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Fragility curves and failure models based on lumped damage mechanics applied to reinforced concrete frames under seismic loads

Rúbia M. Bosse[1], André Teófilo Beck[2]

[1]*Dept. of Civil Engineering, Federal University of Technology - Paraná*
Avenida Professora Laura Pacheco Bastos, 85053-510, Paraná/Guarapuava, Brazil
rubiambosse@utfpr.edu.br
[2]*Dept. of Structural Engineering, University of São Paulo*
Av. Trabalhador Sãocarlense, 400, 13566-590, São Paulo/São Carlos, Brazil
atbeck@sc.usp.br

Abstract. This paper presents a methodology to apply the Lumped Damage Mechanics (LDM) in the Performance Based Earthquake Engineering (PBEE) approach, to evaluate the seismic vulnerability of reinforced concrete frames. The lumped damage model represents the evolution of damage and plasticity lumped in inelastic hinges at the nodes of the elements. The method allows the estimation of material nonlinear effects due to shear and bending moment under static, cyclic and dynamic loads, considering fatigue and hardening. Incremental dynamic analysis using artificial earthquakes are performed to calculate the fragility curves of the RC frames with LDM. The paper proposes a procedure to identify and characterize collapse mechanisms from the local internal variables of damage, using system reliability theory. The main results show that LDM can be efficiently applied to PBEE and it is possible to use the internal variables of damage as engineering demand parameter (EDP) in the vulnerability analysis.

Keywords: Lumped damage mechanics, Performance Based Earthquake Engineering, Internal variables of damage, Fragility curves, Collapse mechanisms.

1 Introduction

The behavior of RC structures under ground motions has been studied by several researches [1–3]. During earthquakes RC elements can present excessive deformations, evolution of cracking or continuous accumulated damage under repeated load reversals. If the damage indexes reach critical levels in elements or connections it can cause structural accidents, due to the formation of collapse mechanisms. In this way, the models used to represent RC structures under earthquakes shall consider properly the nonlinear behavior of the material taking into account the effects of cracking and plastic yielding in the reinforcements.

The lumped damage mechanics (LDM) was proposed by Cipollina et al. [4] and Flórez-López [5] to represent the physical nonlinear behavior of complex RC structures. LDM considers the evolution of damage and development of plastic strains lumped in the elements nodes that can form inelastic hinges. LDM includes the effects of damage and yielding as internal variables of the model and presents a great computational efficiency. Its validation was extensively verified through comparisons with experimental tests [6, 7, 9].

The performance-based earthquake engineering (PBEE) can be defined as a process for designing new structures or evaluate and update existing buildings to achieve a predefined performance objective in earthquakes([10]). In this context, it is necessary to evaluate the building vulnerability during earthquakes, and this analysis is performed through fragility curves that give the probability of exceeding a damage state threshold conditioned on the ground motion Intensity Measure (IM) and results from the structural analysis [11].

In PBEE, it is necessary to define the engineering demand parameter (EDP) applied to characterize the failure or the exceedance of a limit state. The most commonly EDP used is the interstory drift ratio (IDR) of the structure, it is a global parameter obtained through any mechanical model, in this way, the seismic standards provide limits of the capacity levels in terms of the IDR [12]. However, this limits are defined for a given location and the extrapolation for other counties can be not suitable for all design realities [3]. In this way the application of the lumped damage mechanics and evaluation of the seismic vulnerability using the internal variables of damage as EDP can be an advance. The internal damage variables represents the cracking ratio of the cross section and can be generalized for any design scenario. Usually, when a cross section reaches damage values equals to 0.6, a complete hinge is developed, the association of hinges can lead to a failure mechanism.

This study proposes the application of LDM in PBEE approach to estimate the vulnerability of RC frames under stochastic earthquakes. The main objective is to generate fragility curves using the internal variables of damage as EDP. The methodology performs incremental dynamic analysis using synthetic scaled ground motions and the identification of major failure modes for the structure. In this way it is possible to formulate the frame failure scenarios using concepts of system reliability. The final results are the comparison between fragility curves calculated with different methods (empirical, moment and PSDM).

1.1 Lumped Damage Model

The lumped damage mechanics applies concepts of the fracture mechanics, continuum damage mechanics and generalizes the lumped plasticity model to include the effects of damage to represent the nonlinear behavior of framed structures. LDM takes into account the cracking evolution in concrete and stiffness degradation using Griffith criteria. The steel yielding and the development of plastic deformations are described by an yielding function. Figure 1 shows the finite element of lumped damage model.

