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Tuned liquid column damper modeled by pressure based eulerian approach using isoparametric quadrilateral finite element

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Abstract. This paper model sloshing in 2D tuned liquid column damper (TLCD) using a pressure based Eulerian approach. The fluid domain is discretised by isoparametric quadrilateral 4-nodes (Q4) finite elements coded in MatLab. A TLCD was modelled with rigid contours and a free surface to study uncoupled liquid reservoir. It is performed free and forced (harmonic) analysis to determine dynamic parameters. The numerical results were validated using experimental Alkmin's results presenting acceptable errors (inferior of 1.7% relative error) with less than 5k elements. The present numerical implementation presents a good agreement with analytical solution and experimental results.

Keywords: Linear sloshing, 2D, isoparametric FE, Galerkin acoustic fluid, TLCD.

1 Introduction

A passive damper device for vibration control of tall buildings [1], [2], wind turbines[3], [4] and ships [5] is tuned liquid column damper (TLCD) investigated extensively by many researches [6]–[8].

In most of bibliography references, TLCDs are modelled as a reduced 1DoF non-linear (or stochastically linearized) differential equation. However, several papers currently seek to modify the classic TLCD configuration through new geometries and/or fluids [7]–[11]. It's recurrent, in literature, a modified TLCD geometry (rectangular TLCD) which reservoir is composed by the flat plates' assembly. Altunisik Yetisken and Kahya [12] and Silva and Morais [13] observe a good agreement with experimental results and FE modelling of rectangular TLCD, but a great discrepancy by report to analytical natural frequency formula. Chaiviriyawong et al. [14] deduce a modified formula for natural frequency of rectangular TLCD proposing an effectively length with a good agreement with experimental results.

Sloshing problems are modelled by Lagrangian, Eulerian and mixed Eulerian-Lagrangian descriptions [15]. However, a pressure-based Eulerian description [16] is an easy way to study sloshing and fluid-structure problems implemented in most of commercial platforms. This modelling approach could be applied to simulate the performance of TLCDs with new geometries and/or others fluid.

This paper model sloshing in 2D tuned liquid column damper (TLCD) using a pressure based Eulerian approach. The fluid domain is discretised by isoparametric quadrilateral 4-nodes (Q4) finite elements coded in MatLab. A TLCD was modelled with rigid contours and a free surface to study uncoupled liquid reservoir. It is performed free and forced (harmonic) analysis to determine dynamic parameters. The numerical results were validated using experimental results [2] presenting acceptable errors with less than 5k elements. This comparison of TLCD with experimental results and Chaiviriyawong et al. [14] presents a difference with analytical solution undetailed in specialized literature of passive vibration control. By this proposed numerical approach, the effect of fluid inertia due geometry TLCD has an influence as presented by the comparison with numerical, experimental and literature.

2 Pressure based approach for sloshing modelling

The water is discretized as linear compressible and irrotational fluid using acoustic wave equation without mean fluid flow[17], [18]:

$$\nabla^2\left(p + \frac{4}{3}\tau\,\dot{p}\right) = \frac{1}{c^2}\frac{d^2p}{dt^2} \tag{1}$$

where, velocity of sound $c = (\rho/K)^{0.5}$, in fluid medium, is function of density ρ and bulk modulus K, and dissipation constant $\tau = \mu/K$ is function of viscosity and bulk modulus K. But for this work, we suppose that τ is negligeable.

The boundary conditions of fluid domain is defined as described in Figure 1 [18]: (a) rigid wall Γ_{PR}, (b) free surface Γ_{SL} and (c) fluid-structure interface Γ_{FE}, where fluid boundary $\partial\Omega = \Gamma_{\mathrm{PR}} \cup \Gamma_{SL} \cup \Gamma_{FE}$.

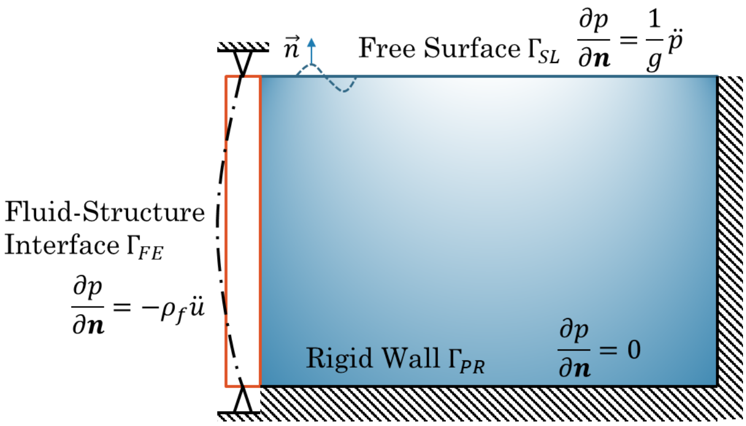


Figure 1. Fluid domain and boundary conditions of sloshing problem

The appropriate boundary conditions for these boundaries are as follows:

