BemLab2D & BemCracker2D: A computational package for modeling and analyzing fracture mechanics problems with boundary elements

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Abstract. The stress analysis in structures with complex geometry as in aircraft fuselages, where geometry is continuously altered by crack growth, usually requires the use of numerical methods, since the presence of cracks in the structure raises difficulties related to the modeling and, consequently, to the calculation of the stress intensity factors (SIF). In this aspect, the dual boundary element method (DBEM) has been applied in this type of analysis, taking advantage over the FEM considered, since there is no need for continuous remeshing at each crack increment. Therefore, this work introduces a software written in C ++ and based on Object Oriented Programming, called BemCracker2D, for two-dimensional analysis by DBEM for crack propagation problems in the field of linear elastic fracture mechanics, as well as its BemLab2D GUI. Parameters as SIF, crack-growth, number of load cycles and others, will be computed and compared with examples from the open literature in order to attest to the program's efficiency and robustness.

Keywords: crack growth, DBEM, bemcracker2d, bemlab2d

1 Introduction

According to De Lacerda and Wrobel [1], regardless of the physical mechanism involved, the crack propagation process invariably requires the use of incremental numerical methods for its analysis. In this regard, finite element methods (FEM) and boundary element methods (BEM) stand out.

The FEM, whose history in fracture mechanics applications is due to Gallagher [2], has been applied in the process of crack propagation [3-4]. However, in most formulations, there is a need for continuous remeshing to track the crack extension, which can make this method quite time-consuming. The BEM has also been applied in incremental analyses of crack propagation problems, through the use of subregions, or more recently, by employing a dual formulation known as DBEM [5-7]. The DBEM has proven to be more efficient and does not present remeshing difficulties as in FEM.

On the other hand, Computer Graphics has been playing an important role as a support tool in the development of computational programs in the field of engineering, especially in the area of geometric modeling, which requires studies and developments of efficient algorithms and sophisticated data structures [8]. This demands innovation both in the programming tools used and in the programming approach. From this perspective, a new programming philosophy has been emerging: Object Oriented Programming (OOP) [9], where the focus is on defining objects - entities involved in the system - and their classes and relationships, rather than decomposing the program into procedures or functions.

An example of a programming language that fully supports Object-Oriented Programming (OOP) is the well-known C++ language, developed by Stroustrup [10]. Gomes [8] developed a data structure for two-dimensional boundary element models based on OOP principles, which was employed in the development of BEMLAB2D [11], a Graphical User Interface (GUI) for drawing and generating the mesh necessary to feed data into the BemCracker2D program [12-13].

This work aims to present the BemCracker2D program, implemented in C++ language and based on OOP principles, as well as its graphical interface BemLab2D, as a computational package for analysis and modeling of crack propagation problems using DBEM.

The article is organized as follows: Section 2 presents the main concepts of Fracture Mechanics, the formulation of DBEM, and the key computational aspects of BemCracker2D and BemLab2D; Section 3 describes the Materials and Methods used; Section 4 showcases the main applications of the computational package; finally, Section 5 presents the concluding remarks.

2 Theoretical Background

2.1 Fracture Mechanics

Linear Elastic Fracture Mechanics (LEFM) is based on energy-based fracture criteria to analyze crack propagation. In this context, the concept of the Rate of Energy Release, introduced by Griffith [14], becomes relevant as it relates the available energy to the energy required for crack propagation. Other important parameters are the Stress Intensity Factors (SIFs), K, proposed by Irwin [15], which quantify the stresses near the crack tip for different opening modes. In general, the value of K is expressed as:

$$K = \sigma\sqrt{\pi a} * Y\left(\frac{a}{W}\right) \tag{1}$$

The formulation of the J-Integral proposed by Rice [16] relates the Rate of Energy Release, $\mathcal{G}$, to the stress and strain field in the vicinity of the crack. The analysis of SIFs is performed through techniques such as the J-Integral, which allows for a precise determination of the elastic field inside the crack. According to Rice and Tracey [17], the relationship between the J-Integral and the SIFs is given by,

$$J = \frac{{K_I}^2 + {K_{II}}^2}{E'} \tag{2}$$

where K_I and K_{II} are the Stress Intensity Factors (SIFs), corresponding to the opening mode (mode I) and the shearing mode (mode II) at the crack tip. E' represent the elastic modulus of the material, being equal to E for plane stress conditions and equal to $E/(1 - \nu^2)$ for plane strain conditions.

The direction of crack propagation in LEFM is a topic of interest, and several criteria using numerical methods are applicable in this context [18]. The Maximum Circumferential Stress (MCS) Criterion states that the crack will propagate normal to the plane with the maximum circumferential stress, provided that this stress is greater than the corresponding value of K_{IC} (without fatigue influence). On the other hand, the Maximum Potential Energy Release Rate (MPERR) Criterion uses the energy release rate as a basis for determining the propagation direction, θ. The Minimum Strain Energy Density (MSED) Criterion, proposed by Sih [19], evaluates the propagation direction based on the magnitude of the strain energy density, S. All these criteria find application in scenarios of linear elastic fracture and provide relevant information about crack propagation. Additionally, the analysis of crack propagation due to fatigue is discussed, which is essential in determining fatigue life, where the Stress Intensity Factors (SIFs) play a crucial role in characterizing the crack propagation rate.

