation angle ( $\psi$ ) and eccentricity ( $\epsilon$ ), related to the yield surface flow rule, the ratio of the second stress invariant on the tensile meridian ( $K$ ), that controls the shape of the yield surface, the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress ( $f_{b0}/f_{c0}$ ) and viscosity parameter ( $\mu$ ).

According to Rewers [8], when the dilation angle is greater than or equal to 25 deg, the numerical model

properly predicts the damage obtained in the experiment. The larger the viscosity parameter, the easier the model is to converge, but the lower the accuracy is.

Dilation angle is related to volume change, and quasi-brittle materials, such as concrete, undergo a significant dilatancy caused by a large amount of inelastic strain [9]. Szczecina and Winnicki [10] describe that by applying higher values of viscosity parameter, it is almost certain that the damaged zone spreads to many finite elements leading to a diffuse pattern of cracking and limiting crack propagation, whereas the higher values of dilation angle may lead to positive volumetric strains in compression zone causing an artificial increase of bearing capacity in the case of confinement.

The paper presents a numerical analysis of a flat slab using ABAQUS software and the CDP material model for concrete, evaluating the dilation angle and viscosity parameter effect on load-deflection compared to experimental results.

## 2 Finite element simulations

The model considered in this study consists of a square flat slab (2500 mm × 2500 mm) with a thickness of 160 mm, tested experimentally by Melges [11], supported by four I-shaped steel sections and eight steel plates (200x200x25,4mm), and the load was applied by a hydraulic cylinder and a load plate (180x180x120mm) located in the center of the slab. The flexural tension reinforcement consists of 16 mm bars placed at a 100 mm distance in orthogonal directions. The compressive flexural reinforcement consists of 8 mm bars placed at 100 mm in both directions. Both reinforcements were placed at 15 mm from the slab face. Reinforcement mechanical properties are presented in Tab 1.

Table 1. Reinforcement mechanical properties.

Bar diameter (mm)	$f_y$ (MPa)	$f_u$ (MPa)	E (GPa)
8	584.3	708.2	214.1
16	601.5	730.2	190.2

The average concrete compressive strength and modulus of elasticity were measured using cylinder specimens, resulting in 26.84 MPa and 29.027 GPa, respectively. The diametral compression test was performed to evaluate concrete tensile strength, resulting in 2.71 MPa.

During the experimental test, cracks were observed on the tension surface of the slab, initiating at the center and reaching the edges of the slab. The failure occurred by punching shear at a load of 441.6 kN and vertical deflection at the center of the top surface equal to 7.71 mm.

A three-dimensional model (Fig. 1) was created using a finite element (FE) technique and simulated in the ABAQUS software using the CDP (Concrete Damage Plasticity) material model. 8-noded hexahedral (brick) elements were used for concrete with reduced integration (C3D8R), and 2-noded linear truss elements (T3D2) were used to model reinforcements. The embedded method was adopted to simulate the bond between the concrete and the reinforcement, assuming a perfect bond. Restraints were introduced at the bottom faces of the support plate specimens in the direction of the applied load, and the sum of the reactions at the support plates, where the boundary conditions were introduced, resulting in the punching shear loads.

The structure was analyzed using static analysis in ABAQUS/Standard, where a 10 mm displacement was applied through the column stub. Among the constitutive models for simulating the behavior of concrete, the concrete damaged plasticity model that ABAQUS offers was chosen, using the stress-strain relationships for reinforced concrete in compression and tension proposed by Carreira and Chu [12] [13], and damage parameters were calculated using equations proposed by Hafezolghorani [6].

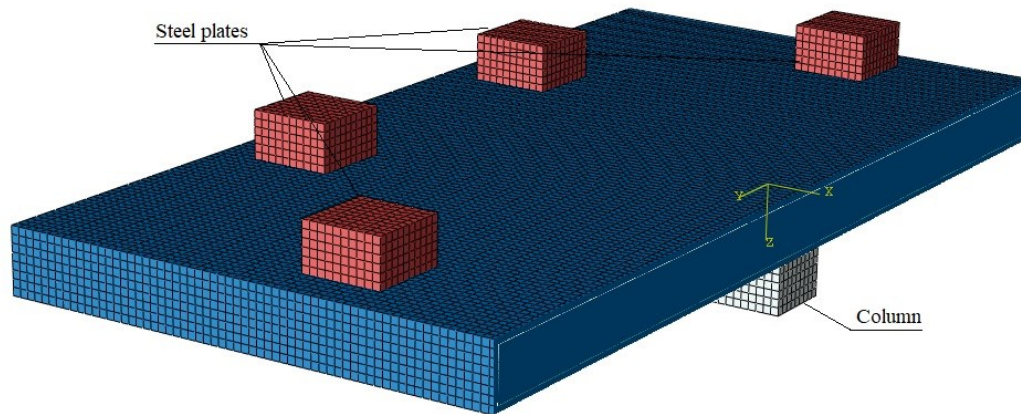


Figure 1. 3D model of the slab with mesh.