i. Free surface boundary condition (Γ_{SL}):

$$\frac{\partial p}{\partial z} = -\frac{1}{g}\frac{d^2p}{dt^2} \tag{2}$$

ii. Fluid-structure interface boundary conditions (Γ_{FE}):

$$\frac{\partial p}{\partial \vec{n}} = \nabla p \cdot \vec{n} = -\rho_f \ddot{u} \tag{3}$$

iii. Rigid wall condition (supposing $\ddot{u} = 0$) (Γ_{PR}):

$$\frac{\partial p}{\partial \vec{n}} = \nabla p \cdot \vec{n} = 0 \tag{4}$$

2.1 Galerkin finite element formulation for acoustic fluid domain

To discretise the wave equation, we adopted approximate function $\hat{\mathrm{p}}$ that satisfy the boundary conditions. Substituting $\hat{\mathrm{p}}$ in Eq. (1) we have the residue function $\mathcal{R}(\hat{\mathrm{p}})$. Applying the weighted residues method, the integral of residue function $\mathcal{R}(\hat{\mathrm{p}})$ plus approximation function $\hat{p}$ is defined as zero, as described:

$$\int_{\Omega_f} \mathcal{R}(\hat{p})\, \hat{p}\, d\Omega = \int_{\Omega_f} \left(\nabla^2 \hat{p} - \frac{1}{c^2}\ddot{\hat{p}}\right) \hat{p}\, d\Omega = 0 \tag{5}$$

Applying the boundary conditions (2)-(4), we obtain the following equation:

$$-\int_{\Omega_f} \nabla\hat{p}\, \nabla\hat{p}\, d\Omega + \oint_{\partial\Omega} \hat{p}\, \frac{\partial p}{\partial \vec{n}}\, d\Gamma - \int_{\Omega_f} \frac{1}{c^2}\ddot{\hat{p}}\, \hat{p}\, d\Omega = 0 \tag{6}$$

Eq. (6) describes the weak form of the sloshing problem in acoustic fluid with mobile boundaries Γ_{FE} . We have the weak form of the equation which is the starting point for discretizing the domain of the fluid by the finite element method. If we consider an incompressible fluid (sound velocity $c \to \infty$), Eq. (6) with only sloshing effect in 'mass' matrix results an ill-conditioned linear system. Then, as a penalty condition, Zienkiewicz et al. [18] propose compressibility inclusion to work around this difficulty.

3 Numerical Implementation

The weak form (6) was discretized using isoparametric finite elements. The elements adopted were: (a) isoparametric L2 (unidimensional element with two nodes) to discretize the free surface contour and fluid structure interface, and (b) isoparametric Q4 (two-dimensional four-node element) to discretize the fluid domain.

Assuming pressure in form of $\boldsymbol{p} \approx \hat{\boldsymbol{p}} \simeq \sum N_i \hat{p}_i$, where $\hat{p}_i$ are the nodal pressure of finite element at fluid domain, and N_i is spatial interpolating shape function, the discretized form of Eq. (6) can be written as:

$$\boldsymbol{K}_f\, \hat{\boldsymbol{p}} + \left\{\frac{1}{c^2}\boldsymbol{M}_f + \frac{1}{g}\boldsymbol{M}_{SL}\right\} \ddot{\hat{\boldsymbol{p}}} = -\rho_f \boldsymbol{C}_{FS}^T\, \ddot{\boldsymbol{u}} \tag{7}$$

where, $\hat{\boldsymbol{p}}$ and $\ddot{\hat{\boldsymbol{u}}}$ are the nodal acoustic pressure in fluid domain Ω_f and nodal solid acceleration in fluid-structure contour Γ_{FS} , respectively.

