Crack detection in 2D structures using wavelet transform and the boundary element method.

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Abstract. Detecting damages in structures is of paramount importance as they can lead to the collapse of structural elements. In the current scenario, two reliable techniques for non-destructive testing of structures, namely ultrasound and X-rays, are practically applicable for precise localization of hidden damages. However, such techniques are costly, time-consuming, and require special procedures to be performed. Currently, research suggests the possibility of using numerical methods to assist in damage detection in structures. Generally, numerical methods for damage detection, such as cracks, are based on finite element modeling and comparisons between the signatures or responses of the structures before and after the occurrence of damage. This article proposes a new approach to damage identification based on the Wavelet Transform (WT) and the Boundary Element Method (BEM), making the numerical modeling of structures more accurate, straightforward, and without requiring the structure's signature before the damage occurs. To validate the adopted methodology, a cantilever beam containing a crack with different orientations will be modeled using the in-house programs BEMLAB and BEMCRACKER2D, as well as the algorithms implemented in MATLAB for handling wavelet coefficients and damage localization. The obtained results indicate that WT is a useful tool for identifying and monitoring damages in structures.

Keywords: Boundary elements; Wavelet transforms; Damage detection; BEMcracker2d.

1 Introduction

A large number of research studies have focused on finding numerical and computational tools to identify damages in structural elements. This technique is extremely useful when inspecting structures in hazardous environments or when visual inspection is impossible. Simply pinpointing where potential damage may exist actually helps narrow down the inspection area before applying more precise and costly tests. The application of these research efforts ranges from civil constructions, bridges, and dams to aircraft and pipelines. However, identifying damages by observing the external behavior of the structure poses a significant challenge due to various factors, such as environmental agents, structural nonlinearities, inaccuracies in numerical modeling, uncertainties in material properties, mass, and boundary conditions, all of which influence the accuracy of the diagnosis.

Computational and numerical methods for damage detection are not yet fully mature for practical use; therefore, new research on structure inspection and monitoring should continue. To enhance confidence in the use of computational and numerical methods for damage detection, it is necessary to conduct basic research applied to simple structures. Structural damage leads to a loss of structural stiffness, resulting in variations in static and dynamic responses [1]. Dynamic non-destructive tests are typically used to provide information and can be easily performed. These response data form the basis for the development of numerical procedures to identify damages.

The problem of damage detection in a structure aims to determine the presence of open cracks based on data provided at some discrete points in the structure. In this context, the data typically consist of experimentally obtained or numerically simulated displacements, stresses, natural frequencies, mode shapes, or accelerations [2]. Generally, the geometric parameters used to describe the damage include the shape, length, and location of the crack. To solve the inverse problem of damage detection, standard approaches involve consecutive direct analyses, where certain geometric parameters describing the damage are assigned, followed by subsequent checks to see if the calculated response matches the provided data.

This approach is based on the premise that structural damages induce variations in structural parameters, which can lead to significant changes in structural responses [3]. The Finite Element Method (FEM) is commonly used to obtain the structural response. To compare the damaged structural response with the intact one, a selected signature is defined as a residual function. The residual function is minimized using deterministic or stochastic algorithms [4][5]. When the residual function reaches a minimum, it can be concluded that the assumed geometric parameters describing the damage represent the actual damage. However, when using numerical computation approaches for damage detection, it is necessary to discretize the entire region between the assumed damage boundary and the structure boundary. Moreover, employing structural signatures to compare two response states requires significant computational effort and storage space for the data between the two states.

In this work, a novel approach based on the Wavelet Transform of the structural displacement obtained using the Boundary Element Method (BEM) is proposed for crack detection. This approach offers several advantages over standard formulations used in the inverse problem [6] of damage detection: (a) the use of BEM requires simplified meshes, (b) BEM produces more accurate structural responses, and (c) the formulation used here only requires the current structural response, and no previous response data is needed. In this study, the Wavelet Transform is numerically applied to detect damages in deep beams and panels under static loads. Additionally, the wavelet coefficient plot reveals peaks that are identified as damage locations along the length of the beam. A parametric study will be conducted to assess the variation of wavelet coefficients with damage, damage location, load intensity, flexural stiffness, and beam length [7]. Finally, all modeling will be performed using the BEMLAB2D software [8], with processing in the BEMCRACKER2D solver, and the implementation of Wavelet Transforms will be done in MATLAB.

