XLIII CILAMCE
21-25, November, 2022
FOZ DO IGUAÇU - PR - BRAZIL

Vibration correlation technique applied to cylindrical and conical shells—an overview of the recent developments

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Abstract. Traditional buckling experiments of imperfection-sensitive structures like cylindrical or conical shells may result in the permanent failure of the specimen. Yet, for validating the numerical models and, consequently, the design of aerospace barrel structures, to perform a qualification test is a crucial step. There is, therefore, interest in non-destructive experimental procedures for predicting the buckling load of these structures from the pre-buckling stage, allowing the use of the same specimen in further qualification tests. An example of these methodologies is the Vibration Correlation Technique (VCT), which allows determining the buckling load without reaching the instability point through a sequence of vibration tests performed at different load levels. In this review, focus will be given to the VCT applied to cylindrical and conical shells, revisiting the analytical foundation supporting its applicability together with experimental and numerical results of simplified downscaled barrel structures with different design details and test conditions, highlighting its non-destructive characteristic. The state of the art corroborates the robustness of the VCT when applied to imperfection-sensitive thin-walled structures; however, more test results, especially for real-scale barrel structures, are needed for expanding the experimental database and confirming the application of VCT to predict the buckling load these structures.

Keywords: buckling, vibration correlation technique, thin-walled structures.

1 Introduction

Prominently, thin-walled cylindrical shells dominate the design of launch vehicles, e.g., one may refer to Ariane 6, developed and manufactured by ArianeGroup under the authority of ESA. In such applications, the operational load envelope, combined with the simultaneous lightweight and high-performance requirements, recurrently leads to a buckling-critical shell design.

Traditional buckling experiments have a destructive, terminal nature; in addition to that, typical barrel components for aerospace application involve long production and test preparation cycles, requiring a specimen solely assigned to the static qualification test. This scenario substantiates financial and time-consuming interests in the development and validation of nondestructive methods to obtain the in-situ buckling load from the pre-buckling stage, permitting the use of the same specimen in further qualification tests. Typically, these methods consist of tracking a response of the structure at different axially applied load levels for extrapolating the instability point from the measured data. A consolidated methodology aiming at structures exhibiting a complex buckling behavior like curved panels and cylindrical shells is the vibration correlation technique (VCT) [1].

In this review article, focus will be directed to the VCT applied to cylindrical and conical shells. **First**, a succinct historical background addressing the establishment and consolidation of the methodology is presented. **Second**, the recent developments concerning cylindrical and conical shells emphasizing the recent efforts through analytical, experimental, numerical, and sensitivity analysis conducted to expand its applicability and robustness are reported.

2 Historical background

For the first half of the 20th century, several researchers were simultaneously addressing the "apparently different" problems of vibration and elastic stability [2]. In this historical background, an overview of a relevant part of this literature dedicated to estimating the buckling load from the vibration measurements is provided. To the best of the author's knowledge, the concept of relating the axially applied load to natural frequencies for identifying the buckling load is accredited to Sommerfeld [3]. The author verified experimentally that the first natural frequency of a clamped-free column loaded by a variable mass at the free end decreases, approaching zero, as the mass increases to the amount required to buckle the structure. In the following decades, the foundation of the VCT, i.e., the analytical formulation between the applied load and the loaded natural frequency, was established for several structures [1]. The author demonstrated for simply supported beams, plates, and cylindrical shells for which the vibration and buckling modes are identical that the linear relationship between the applied load and the squared loaded natural frequency holds:

$$f^2 + p = 1, \tag{1}$$

where $f = \bar{\omega}_{mn}/\omega_{mn}$, being $\bar{\omega}_{mn}$ the loaded natural frequency and ω_{mn} the unloaded natural frequency, both associated with the same vibration mode specified by m axial half-waves and n circumferential waves (for cylindrical shells), and $p = P/P_{\mathrm{CR}}$, being P the axially applied load.