The matrix $\boldsymbol{K}_f$ and $\boldsymbol{M}_f$ are the acoustic 'stiffness' matrix (first term of Eq.(7)) and acoustic 'mass' matrix (last term of Eq. (7)), respectively using isoparametric Q4 finite element, given by:

$$\left(K_f\right)_{ij} = \int_{-1}^{+1}\int_{-1}^{+1} \left(\frac{\partial N_i}{\partial \xi}\frac{\partial N_j}{\partial \xi} + \frac{\partial N_i}{\partial \eta}\frac{\partial N_j}{\partial \eta}\right) |J|\, d\xi\, d\eta \tag{8}$$

$$\left(M_f\right)_{ij} = \frac{1}{c^2}\int_{-1}^{+1}\int_{-1}^{+1} N_i N_j\, |J|\, d\xi\, d\eta \tag{9}$$

where, $\boldsymbol{J}$ is the jacobian matrix to transform global coordinate system (x, y) to natural coordinate system (ξ, η); $|J| = \det \boldsymbol{J}$; the Q4 shape form $N_i = 0.25(1 \pm \xi)(1 \pm \eta)$; and $i, j \in [1,2,3,4]$.

The matrix $\boldsymbol{M}_{SL}$ is the free-surface boundary condition, and $\boldsymbol{C}_{FS}^T$ is the fluid-structure coupling matrix using isoparametric L2 finite element, given by:

$$(M_{SL})_k = \frac{1}{g}\int_{-1}^{+1} N_k\, N_k\, |J|\, d\xi \tag{10}$$

$$(C_{FS}^T)_k = \rho_f \int_{-1}^{+1} N_k \cdot \boldsymbol{n}\, N_k\, |J|\, d\xi \tag{11}$$

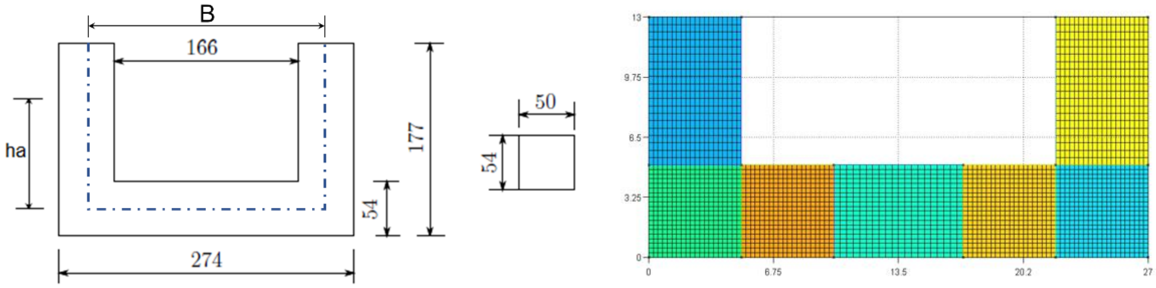
where, $\boldsymbol{n}$ is the outwardly directed normal to the element surface along the interface; and $k \in [1,2]$.

The shape functions are linear with continuity C_o (infinitely continuous interpolation functions inside the elements and continuous at the element interface or first discontinuous derivative). The numerical implementation was developed in MATLAB (MatLab R2015b).

4 Free Vibration of TLCD – Numerical Example

The present study analyses the sloshing frequencies of a TLCD device. We perform modal analysis and subject both configuration to harmonic forced movement. The present simulations are compared to analytical solution, literature and ANSYS Student 19.0 (Mechanical APDL) commercial platform.

Finite element meshes were building using mesh generator GMSH [19]. TLCD is divided into seven distinct parts, as described in Figure 2b. A mesh convergence study was done using a reference water level h_a. And B and L $(= 2h_a + B)$ correspond to the horizontal and total length to TLCD, respectively. With an identical aspect ratio for all elements, we observe a good convergence from 20 elements (counting water level $h_a = 105mm$). We use the same element aspect ratio for all meshes. Using a convergent mesh, we carry out the numerical determination of TLCD's natural frequencies as function of the water height level h_a (mm).



(a) (b)

Figure 2. TLCD geometric description (in mm) (a) and FE mesh example (b).

Table 1 and Figure 3 compare the results of free-surface fundamental frequency obtained by present simulation, experimental results [2], analytical formula $\omega = \sqrt{g/L}$, and modified length L_e analytical function [14]. The numerical solution presents a good agreement with experimental results. There was no discrepancy superior to 1.7%.

Figure 4 shows the first four modal shapes of fluid contained. The fundamental modal form reproduces the expected U-tube behaviour. And the following three modal shapes behaves as two tanks with aspect ratio $h/L = (h_a + 27)/50$. We can also observe that the free surface modal shape (with low frequencies) is uncoupled to cavity modes (with higher frequencies).