The paper analyses the influence of material input parameters on the results of numerical computations. The global mesh size of the concrete slab was 25 mm, chosen to be larger than the aggregate size (19 mm) and considering the minimum of 5 elements through the thickness of the slab, recommended by Geninkomsou and Polak (2015) and Milligan et al. (2020).

Three values of the dilation angle ( $30^\circ$ ,  $40^\circ$ , and  $50^\circ$ ) and three viscosity parameters (0.001, 0.005, and 0.0005) were combined, generating nine slabs for the parametric investigation (Tab. 2). For all models, the eccentricity ( $\epsilon$ ), the ratio of the second stress invariant on the tensile meridian ( $K$ ) and the ratio of initial equibiaxial compressive yield stress to initial uniaxial compressive yield stress ( $f_{b0}/f_{c0}$ ) were constant, equal to 0.1, 0.6667 and 1.16, respectively.

Table 2. Numerical models and CDP parameters.

Model	Dilation angle ( $^\circ$ )	Viscosity parameter (s)
M1	30	0.0005
M2	30	0.001
M3	30	0.005
M4	40	0.0005
M5	40	0.001
M6	40	0.005
M7	50	0.0005
M8	50	0.001
M9	50	0.005

Load-deflection obtained in numerical analyses was compared to experimental results presented by Melges [11].

### 3 Results and discussion

Load-deflection results from the three-dimensional FE analysis of the models are presented in Fig. 2, grouped by dilation angle. Similar to experimental results, FE analysis showed a linear load-deflection response in the first phase until the load reaches approximately 100 kN, and nonlinear response, with rigidity reduction due to crack formation. The analysis shows that lower values of dilation angle resulted in curves more accurate to experimental results (Fig. 2a), similar to results obtained by Najafgholipour et al. [14], where  $35^\circ$  shows reasonably results, and increasing the value of dilation angle the displacement capacity and the ultimate failure load increased subsequently.