Figure 1. Inelastic hinges in lumped damage finite element.

The generalized strains Φ for the plane frame finite element, are given by $\Phi = [\phi_i, \phi_j, \delta]$, where ϕ_i and ϕ_i are the relative rotations, and δ is the stretching of the element.

Considering a 2D plane frame finite element with six degrees of freedom (horizontal u, vertical w displacements and rotations θ), the vector of generalized displacements is given by $\{U\} = \{u_i, w_i, \theta_i, u_f, w_f, \theta_f\}$. The deformations are obtained by the kinematic equations that relate the increments of generalized deformations Φ with the increments of displacements ΔU, in the form of $\Delta\Phi = [B_0]\Delta U$, where [$B_0$] is the kinematic transformation matrix.

The constitutive law relates the generalized matrices of deformations and stresses, according to Equation 1:

$$\Phi_b - \Phi_b^p - \gamma_b^p = [F_f(D^-)]M_b^- + [F_f(D^+)]M_b^+ + \frac{1}{G}A_b(1-{d_s}^-)V_b^- + \frac{1}{GA_b(1-{d_s}^+)}V_b^+, \tag{1}$$

$\left[F_f(D^{+/-})\right]$ is the flexibility matrix, L_b is the element length, EI_b is the stiffness, G is the shear modulus, A_b is the area, $\langle V\rangle_b{}^{+/-}$ is the vector of shear forces given by the median of the bending moments in each node of the element.

$$[F_f(D^{+/-})] = \begin{bmatrix} \frac{L_b}{3EI_b(1-{d_i}^{+/-})} & \frac{-L_b}{6EI_b} & 0 \\ \frac{-L_b}{6EI_b} & \frac{L_b}{3EI_b(1-{d_j}^{+/-})} & 0 \\ 0 & 0 & \frac{L_b}{EA_b} \end{bmatrix}, \quad \langle V\rangle_b^{+/-} = \begin{bmatrix} \langle V\rangle^{+/-} \\ \langle V\rangle^{+/-} \\ 0 \end{bmatrix}. \tag{2}$$

The damage evolution can be described using the Griffith criterion. Equation 3 describes the complementary deformation energy of a damaged frame element,

$$W_b = \frac{1}{2}\{M\}_b^t\{\phi - \phi^p - \gamma^p\}_b = \frac{1}{2}\{M\}_b^t[F(D, d_s)]_b\{M\}_b. \tag{3}$$

According to Griffith criterion, the released energy depends on the bending moment, while the shear damage varies with the shear force. The evolutin of damage is obtained as

$$\begin{cases} d_i = 0, \ se\ G_i < R_i \\ G_i = R_i, \ se\ d_i > \ 0 \end{cases} \quad \begin{cases} d_j = 0, \ se\ G_j < R_j \\ G_j = R_j, \ se\ d_j > \ 0 \end{cases} \quad \begin{cases} d_{cis} = 0, \ se\ G_{cis} < R_{cis} \\ G_{cis} = R_{cis}, \ se\ d_{cis} > \ 0 \end{cases}, \tag{4}$$

G are the energy release rates, R are the crack resistance functions of inelastic hinges i and j, d is the internal variable of damage.

$$G_i = \frac{\delta W}{\delta d_i} = \frac{F_{11}^0 m_i^2}{2(1-d_i)^2}, \quad G_j = \frac{\delta W}{\delta d_j} = \frac{F_{22}^0 m_j^2}{2(1-d_j)^2}, \quad R(d) = R_0 + q\frac{\ln(1-d)}{(1-d)}. \tag{5}$$

$R(d)$ is calibrated using experimental tests, with the hardening term $q\ln(1-d)/(1-d)$. R_0 and q depend on the characteristics of the element and are calculated with the cracking and ultimate bending moments. Equaling the expressions of G and R, it is possible to evaluate the relation between the bending moment and the internal variable of damage. When $m \leq M_{cr}$, $(d = 0)$ is assumed and R_0 is calculated.

$$m^2 = \frac{6EI(1-d)^2}{L}R_0 + \frac{6qEI}{L}(1-d)\ln(1-d), \quad R_0 = \frac{M_{cr}^2 L}{6EI}. \tag{6}$$