In the early 1950s, researchers from Caltech applied eq. (1) for direct estimations of the buckling load, establishing the classic VCT [2]. The technique consists of plotting the experimental data in the classic characteristic chart, i.e., between f^2 and p, and adjusting a linear best-fit relationship. From this function, one extrapolates the buckling load as the load level where the loaded natural frequency is zero. This procedure has been straightforwardly applied for evaluating columns and imperfection-insensitive thin-walled structures since the beginning, even when tested with different boundary conditions. Already in the 1960s, it became an accepted practice in the industry [1], e.g., vide [4] for the procedures used in the Northrop Corporation.

For imperfection-sensitive structures, like curved panels, cylindrical and conical shells, where the contributions of a non-destructive experimental procedure for determining the buckling load would be even more appreciated, there is no consensus on a mature technique suitable for practical scenarios [5]. In this context, several authors devised modified VCT approaches addressing such structures; for comprehensive reviews covering most of this effort, see chapter 15 of [1], chapter 2 of [6], and the review paper [7].

The following paragraphs present two correlated methodologies that laid the foundation for the recent developments. The first consists of a semi-empirical approach proposed by Souza et al. [8], and the second is the empirical modification of the first suggested by Arbelo et al. [9]. In their studies, Souza et al. defined a simplified model for the compressed cylinder consisting of a strut and an arch interconnected at their centers. The authors devised the following equation for the parametric form of the applied load in terms of the loaded natural frequency:

$$(1-p)^2 + (1-\xi^2)(1-f^4) = 1, \tag{2}$$

where ξ^2 is the square of the drop of the buckling load due to initial imperfections, and it is estimated by evaluating the magnitude of $(1-p)^2$ when f is equal to zero, i.e., $1-f^4$ is equal to one, see the schematic view in Fig. 1(a). As anticipated, Arbelo et al. [9] suggested an empirical modification of Souza's method. The authors represented the parametric form $(1-p)^2$ as a quadratic function of $1-f^2$, yielding the characteristic curve reproduced in Fig. 1(b). The second-order best-fit equation for the experimental data is expressed as:

$$(1-p)^2 = \mathrm{A}(1-f^2)^2 + \mathrm{B}(1-f^2) + \mathrm{C}, \tag{3}$$

where A, B and C are fitting coefficients and ξ^2 is evaluated as the minimum value of the $(1-p)^2$ axis:

$$\min(1-p)^2 = \xi^2 = -\frac{\mathrm{B}^2}{4\mathrm{A}} + \mathrm{C}. \tag{4}$$

Accordingly, both methods predict the buckling load P_{VCT} using the positive square root of ξ^2 [8]:

$$P_{\mathrm{VCT}} = P_{\mathrm{CR}}(1-\xi), \tag{5}$$

being $1-\xi$ comparable to the KDF γ traditionally used when sizing cylindrical shells [10].

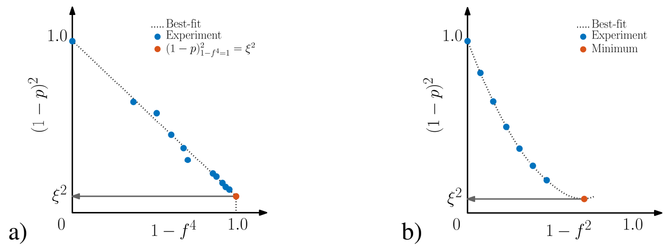

Figure 1. Schematic view of the methodologies published in [8] (a) and [9] (b).

The modified method [9] is grounded on the effects of the initial imperfections in the vibration response; typically, it considers the first two or three natural frequencies for estimating the buckling load. The author verified the methodology for composite laminated cylinders, and following this development, a profusion of experimental campaigns was published as listed in [11].

3 Recent developments

Taking into account just the recent effort, i.e., after [9] was published, at least 28 cylindrical shells were tested for VCT considering different geometries, materials, design details, and load and boundary conditions [11]. This chapter addresses recent prominent developments, and it is split into three main sections: **first,** an overview of the analytical verification and further experimental validation is given in Section 3.1, **second,** the recent efforts for extending [9] to conical structures are presented in Section 3.2, and **third,** an uncertainty quantification study is discussed toward practical applications in Section 3.3.