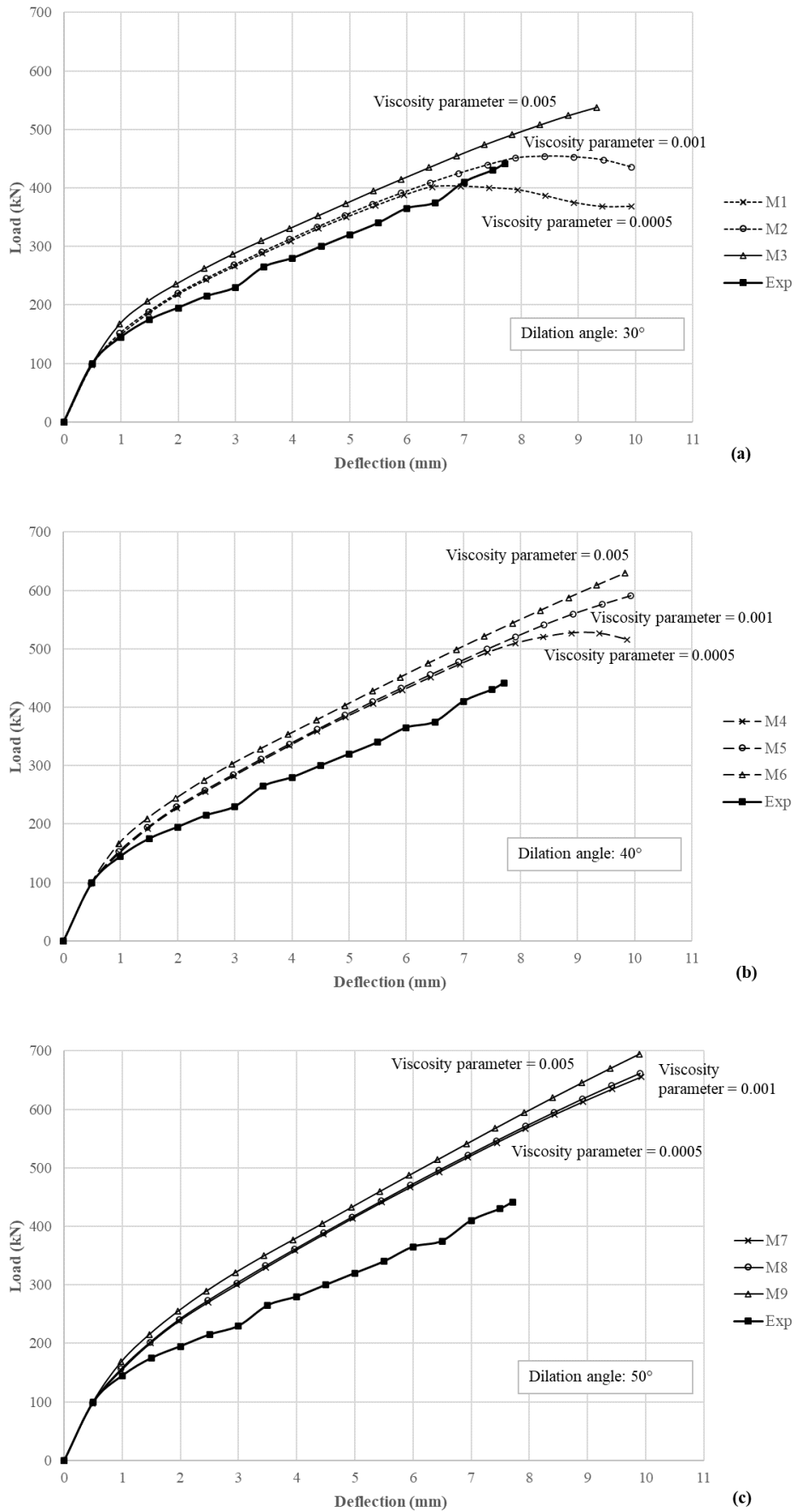


Figure 3. Load-deflection responses for dilation angle equal 30°(a), 40°(b), and 50°(c).

According to Genikomsou and Polak [15], the value of the viscosity parameter depends on the time increment step. In this study, the time increment step was set as automatic, and the viscosity parameter was varied to investigate its influence on the numerical response. With lower values of viscosity parameter, results were more accurate with the experimental results, and a post-peak behavior was observed in models M1, M2, and M4, which were not achieved in the laboratory. The best fit of the experimental load-deflection value was identified with the combination of dilation angle  $30^\circ$  and viscosity parameter 0.001 seconds, resulting in load as 446.38 kN for deflection equal to 7.71 mm, maximum value experimentally measured, a difference less than 2%.

Due to best-fit results, model M2 was used to evaluate the cracking pattern, shown in Fig. 4, through the contours of maximum principal plastic strain for the ultimate load. It was observed that the crack pattern obtained numerically is similar to the experimental, obtained by Melges [11], propagating from the central column radially to the board as the load increases. Also, the punching shear cone is visible at ultimate load in Fig. 5, indicating that the simulation was able to capture the punching shear failure.

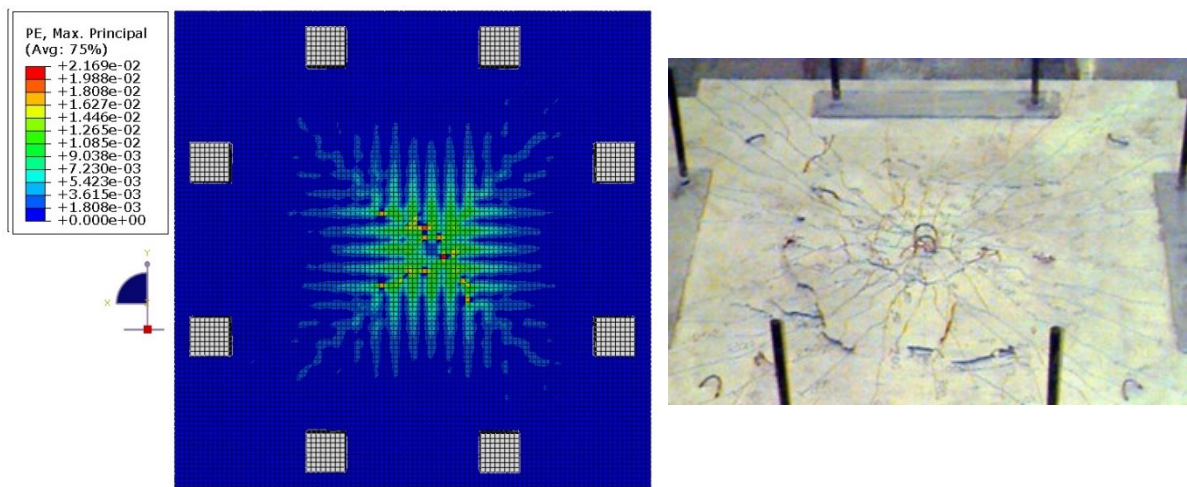


Figure 4. Predicted crack patterns obtained numerically and experimental.