An evolution law for the plastic deformation is added in the damage model assuming the hypothesis of equivalence in deformation. The equivalent moment $\overline{m}$ on a plastic hinge i is defined in Eq. eq. (7). Also, the resistance function in the inelastic hinge with damage and plasticity, is given by:

$$\overline{m}_i = \frac{m_i}{1-d_i}, \quad f_i = |\overline{m}_i - c_i\phi_i^p|k_{0i} = \left|\frac{m_i}{1-d_i} - c_i\phi_i^p\right| - k_{0i} \leq 0, \tag{7}$$

m_i is the bending moment in the node i, $\overline{m}_i$ is the effective moment at i, c_i and k_{0i} depend on the characteristics of the element. In this way the plastic deformation evolution law is given by Eq. 8. The bending moment is related to the inelastic hinge i through Eq. 8, where the plastic moment is always greater than the cracking moment M_{cr}, in this way the damage d_p in which the plastic strains starts is calculated as:

$$\begin{cases} d\phi_{i/j}^p = 0, & se \quad f_{i/j} < 0 \\ f_{i/j} = 0, & se \quad d\phi_{i/j}^p \neq 0 \end{cases}, \quad M_p^2 = \frac{2(1-d_p^2)}{F^0}R_0 + \frac{2q}{F^0}(1-d_p)\ln(1-d_p), \tag{8}$$

d_p is usually in the range between $(0,3-0,4)$. When the resistance function is equal to zero, the parameter k_0 (Eq. eq. (9)) is the plastic bending moment. The resistance function is also equals to zero when the moment is the ultimate bending moment, so it is possible to calculate the parameter c (Eq. eq. (9)):

$$k_0 = \frac{M_p}{1-d_p} = \overline{M}_p, \quad c = \frac{1}{\phi_u^p}\left(\frac{M_u}{1-d_u} - k_0\right) = \frac{\overline{M}_u - \overline{M}_p}{\phi_u^p} \tag{9}$$

where ϕ_u^p is the ultimate plastic rotation. For more information and details on lumped damage model, the reader is advised to consult the works of Cipollina et al. [4], Flórez-López [5], Perdomo et al. [6], Flórez-López et al. [9].

1.2 Fragility curves

The main objective of PBEE is to develop methods to design structures with predictable levels of safety. In PBEE, the mean annual rate of exceedance of a damage state is obtained by integrating the fragility curve with respect to the seismic hazard curve. There are different techniques reported in the literature to calculate the fragility curves that can be based on expert opinion, experimental, analytical, hybrid or empirical curves. The precision of the fragility curves obtained with each method depends basically on the behavior and nature of the demand parameter chosen. In this paper four techniques are evaluated: Empirical, Moment method, Probabilistic Seismic Demand Model (PSDM) with linear and bi-linear regression laws.

Empirical method

With this technique, the probability an EDP has to exceed a particular performance level for a given IM is estimated as the sum of those events which exceed the capacity C over the total number of seismic excitation. The empirical CDF method provides discrete probability data points at a given value of IM, and can be considered the most precise method if an adequate number os simulations and ground motions are used in structural analysis.

$$P[EDP \geq C] = \frac{m}{n} \tag{10}$$

where, m is the total number of simulations that exceed the capacidade level C and n is the total number of simulations performed.

Moment Method

In the moment method a lognormal distribution is used to represent EDP results for each IM testes. In this way the median μ and the dispersion β of the structural response can be estimated analytically according to Eq.11, and the fragility curve (P[EDP $\geq$ C]) is calculated as:

$$\mu = \frac{1}{n}\sum_{i=1}^{n}\ln(\text{EDP}), \quad \beta = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\ln(\text{EDP}/\mu)}, \quad P[\text{EDP} \geq \text{C}] = \Phi\left(\frac{\mu(\text{IM}) - \ln(C(\text{IM}))}{\beta(\text{IM})}\right) \tag{11}$$

where, n is the number os simulations performed for each tested IM.

Probabilistic Seismic Demand Model (PSDM)

Cornell et al. [19] were the first to relate the IMs to the EDPs in a closed form, with this technique the variance of the structural demand for a wide range of earthquakes is described by regression models known as Probabilistic Seismic Demand Model (PSDM). Usually linear regressions are applied in IDA results transformed to lognomal space, however other regressions laws can be applied depending on the nonlinear behavior of the EDP chosen. In this paper we are comparing linear and bilinear regression laws applied to PSDM, based on the works of [2, 21]. For PSDM the following hypothesis are assumed: i. lognormal distribution of IDA results; ii. Existence of a regression law to represent the median of the structural demand as a function of IM; iii. Homoscedasticity assumption: constant logarithm standard deviation of the EDPs along the IMs. With PSDM the fragiity curves are calculated as P_f in Eq. 12.Applying a linear regression low in the lognormal space it is possible to use the median $\hat{D}$ of the structural demand (Eq. 12).

$$P_f[D \mid \mathrm{IM} \geq \mathrm{C} \mid \mathrm{IM}] = 1 - \phi\left(\frac{\ln(\hat{C}/\hat{D})}{\sqrt{\beta^2_{D|IM}}}\right), \quad \hat{D} = a\ (IM)^b \tag{12}$$

where $\hat{C}$ is the median of the structural capacity, $\hat{D}$ is the median of the demand, β_C is the dispersion of the demand, a and b are constant parameters of the regression model.