2 Literature Review

Damages in structures can compromise the service life of the system, which is defined as the period of time during which the structure is capable of performing the functions for which it was designed, without the need for unforeseen interventions [9]. This is particularly concerning when it comes to bridges, as they are exposed to environmental conditions that alter their physical and chemical properties, undermining their durability and promoting the emergence of pathologies. Durability is defined as the structure's ability to withstand environmental influences specified by the project designer and the contractor during the project's development [10]. Pathology, on the other hand, is understood as the deterioration of performance over time in a product, component, or construction due to planning, design, execution, usage errors, and deterioration caused by interaction with the environment [11].

When designing a structure, loads are an essential element to be taken into consideration. This is because they directly influence durability, serviceability, stability, and strength. When a failure occurs, it is important to identify its origin. Often, this can be related to an excessive load on the structure [12].

Damage detection methods are classified into four levels: Level I (damage detection), Level II (damage localization), Level III (assessment of damage severity), and Level IV (determination of remaining service life due to damage) [13]. Damage is defined as changes in the material, geometric properties of structures, boundary conditions, connectivity between elements, cross-sectional geometry, loading, and material properties [14]. These factors can directly affect the structural behavior of a structure.

There are several methods for detecting damages, including destructive methods that require extracting part of the structure for damage identification and evaluation. Non-destructive methods for damage detection include acoustic emission, fiber optic sensors, guided ultrasonic waves, radiography, visual inspection, and vibration-based methods. Local damage detection methods are better suited for assessing the structural performance in small areas of the structure, while global detection methods leverage the overall changes caused by damage. Additionally, damages can also be detected using linear or nonlinear numerical methods. Linear methods assume that the structure remains in a linear elastic regime even after the damage occurs, whereas nonlinear methods consider that the structural behavior becomes nonlinear after the introduction of damage.

Most damage detection methods are based on vibration monitoring, as they consider that structural damages lead to changes in the structural and dynamic parameters of the structure. Some methods rely on static or dynamic analysis and are capable of locating the damage solely with the information provided by the already damaged structure. Wavelet analysis methods are also used. Other damage detection methods include modal analysis, monitoring technologies, and the comparison of the structural behavior before and after the damage [15].

2.1 Boundary Element Method (BEM)

The basic concepts of BEM are presented, described as the application of the classic Somigliana's Identity [16], which is the integral equation written in eq. (1).

$$C_{lk}^{i} u_{k}^{i} = \int_{\Gamma} u_{lk}^{*} u_{k} \, d\Gamma - \int_{\Gamma} p_{lk}^{*} u_{k} \, d\Gamma + \int_{\Omega} u_{lk}^{*} u_{k} \, d\Gamma\Omega \tag{1}$$

In eq. (1), the indices "k" represent the direction, and "i" denotes a generic point; u_k^i represents displacements; bk denotes body forces; u_k and u_k are, respectively, displacements and tractions along the boundary Γ; u_l^*k and p_l^*k are fundamental solutions [17]; for the point-i ∈ Γ, c_l^ik = δlk/2, where δlk is the Kronecker delta for a smooth Γ, and for the point - i ∈ Ω, c_l^ik = 1. In eq. (1), the fundamental solutions u_l^*k and p_l^*k, as well as more details, can be found in [18]. Figure 1 illustrates a boundary element model containing a crack and its discretization.

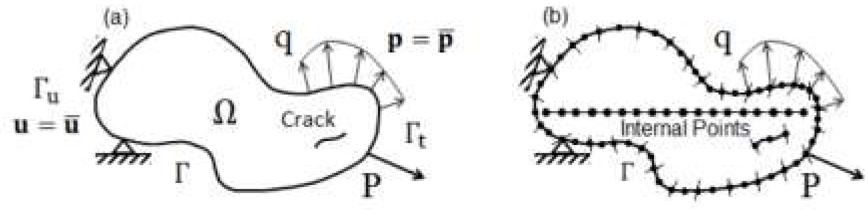


Figure 1 - (a) Body with damage and boundary conditions. (b) BEM modeling. [3]

The BEM using quadratic elements was implemented in the graphical interface BEMLAB2D [8] for drawing and meshing, as well as in the solver BEMCRACKER2D [18] for elastostatic analysis. Once all the unknowns $\{\mathbf{X}\}$ = {û|p̂}T are determined, all the boundary values uk $\{\boldsymbol{u}\}$ and $pk \in \{\boldsymbol{p}\}$ within the integral in eq. (1) are known. Therefore, displacements for any internal point, such as those represented in Fig.1b, which are on an arbitrary reference line, can be calculated. These internal displacements are the quantities that will be transformed using the Discrete Wavelet Transform also available in MATLAB. In the following sections, this technique will be explained.