3.1 DLR efforts for analytical and experimental verification

The empirical method [9] is grounded on the rearrangement of eq. (1) as demonstrated for isotropic cylinders in [6, 12]:

$$1-p=f^2=1-\left(1-f^2\right)\Rightarrow\left(1-p\right)^2=\left[1-\left(1-f^2\right)\right]^2, \tag{6}$$

Moreover, due to the typical imperfection-sensitive behavior of cylindrical and conical shells, a sudden drop in the load-carrying capacity is expected. At this point, the parametric form $(1-p)^2$ reaches its minimum magnitude; therefore, assuming that eq. (6) would be a good representation of $(1-p)^2$, even for discrepant buckling and vibration modes, the minimization of this relationship provides an estimation of this minimum magnitude. This is illustrated in Fig. 2(a), which presents the load-shortening curve of a typical imperfection-sensitive cylindrical shell together with $(1-p)^2$; summing up, Fig. 2(b) brings the sequence of a VCT experiment.

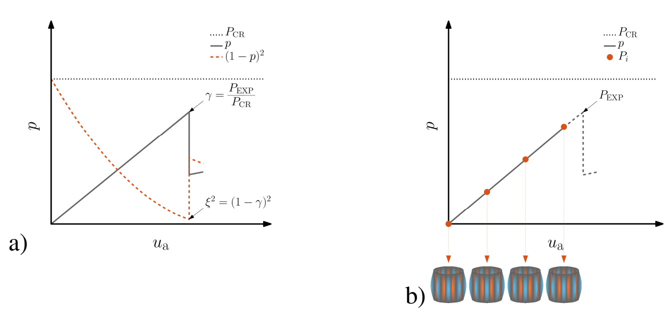

Figure 2. Typical load-shortening curve of a cylindrical shell (a) and steps for a VCT experimental campaign (b).

Besides providing the analytical meaning of the empirical method [9], DLR's recent efforts further extended the applicability of the methodology. Specifically, three experimental campaigns [6, 13, 14] were performed. The first, available in [13], evaluated a metallic cylinder with closely-spaced stiffeners, named Z38, with different pressure levels. The second assessed three nominally equal unstiffened composite shells, ZD27, ZD28, and ZD29, at different buckling test facilities (ZD27 at TU Delft and ZD28 and ZD29 at DLR Braunschweig), see [14]. The third assessed an unstiffened thin-ply composite cylinder, Z42, with mounting fluxes and internal pressure [6].

A complete description of the geometric characteristics of the specimens and experimental campaigns is available in [6, 13, 14]. All of the specimens were sized for elastic buckling, providing a direct comparison between the experimental buckling load P_{EXP} and its corresponding VCT prediction P_{VCT}. Table 1 summarizes the experimental buckling loads P_{EXP}, their predictions P_{VCT} together with the deviation δ, and the maximum applied load P_{MAX}, both as related to P_{EXP}, while Fig. 3 shows the characteristic charts with the experimental results, where the legend of the charts contains the coefficients of determination R_S^2 and minima along the $(1-p)^2$ axis ξ^2.

Table 1. Summary of the VCT predictions of the experimental campaigns [6, 13, 14]

Cylinder	Z38				ZD27	ZD28	ZD29	Z42		Z42*	
Pressure [bar]	-	0.01	0.02	0.03	-	-	-	-	0.01	-	0.01
P_{MAX} [%]	84.1	83.7	79.3	78.8	81.8	94.9	95.7	86.8	83.9	83.0	88.0
P_{EXP} [kN]	86.5	104.3	116.8	127.8	15.9	21.5	21.9	12.4	15.1	10.6	13.3
P_{VCT} [kN]	80.2	94.0	112.3	122.2	15.2	19.7	20.5	11.8	13.5	9.5	12.8
δ [%]	-7.3	-9.9	-3.9	-4.4	-4.4	-8.3	-6.3	-5.0	-10.5	-10.5	-3.7

* With mounting fluxes (in-plane imperfection – IPI).