1.3 Results

The methodology was tested in a typical RC regular bare frame of six-story and 2 bays with a uniform story height of 3.2 m, an uniform bay width of 5 m, the building is symmetric in plane and elevation. The characteristic strength of concrete and steel is taken as 25 MPa and 415 MPa, respectively. The 6-story frame was modeled with LDM using one finite element each bar. The fundamental period of 0.74 s was calculated with modal analysis. This frame was subjected to 50 earthquakes artificially generated for the soil of California, ($38.123°$ north and $121.123°$ west), $F_a = 1.293, F_v = 1.856, S_s = 0.634g, S_1 = 0.272g$ with spectral response compatible with the target spectra given by ASCE [23], the duration of the earthquakes was set to 30 s with $dt = 0.01$ s. The generated earthquakes were scaled to intensity measures from $0.1g$ to $1.8g$ for $Sa(T_1, 5\%)$ to be used in the incremental dynamic analysis.

The following steps are suggested to meet the objectives: i. Run incremental dynamic analysis; ii. Post process the damage maps from the internal variables of damage in the nodes of the elements; iii. Identify and formulate the failure mechanisms according to the system reliability theory; iv. Evaluate the fragility curves using the internal variables of damage as EDP in PBEE methodology.

Fig. 2 shows the damage maps of 3 simulations that presented failure due to the formation of collapse mechanisms. Fig. 3 presents the damage values of the columns and beams along the 6 floors for the simulations in collapse prevention limit state characterized by interstory drift ratios higher than 4%. Observing Fig. 2 - Fig. 3 it is possible to verify that the frame presents two main failure mechanisms: a soft-story in the columns of the first floor and a global mechanism with all beams over the first floor and the base of the columns forming complete hinges, as illustrated in Fig. 4.

To formulate the system failure, consider the events of failure in each node of the elements given by $E^{node_j}_{V_i} \equiv d^{node_j}_{V_i} \geq d_{ult}, \quad E^{node_j}_{P_i} \equiv d^{node_j}_{P_i} \geq d_{ult}$, with $d_{ult} = 0.60$. It is possible to represent the soft-story failure at the k-th floor as:

$$F_{SS_k} = \left[\cap_{i=1}^{n} E^{nó1}_{P_i}\right]_k \cap \left[\cap_{i=1}^{n} E^{nó2}_{P_i}\right]_k, \quad i = 1, ..., n \tag{13}$$

where n is the columns number at each floor and k is the story number, $\cap$ represents the intersection of the events. Eq. 13 shows that the soft-story is given by an association in parallel of the events of failure at nodes 1 and 2 of the columns. Parallel failures are governed by the toughest elements in the system, in this way soft-story occurs when the node 2 of the columns of the first floor reach the ultimate damage.

The global failure mechanism evolving the beams and columns can be represented as:

$$F_{G_l} = \left[\cap_{k=1}^{n} \left[E^{node1}_{V_j} \cap E^{node2}_{V_j}\right]\right]_l \cap \left[\left[E^{node1}_{P_j}\right]_l \cup \left[E^{node2}_{P_j}\right]_l\right] \tag{14}$$

where n is the floor number, j is number of elements at each story, $\cap$ is the intersection of the failure events and $\cup$ is the union between events. In global failure the most resistant elements are the beams of the first floor, and these are the nodes that control the formation of the global mechanism.

Fig. 5 shows the correlation between internal variables of damage on the beams (1st floor) and columns (node 2 - 1st floor) - elements that control Global and soft-story failures, respectively. It can be observed a higher number of simulations failing with global mechanism (green area in Fig. 5) and some samples in a combined failure mode (yellow area in Fig. 5).