2.2 Wavelet Transform

The Wavelet Transform is a signal analysis tool useful for identifying specific frequency-time characteristics of a signal. This technique was originally proposed by Newland and has become an important tool for vibrational signal processing [19-20]. The wavelet transform divides the input signal, for example, in the time domain, into a series of localized base functions or "wavelets" along the time axis. These wavelets allow the identification of local features, ranging from their scale to their position [21].

Wavelets are a class of functions used to describe and localize a given function in both space and scale. From the "mother" function ψ(x), it is possible to create a family of wavelets. In this study, the Wavelet Transform was applied to a set of static displacements u(x) varying along a reference line of two-dimensional structures. The mother wavelet function ψ(x) in eq. (2) can be used to calculate the Wavelet Coefficient ψa,b (x_0) at any point x = x_0. The translation parameter "b" and the dilation or contraction parameter "a" define the term ψa,b (x_0) as the "Daughter Wavelets" [22].

$$\Psi_{a,b}(x_0) = \frac{1}{\sqrt{a}}\Psi\left(\frac{x_0 - b}{a}\right) \tag{2}$$

2.3 Discrete Wavelet Transform (DWT)

In practical signal processing, the Wavelet Transform is often discretized, with the dilation parameter "a" and translation parameter "b" being defined discretely, as shown in eq. (3). The use of discrete values for "a" and "b" makes the procedure more efficient [16].

$$a = 2^j, \qquad b = 2^j k, \qquad (j,k) \in Z \tag{3}$$

Where Z is a set of integers. This procedure is called Discrete Wavelet Transform (DWT), and the wavelets ψ(x) are then given by eq. (4).

$$\Psi_{j,k}(x) = 2^{\frac{j}{2}}\Psi(2^j x - k) \tag{4}$$

In this case, the DWT can be expressed as follows, as shown in eq. (5):

$$C_{j,k} = C_{j,k}(x_0) = \int_{-\infty}^{\infty} u(x)\, \Psi_{j,k}(x) \tag{5}$$

The DWT coefficients are computationally less complex and extremely sensitive to the sudden stiffness loss in the displacement signal u(x) of a damaged structure. In the next section, the DWT from eq. (3) will be applied to displacement signals u(x) obtained from similar crack-like damages induced in 2D structures. There are many types of mother wavelet functions, but a detailed analysis of their properties is beyond the scope of this study. The objective here is to use a mother wavelet function that can effectively detect the position of damages in the structure.

2.4 BEMLAB2D and BEMCRACKER2D

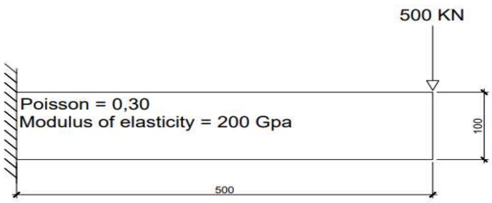
The software BEMLAB2D [8] was developed with the aim of providing a graphical interface for the pre-processing and post-processing of two-dimensional problems, making it a GUI - Graphical User Interface - type tool. It allows users to generate meshes and visualize the results of elastostatic analysis processed by the BEMCRACKER2D program [20].