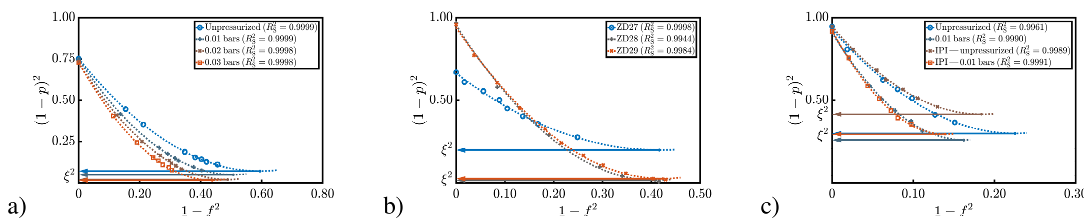

Figure 3. VCT implementation for Z38(a), ZD27, ZD28, and ZD29 (b), and Z42 (c).

Analyzing the results in Tab. 1, the VCT provides consistent estimations for the buckling load for cylindrical shells with different design details and under varying testing conditions. Noticeably, for all cylindrical shells the predicted buckling load is conservative since its magnitude is smaller than the corresponding experimental buckling load. A good range of deviation magnitude is observed with the smallest and greatest deviations, 3.7% and 10.5%, obtained for Z42 with IPI and 0.01 bar of internal pressure and in the unpressurized condition, respectively. From Fig. 3, the adjusted curves are associated with high magnitudes of R_S^2, corroborating their goodness of fit, being the smallest 0.9944 for ZD28.

3.2 VCT applied to CFRP truncated cones

To the best of the authors' knowledge, only recently, the VCT was explored for conical shells and three scientific papers consisting of an analytical verification (2021) [15], an experimental validation (2022) [16], and an extensive numerical study (2022) [17] are available. This chapter summarizes the work done in [17], where the predictive capabilities of the VCT were investigated for two nominally identical composite truncated cones named K01 and K06, having only different ply topologies between the designs.

In the mentioned paper, two types of numerical models are used: one with an accurate representation of the boundary conditions (1), and another with a simplified representation, i.e., clamped boundary conditions (2). These models have measured mid-surface and thickness imperfections, and based on the latter, the material properties assigned element-wise were adjusted accordingly. Figure 4(a-b) presents an example of such imperfections applied to the finite element model (in mm), and Fig. 4(c) a direct comparison between the nonlinear static results with the buckling experiments for K06. Besides, ten imperfection sets measured for thin-walled, unstiffened composite cylinders were also applied to the tested conical shells. Furthermore, for one of the truncated cones, K06, a parametric study evaluated conical shells with lengths 300, 500, 750, 1000, 1250, 1500, 1750, and 2000 mm, semi-vertex angles 0, 5, 10, 15, 20, 25, 30, and 35°, and bottom radii 100, 200, 300, 400, and 500 mm.

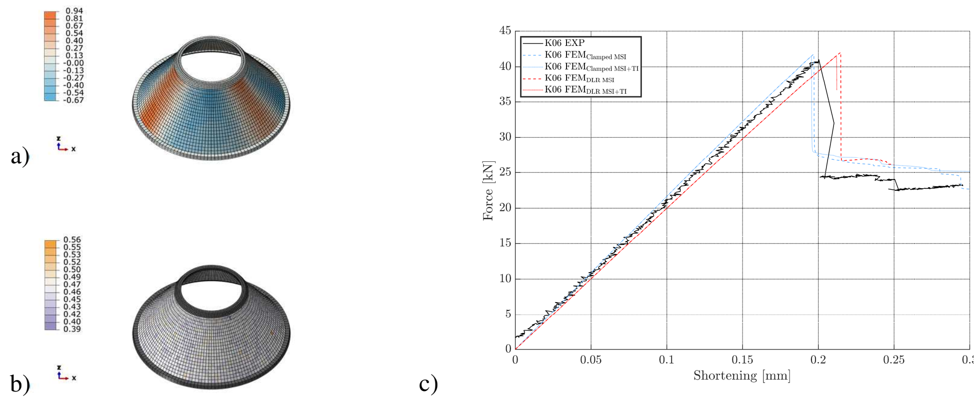

Figure 4. Mid-surface (a) and thickness (b) imperfections applied to the finite element model (mm) and validation of the nonlinear static results for K06 (c).