From the formulation of failure mechanisms it is possible to calculate the fragility curves using as EDP the internal variable of damage in each node of the elements that control the mechanisms. Fig. 6 shows the mean and dispersion used to calculate the fragility curves for soft-story failure. It is possible to verify the differences between the values estimated through moment method (moving averages and variable dispersion) and PSDMs with linear and bilinear regressions. Fig. 7 shows the fragility curves for soft-story failure calculated for capacity levels of 0.30, 0.40 and 0.60. It is clear that the method of PSDM with linear regression is not precise enough to represent fragility curves with EDPs of damage, once its results are clearly different from others. PSDM with bilinear regression was precise for capacity levels of 0.30 and 0.40 while moment method showed a good correspondence for all curves with the results of empirical method, considered more precise.

Fig. 8 shows the mean and dispersion used to calculate the fragility curves for global failure. Fig. 9 shows the fragility curves for global failure calculated for capacity levels of 0.40, 0.50 and 0.60. It is clear that PSDM didn't represent well the fragility curves if compared to the empirical response, moment method showed a good precision to represent results of empirical estimation.

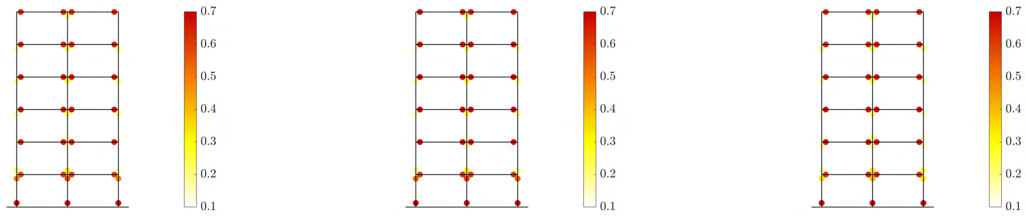

Figure 2. Damage maps of the simulations in collapse prevention limit state, a) Sa = 1.6g, b) Sa = 1.7g and c) Sa = 1.8g.

2 Conclusions

This paper shows the application of LDM in a PBEE approach to estimate the seismic vulnerability of RC frames. It was shown that it is possible to use the internal variables of damage as the EDP in the calculation of the fragility curves by identifying and formulating the failure mechanisms using system reliability theory. For the studied frame, the failure mechanisms are formed by double parallel associations of failures in the elements resulting failure mechanisms of soft-story in the columns first floor and a global failure mode evolving all beams of the structure and the columns of the first floor. It was clear that to calculate the fragility curves using as EDP the internal variables of damage, it is necessary a detailed evaluation of the hypotheses assumed for PSDM once the damage results of incremental dynamic analysis showed to be more nonlinear than what is usually observed for IDRs. In general it is indicated to use the empirical method or the moment method to build the fragility curves with variables of damage, these methodologies can be considered more precise although they require a higher number of simulations and give a discrete description of the fragility function.

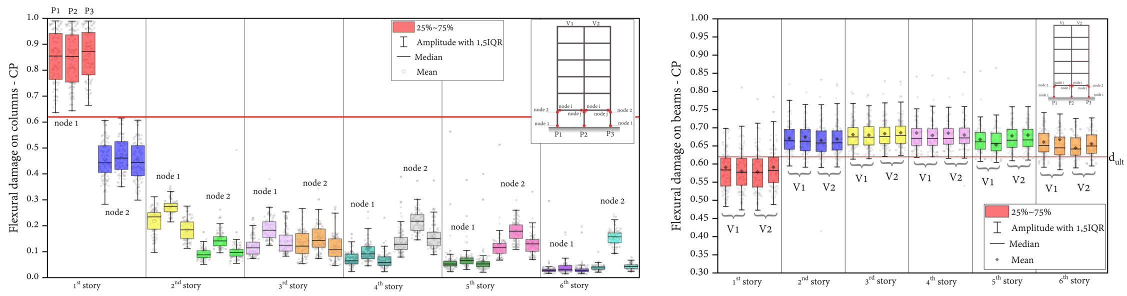

Figure 3. Flexural damage a) on columns; b) on beams - Collapse prevention (CP).

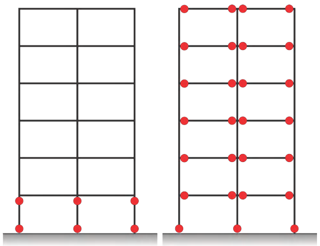

Figure 4. Failure modes.

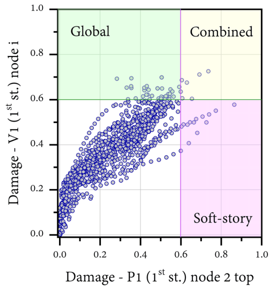

Figure 5. Correlation of damage on columns and beams.