We analyse the TLCD subjected to a forced acceleration $(\ddot{u} = \ddot{u}_o \exp i\Omega t)$. Figure 5 shows frequency response function (FRF) of a node pressure on free surface of TLCD with water level height $h_a = 105mm$. We observe five peaks in frequency range $\Omega \in [0, 6]Hz$. This resonant frequency is near to the natural frequencies. Figure 5 shows the first four operational modal shapes of TLCD with water level height $h_a = 105mm$.

Table 1. Coefficients in constitutive relations

Column height h_a (mm)	**Present Results**	**Experimental** [2]	$\frac{(f_n - f_e)}{f_e/100}$	**Analytical**	**Reference** [14]
35	1.39	1.38	0.72	1.31	1.34
45	1.34	1.33	0.75	1.27	1.29
55	1.29	1.27	1.57	1.23	1.25
65	1.25	1.23	1.63	1.19	1.21
75	1.21	1.20	0.83	1.16	1.18
85	1.18	1.17	0.85	1.13	1.14
95	1.14	1.15	0.87	1.10	1.12
105	1.11	1.12	0.89	1.08	1.09

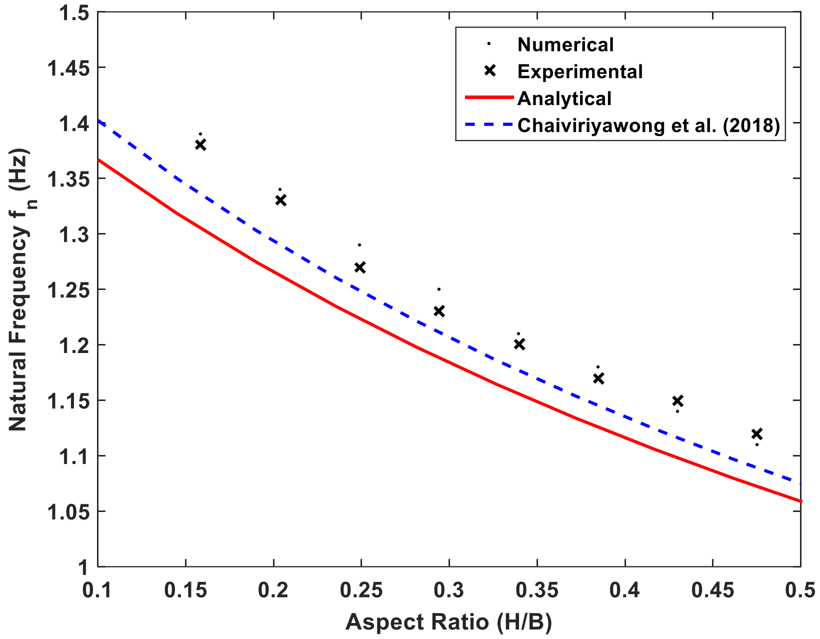


Figure 3. Evolution free-surface fundamental natural frequency (Hz) of TLCD as function of aspect ratio h_a/b for: present simulation ('.' dot), experimental results [2] ('x' cross), analytical formula $\omega = \sqrt{g/L}$ (continuous red line), and effective-length formula [14] (dashed blue line).