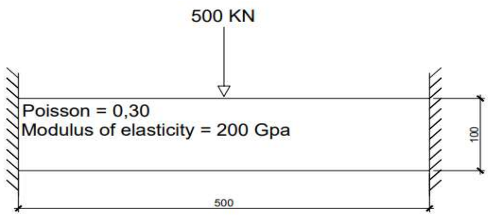
BEMCRACKER2D is a software written in C++ using Object-Oriented Programming (OOP) concepts, aiming to analyze two-dimensional elastostatic problems for plane stress and plane strain, based on the boundary element method [3]. The program has three calculation modules: 1) Standard BEM (Module I); 2) BEM without propagation (Module II); and 3) BEM with propagation (Module III). Therefore, the procedures executed by the program include stress analysis using conventional BEM and BEM with crack propagation for crack problems, in which it evaluates Stress Intensity Factors (SIFs) using the J integral method, assesses crack propagation direction/correction with various direction criteria, as well as evaluates fatigue life using the Paris Law.

3 Materials and Methods

This study aims to locate damages in beams, and in this regard, the proposed methodology consists of three main steps: a) modeling of beams using BEMLAB2D; b) obtaining displacements/deformation using BEMCRACKER2D; and c) applying the Wavelet Transform to find the location of the damage.

To validate the proposed methodology and locate the damage, two beams will be modeled, as illustrated in Figure 2. In the next section, the following results will be presented: elastic deformation, aiming to validate BEMCRACKER2D, as well as graphs of the Discrete Wavelet Transform (DWT) versus the length of the beam, to illustrate the location of the damage.





(a) (b)

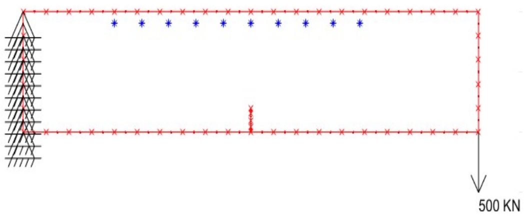
Figure 2 - Beam models: a) Simply supported and cantilever; b) Double-supported.

3.1 Results and Discussion

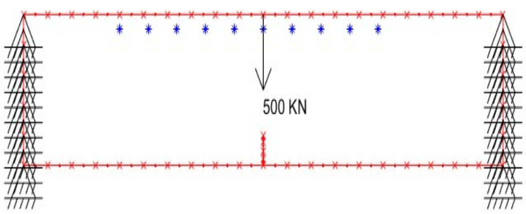
The following examples were modeled in BEMLAB2D and analyzed by BEMCRACKER2D to obtain the displacements. Both meshes consist of 98 quadratic elements, with 90 continuous elements on the boundary and 8 discontinuous elements at the crack (4 per face). It is essential to emphasize that the crack modeled here is purely illustrative and fictitious, and does not exist in the physical model. It serves as a verification parameter with the Wavelet Transform.

A reference line was used in both examples to represent the vertical displacement signal d(y). This line is composed of equally spaced internal points, starting 100 mm from the left end and ending 130 mm from the right end of the beam, at a standard height of 90 mm from the bottom line (boundary).

Figure 3 illustrates the discretized model of the beams under study, as well as the reference line, generated by BEMLAB2D.



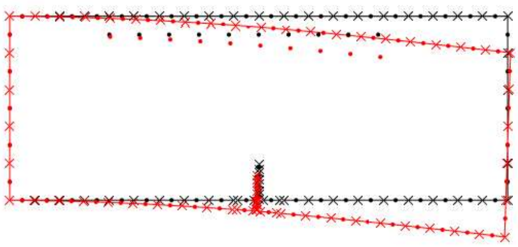
(a)



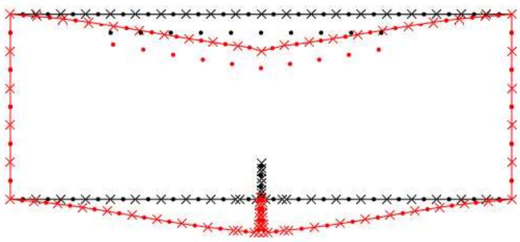
(b)

Figure 3 - Discretized beam models: a) Simply supported and cantilever; b) Double-supported.

Figure 4 shows the deformation of the beams, which is as expected, with maximum displacement at the end and middle of the beam, respectively.



(a)



(b)

Figure 4 - Deformation of the beam: a) Simply supported and cantilever; b) Double-supported.

The Wavelet Transform was applied to the vertical displacement signal d(y) obtained from the internal points of the reference line, and the MATLAB wavetoolbox was used to calculate the Biorthogonal 3.7 Wavelet Transform (bior3.7). In Figures 5 and 6, we have, respectively, the DWT x Beam Length, for the beams under study, where the largest peak corresponds to the exact location of the damage, i.e., 250 mm.