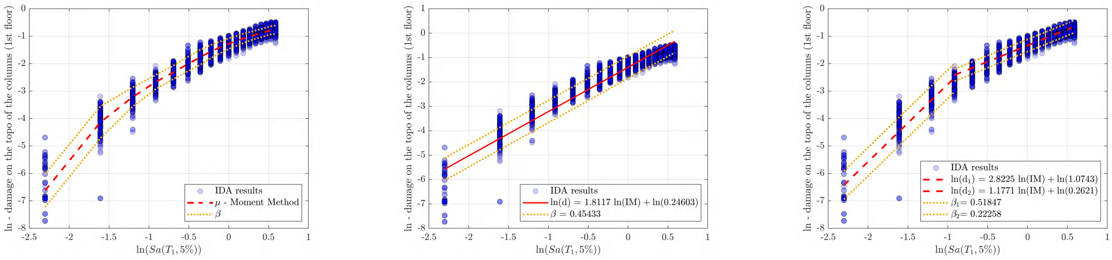

Figure 6. Mean and dispersion for soft-story failure calculated with a) Moment method, b)PSDM - linear regression and c) PSDM - bilinear regression.

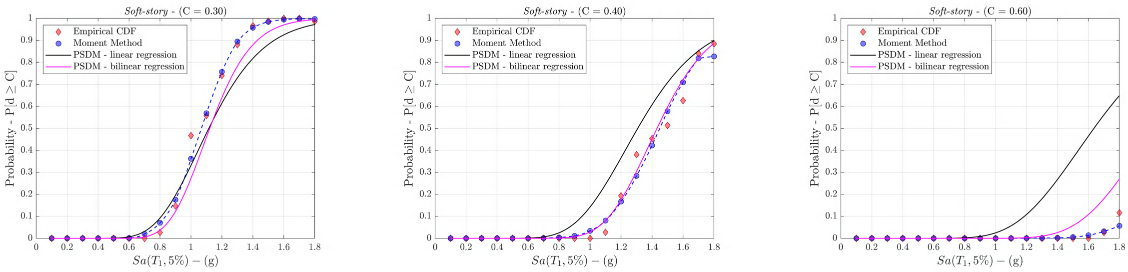

Figure 7. Fragility curves - soft-story failure, a) C = 0.30, b) C = 0.40, c) C = 0.60.

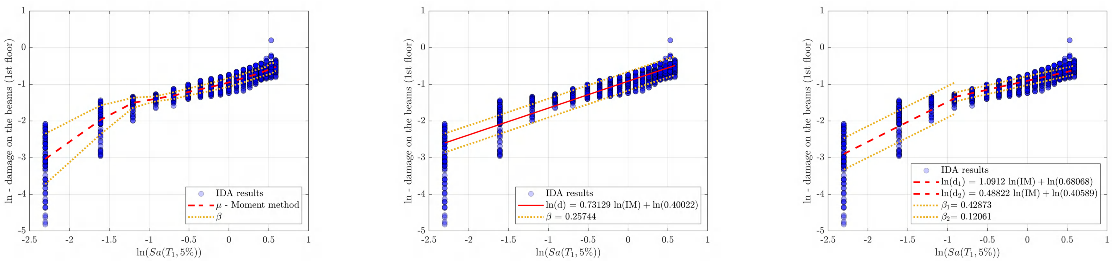

Figure 8. Mean and dispersion for global failure obtained with a) Momento method, b) PSDM - linear regression and c) PSDM - bilinear regression .

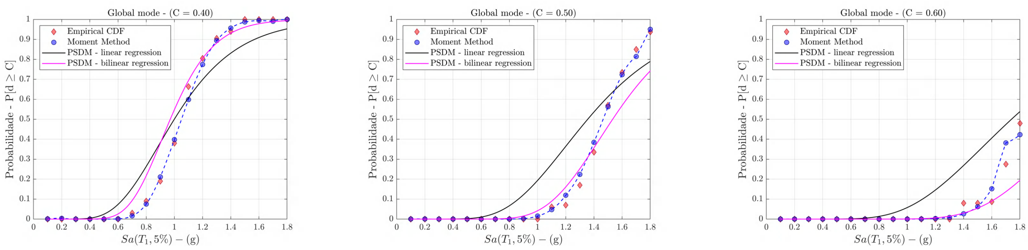

Figure 9. Fragility curves - global failure mode a) C = 0.40, b) C = 0.50, c) C= 0.60.

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