For the VCT evaluation, the predictive capabilities were checked for the first six vibration modes, taking into account 20 load steps varying from 5% to 100% of the nonlinear buckling load P_{NL}. Altogether the study comprises a total of 2,826 VCT predictions. The results are analyzed in terms of convergence surface plots defined for a given bottom radius. Figure 5 reproduces the convergence plots considering load levels up to 25%, 50%, 75%, and 95% of the P_{NL} for the VCT predictions of cones defined by a bottom radius equal to 500 mm.

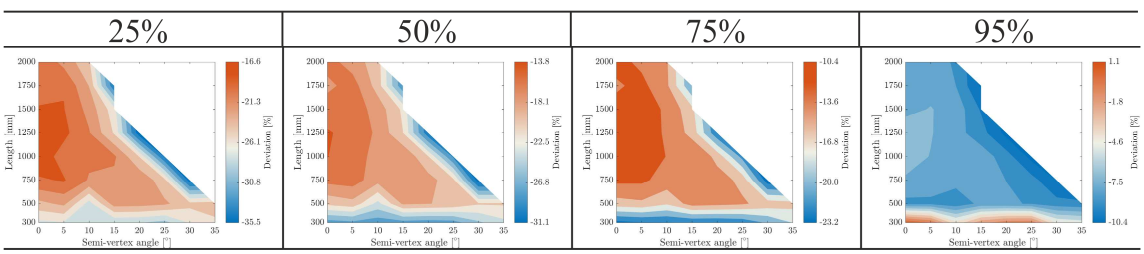

Figure 5. Deviation of the VCT predictions from parametric analysis for different load levels used.

This parametric study obtained conservative buckling load predictions for the vast majority (99.93% of 2,826 predictions), and found no significant influence of thickness imperfections on the predictive capabilities of the VCT (not more than 3%). In general, the best predictions were obtained for cylindrical structures and cones with small semi-vertex angles. The results of the parametric analysis also showed cases where the generated discrepancies in the VCT predictions are related to the geometric aspect and a very small difference between the bottom and upper radius of the cone.

3.3 Uncertainty quantification toward practical applications

The VCT is consolidating as a non-destructive method for in-situ predictions of the buckling load of imperfection-sensitive structures. Nevertheless, publications providing recommendations on how to set up the experimental test for VCT usage are relatively scarce. Given these points, Baciu et al. [11] performed a sensitivity analysis toward the number of load steps and maximum axial load level used on the VCT predictions, quantifying the uncertainty associated with the measured natural frequency.

Besides a numerical study based on ten cylindrical shells, the authors also gathered a total of 48 VCT data sets from 28 nominally different cylinders covering different materials (Al, GFRP, CFRP), diameters (160 – 801 mm), heights (150 – 1000 mm), thicknesses (0.273 – 2.5 mm), aspect ratios (0.5 – 4), R/t (32 – 800 mm), boundary conditions, loading, or with/without stiffeners. Each numerical or experimental VCT data set was disturbed by random deviations within ±0.1Hz, ±0.25Hz, and ±0.5Hz added to the frequencies regardless of the load step, generating additional 50 data sets out of the pristine ones. The VCT combinations were defined following some criteria allowing only more realistic load steps combinations: maximum load ratio in the 50% – 85% interval (1), bias toward higher load levels (2), minimum load ratio within 15% (3), and maximum load ratio gap of 20% (4).