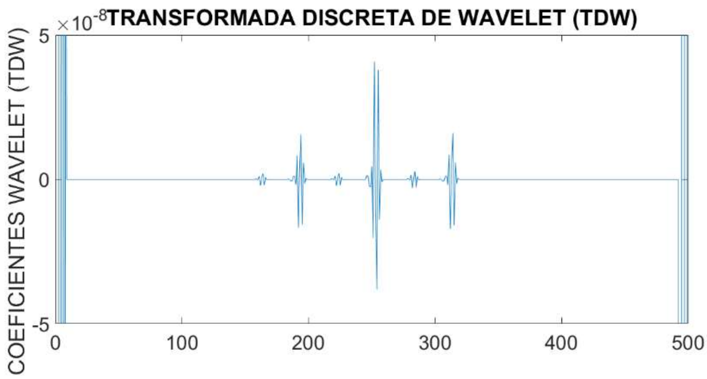


Figure 5 - Wavelet Coefficient for the simply supported and cantilever beam.

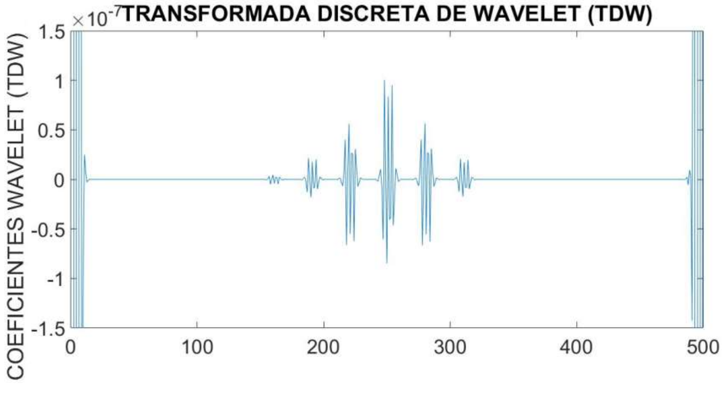


Figure 6 - Wavelet Coefficient for the double-supported beam.

Examining fig. 5 and fig. 6, peaks can be observed near the position of the fictitious crack, corresponding to the node numbers on the x-axis of the figure. The highest peaks correspond approximately to the position indicated by the internal point number "10 node 3" (or 250 mm) of the reference line on the x-axis and, consequently, very close to the exact position of the modeled crack. Therefore, the bior3.7 mother wavelet was able to indicate the location of the induced damage with reasonable accuracy.

It can also be observed that the DWT coefficients, in both graphs, showed undesired peaks that do not correspond to the location of the damage. The undesired peaks are at the ends of the beam due to the prescribed boundary conditions at the left end support with zero displacement and at the right end with a concentrated force. There are also other smaller spikes near the vertical damage. Such peaks exhibit small discontinuities in the vertical displacement signals d(x) at the points of the reference line in the vicinity of the crack. These DWT signal noises have also been observed by many other researchers using FEM modeling for the discretization of embedded damages in beams [23], [24], [25].

4 Conclusions

In this paper, a methodology addressing the inverse problem of damage detection was presented by combining Wavelet Transforms with Boundary Elements. By applying Wavelet Transforms to a static response treated in a model using the Boundary Element Method (BEM), it is possible to find the precise location of the damage. The use of BEM by the BEMCRACKER2D program, along with the Wavelet families, proves to be a viable alternative for obtaining the numerical response of the damaged structure due to its proven efficiency in this damage detection process.

References

[1] Buda, G., Caddemi, S.: Identification of concentrated damages in Euler Bernoulli beams under static loads. J. Eng. Mech., Vol. 8, 2007, pp. 942-956.

[2] Damage Models and Algorithms for assessment of structures under operating conditions. Taylor & Francis Group, London, UK, 2009.

[3] SILVA, Ramon S. Y. R. C. , BEZERRA, Luciano M. , PEÑA, Luiz A. P., GOMES, Gilberto & DA SILVA, Welington V. “Boundary element and wavelet transform methods for damage detection in 2D structures”. INTERNATIONAL JOURNAL FOR COMPUTATIONAL METHODS IN ENGINEERING SCIENCE AND MECHANICS 2019, VOL. 20, NO. 3, 242–255.