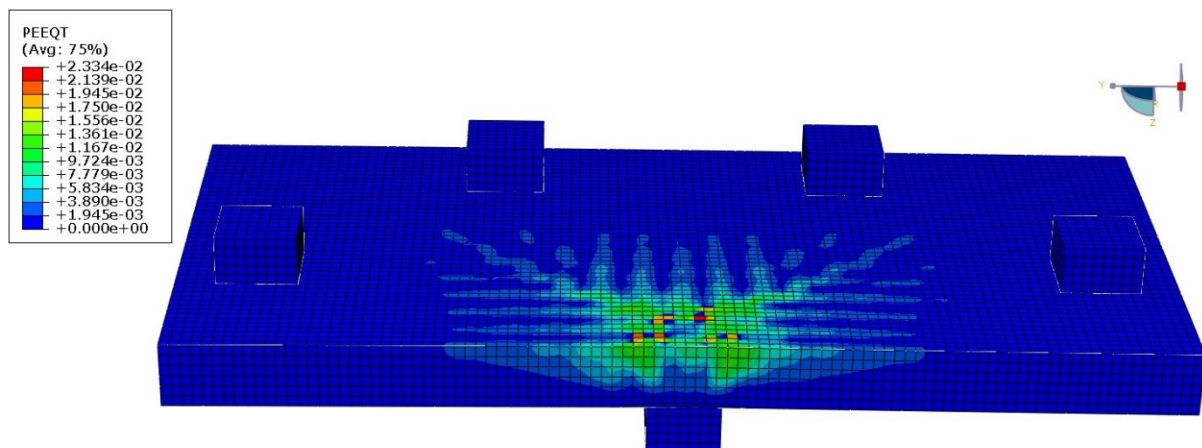


Figure 5. Punching shear cone obtained numerically.

## 4 Conclusions

In this paper, the finite element analysis with the concrete damaged plasticity model was used for predicting the punching shear response of a slab without shear reinforcement. Nine models were evaluated, with varied dilatation angles and viscosity parameters, aiming to calibrate the numerical model, and the main conclusions are:

The finite element analysis using the CDP model showed good agreement with experimental data, demonstrating the model's ability to predict the punching shear failure in concrete slabs without shear reinforcement.

The dilation angle and viscosity parameter appear to be critical for the accurate definition of the concrete modeling, and the combination of dilation angle equal to 30° and viscosity parameter equal to 0.001 seconds resulted in a load-deflection response closer to experimental results.

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## References

- [1] G. Toniolo and M. Prisco, Reinforced Concrete Design to Eurocode 2. *Springer Tracts in Civil Engineering*, 2017.
- [2] S. Guandalini, O. Burdet and A. Muttoni, “Punching tests of slabs with low reinforcement ratios”. *ACI Structural Journal*, v. 106, n. 1, 2009.
- [3] M. F. Ruiz, Y. M. and A. Muttoni, “Post-Punching Behavior of Flat Slabs”. *ACI Structural Journal*, vol. 110, n. 5, September-October, 2013.
- [4] D. Salman, R. Allouzi and N. Shatarat, “Punching shear behaviour of flat slabs with different reinforcement schemes: openings and rectangularity effects”. *International Journal of Structural Integrity*, vol. 12, n. 4, 2021.
- [5] G. J. Milligan, M. A. Polak and C. Zurell, “Finite element analysis of punching shear behaviour of concrete slabs supported on rectangular columns”. *Engineering Structures*, vol. 224, 2020.
- [6] M. Hafezoghori, F. Hejazi, R. Vaghei, M. S. B. Jaafar and K. Karimzade, “Simplified Damage Plasticity Model for Concrete”. *Structural Engineering International*, n. 1, 2017.
- [7] L. Qingfu, G. Wei and K. Yihang, “Parameter calculation and verification of concrete plastic damage model of ABAQUS”. *IOP Conf. Series: Materials Science and Engineering*, vol. 794, 2020.
- [8] I. Rewers, “Numerical Analysis of RC beam with High Strength Steel Reinforcement using CDP model”. *IOP Conf. Series: Materials Science and Engineering*, vol. 471, 2019.
- [9] J. Lee and G. L. Fenves, “Plastic-damage model for cycling loading of concrete structures”. *Journal of Engineering Mechanics*, August, 1998.
- [10] M. Szczecina and Andrzej Winnicki, “Calibration of the CDP model parameters in Abaqus”. *Advances in Structural Engineering and Mechanics*, 2015
- [11] J. L. P. Melges, *Análise experimental da punção em lajes de concreto armado e protendido*. PhD thesis, University of São Paulo, 2001.
- [12] D. J. Carreira and K. H. Chu, “Stress-strain relationship for plain concrete in compression”. *ACI Journal*, n. 82-72, 1985.
- [13] D. J. Carreira and K. H. Chu, “Stress-strain relationship for plain concrete in tension”. *ACI Journal*, n. 83-3, 1986.
- [14] Najafgholipour et al.