All VCT data sets (numerical or experimental) were clustered in terms of the corresponding load levels. The authors evaluated the deviations of the VCT predictions through violin plots defined in terms of the number of load steps and maximum load levels. Furthermore, the correlation between the two parameters of interest and the VCT predictions is evaluated through Kendall's correlation coefficient τ. Figure 6 presents the violin plot defined in terms of the number of load steps (a) and the maximum load level (b), and the heat map for Kendall's correlation coefficient for the pristine experimental data (c).

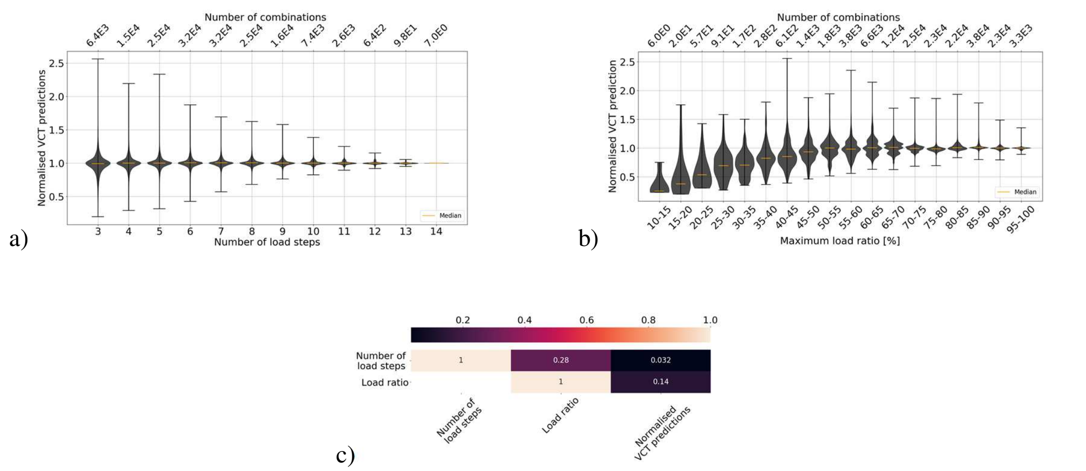

Figure 6. Violin plots for the number of load steps and maximum load ratio, and τ heat map for the pristine experimental data.

The authors observed that higher load ratios tend to prevent over-conservative predictions and that the variation of the VCT predictions tends to decrease with increasing the maximum load ratio, regardless of the number of load steps used. Also, the analysis based on τ resulted in a non-existent correlation between the number of load steps and the VCT prediction. This study provides statistical meaning to similar findings [12 – 14, 17], substantiating the strategy of simultaneously increasing the maximum load level and number of load steps when planning the VCT test campaign based on numerical results.

4 Conclusions

The analytical, numerical, experimental, and sensitivity analysis results reviewed in this article consolidate the VCT devised in [9] as a reliable non-destructive experimental procedure for predicting the buckling load of imperfection-sensitive structures, especially for cylindrical shells. The review started by highlighting some aspects of the analytical meaning of this methodology and some experimental campaigns that further validated the applicability and robustness of the methodology by addressing design details and loading conditions, such as closely spaced stiffeners, internal pressure, realistic R/t, and in-plane induced stresses. Second, it summarized an extensive parametric study addressing the application capabilities of the method considering conical shells, where

2,826 VCT predictions are appraised. Third, an overview of a sensitivity analysis considering the number of load steps and maximum load level and the uncertainty associated with the measured natural frequency was shown.

Given the current maturity, the authors claim that the VCT is ready to be included in qualification tests of actual barrel structures. Yet, some aspects pose as additional topics for industrial applications: a broader range of realistic design solutions and loading conditions, e.g., double-curved shells and axial load cases combined with bending (1), experimental validation for conical shells (2), and to investigate real-scale barrel structures (3).

Acknowledgements. The research leading to these results has received funding from the European Community's Eighth Framework Programme (FP8/H2020), under Priority ERDF (European Regional Development Fund), Grant Agreement Number ZW 6-85042584, and internal funding from the German Aerospace Center DLR. All support is gratefully acknowledged. The information in this paper reflects only the author's views and the European Community is not liable for any use that may be made of the information contained therein.

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