[4] Gill P.E., Murray W. Write M.H.: Pratical optimization. London, Academic Press, 1982.

[5] Reed H. M.: Stochastic & deterministic inverse problem solutions for structural damage condition assessment in a multi-physics context. PhD Thesis, Cornell University, 2014.

[6] Karageorghis A., Lesnic D., Marin L.: A moving pseudo-boundary method of fundamental solutions for void detection. Numerical Methods for Partial Differential Equations, Vol. 29, 2012, pp.935-960.

[7] P.K. Umesha,∗ R. Ravichandran & K. Sivasubramanian. Structural Engineering Research Centre, Council of Scientific and Industrial Research, Chennai 600113, India.

[8] DELGADO NETO, Á. M. "BEMLAB2D: Interface Gráfica de Modelagem, Visualização e Análise com Elementos de Contorno – Uma Aplicação em Problemas Elastostáticos". Brasília: Dissertação (Mestrado em Engenharia Civil) – Programa de Pós-graduação em Estrutura e Construção Civil, Universidade de Brasília, UnB, 2017. 112 p.

[9] CEB (1990), CEB - FIP Model Code 90 (MC - 90) Design Code. Lausanne, May 1993. 437p. (Bulletin DíInformation, 213-214).

[10] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. NBR 6118: Projeto de estruturas de concreto - Procedimento. Rio de Janeiro, 2014.

[11] Nepomuceno, A. A., Teatini, J. C.(2009). Notas de aula da disciplina de Patologia, Recuperação e Manutenção de Estruturas da Universidade de Brasília.

[12] Helene, P. R. L. ; Pereira, F.(2007). Rehabilitación y mantenimiento de estructuras de concreto. São Paulo: Cargraphics, 598p.

[13] Rytter, A.(1993). Vibration based inspection of civil engineering structures, Ph.D. Dissertation, Department of Building Technology and Structural Engineering, Aalborg University, Denmark.

[14] Doebling, S.W., Farrar, C.R., Prime, M.B., Shevitz, D.W.(1996). Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: A literature review, Los Alamos National Laboratory report no. LA-13070-MS.

[15] Ramos, L.F.(2007). Damage identification on masonry structures based on vibration signatures, PhD dissertation, University of Minho, Guimarães, Portugal.

[16] Brebbia C. A., J. Dominguez.: Boundary elements an introductory course. WIT Press/Computational Mechanics, 1992.
[17] Bezerra L. M., Saigal S.: A boundary element formulation for the Inverse Elastostatics Problem (IESP) of flaw detection. International Journal for Numerical Methods in Engineering, Vol. 36, 1993, pp. 2189-2202.
[18] GOMES, G.; DELGADO NETO, Á. M.; WROBEL, L. C. "Modelagem e visualização de trincas 2D usando equação integral de contorno dual". XXXVII IBERIAN LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING, Brasília, v. 37, 2016.
[19] Dilena M., Morassi A.: Dynamic testing of damaged bridge. Mechanical Systems and Signal Processing, 2011, pp. 1485-1507.
[20] GOMES, G.; DELGADO NETO, Á. M.; WROBEL, L. C. "Modelagem e visualização de trincas 2D usando equação integral de contorno dual". XXXVII IBERIAN LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING, Brasília, v. 37, 2016.
[21] Newland, D. E.: Random Vibrations, Spectral and Wavelet Analysis. 3rd ed. Longman, Harlow and John Wiley. New York. 1993.
[22] Newland, D. E.: Wavelet Analysis of Vibration, Part I: Theory. Journal of Vibration and Acoustics. Vol. 116, pp. 409-416.
[23] Ovanesova, A. V., Suárez L. E.(2004). “Applications of wavelet transforms to damage detection in frame structures.” Journal Engineering Structures, Volume 26, Pages 39-49.
[24] Polikar, R., (1994). Wavelet tutorial, Ames, IA.
[25] X. L. Peng, H. Hao, and Z. X. Li, “Application of wavelet packet transform in subsea pipeline bedding condition assessment,” Eng. Struct., vol. 39, pp. 50–65, 2012. DOI: 10.1016/j.engstruct.2012.01.017.