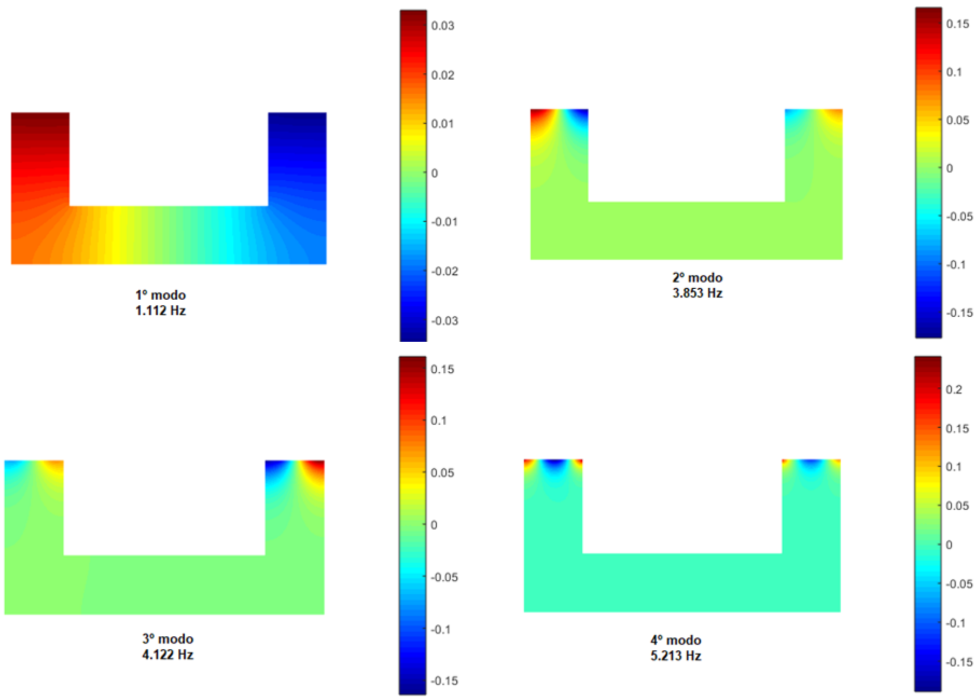


Figure 4. First four natural frequencies and modal shapes of TLCD with $h_a = 105mm$.

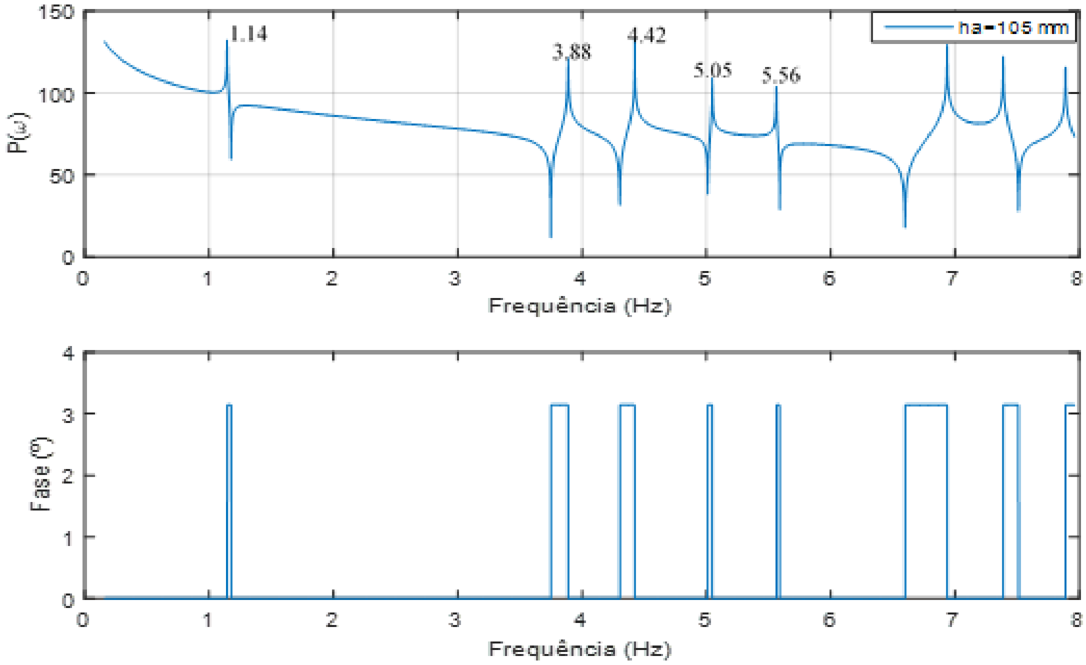


Figure 5. Pressure FRF on free surface node of TLCD with water level height $h_a = 105mm$.

5 Conclusions

An isoparametric 2D finite element was implemented to analyse sloshing in 2D tuned liquid column damper (TLCD) using a pressure based Eulerian approach. The fluid domain is discretised by isoparametric quadrilateral 4-nodes (Q4) finite elements coded in MatLab.

The present numerical code was applied to determine dynamic parameters of a TLCD containing liquid water analysing free and forced (harmonic) vibration. For free vibration analysis, the numerical solution is compared to analytical solution, literature (modified length solution) [14] and experimental results of TLCD's fundamental frequency as a function of water column height h_a. The numerical solution shows a good agreement (less than 1.7% relative error) with modified length solution and experimental results with less than 5k elements. Finally, a harmonic analysis was done by acceleration container liquid of a TLCD with $h_a = 105mm$. The FRF's resonance pic are remarkably close to two first natural frequency.

As perspectives, it is important: (a) coupling structural to acoustic fluid domain to perform fluid-structure interaction studies, and (b) implement a viscous-fluid damping model.

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