2.2 Dual Boundary Element Method (DBEM)

The solution of problems involving cracks cannot be achieved with the conventional method of boundary elements due to the coincidence of crack contours, leading to an ill-posed problem. This coincidence results in identical algebraic equations for a pair of coincident source points on the crack contour since the same integral equation of displacement is applied at both points along the entire contour of the problem. To overcome this difficulty, the dual boundary element method (DBEM) is employed. The DBEM considers two independent equations - the integral equations of displacement and traction - with the same integration path for each pair of coincident source points, but with distinct integral equations. This method generates independent algebraic equations, forming the basis of DBEM, and allows for the efficient determination of displacements and tractions on the contour points [20-21].

The method considers two different equations on each face of the crack, which are the integral equations of displacement and traction. Assuming the absence of surface forces and continuity of displacements at a point x' on the contour, the integral equation of displacement is given by:

$$c_{ij}(x')u_j(x') + CPV \int_\Gamma T_{ij}(x',x)u_j(x)d\Gamma(x) = \int_\Gamma U_{ij}(x',x)t_j(x)d\Gamma(x) \quad (3)$$

where i and j are the Cartesian components; $T_{ij}(x',x)$ and $U_{ij}(x',x)$ represent Kelvin's fundamental solutions for traction and displacement, respectively, at a point x on the contour. The $CPV \int$ is the Cauchy Principal Value integral, and $c_{ij}(x')$ are coefficients given by $\delta_{ij}/2$ for a smooth contour at the point x', where δ_{ij} is the Kronecker delta.

When the same assumptions are considered, now with continuity of tractions at point x' on a smooth contour, the integral equation for traction is given by:

$$\frac{1}{2}t_j(x') + n_i(x')HPV \int_\Gamma S_{ijk}(x',x)\, u_k(x)d\Gamma(x) = n_i(x')CPV \int_\Gamma D_{ijk}(x',x)\, t_k(x)d\Gamma(x) \quad (4)$$

where the $HPV \int$ is the Hadamard Principal Value integral. The tensors $S_{ij}(x',x)$ and $D_{ij}(x',x)$ contain the derivatives of $T_{ij}(x',x)$ and $U_{ij}(x',x)$, respectively, from Eq. (3). The term $n_{ij}(x')$ is the i-th component of the unit normal vector to the boundary at point x', and $t_{ij}(x')$ contains the traction components.

2.3 BemCracker2D and BemLab2D Softwares

In this study, programs were developed with the aim of automating the entire crack propagation analysis process, from defining the problem's geometry and mesh to presenting the results. To achieve this goal, [13] created a C++ program called BemCracker2D, based on the Dual Boundary Element Method (DBEM) and grounded in the concepts of Object-Oriented Programming (OOP), capable of analyzing two-dimensional elastostatic problems. In a subsequent step, [11] developed BemLab2D, a Graphical User Interface (GUI) written in MatLab, which allows for pre and post-processing and generates data such as geometry, mesh, and boundary conditions for the BemCracker2D processor. BemCracker2D is responsible for performing the entire problem analysis and then forwarding the results to BemLab2D, where it can be visualized during the post-processing phase, including graphs, diagrams, and crack path representation. The combined workflow of these programs is schematically presented in Figure 1.

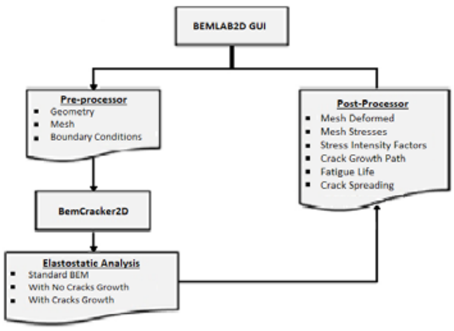


Figure 1. Flowchart of the automation process for crack propagation analysis.

The BemLab2D GUI was designed as a pre and post-processor. Its purpose is to facilitate the generation and visualization of two-dimensional solids, providing a user-friendly platform for entering problem data through buttons, mouse, and dialog boxes. In addition, the program also includes a module dedicated to the post-processing step, allowing users to request and visualize the results generated by the BemCracker2D processor.

The BemCracker2D program consists of three main processing modules: Standard DBEM (Module I), Dual Boundary Element Method without propagation (Module II), and DBEM with propagation (Module III). The Module III plays a fundamental role in crack propagation analysis, where stresses are analyzed using the DBEM, SIF's are calculated through the J-Integral, the crack growth direction is determined based on the MCS criterion, and fatigue life is estimated according to the modified Paris Law, as suggested by Tanaka [22].

3 Material and Methods

The methodology adopted in this article involves numerical simulation using the BemCracker2D and BemLab2D programs. Parameters such as Stress Intensity Factors (SIFs) and crack propagation (growth), among others, will be calculated and compared using examples from the available literature. The objective is to demonstrate the efficiency and robustness of the program through these comparative analyses.

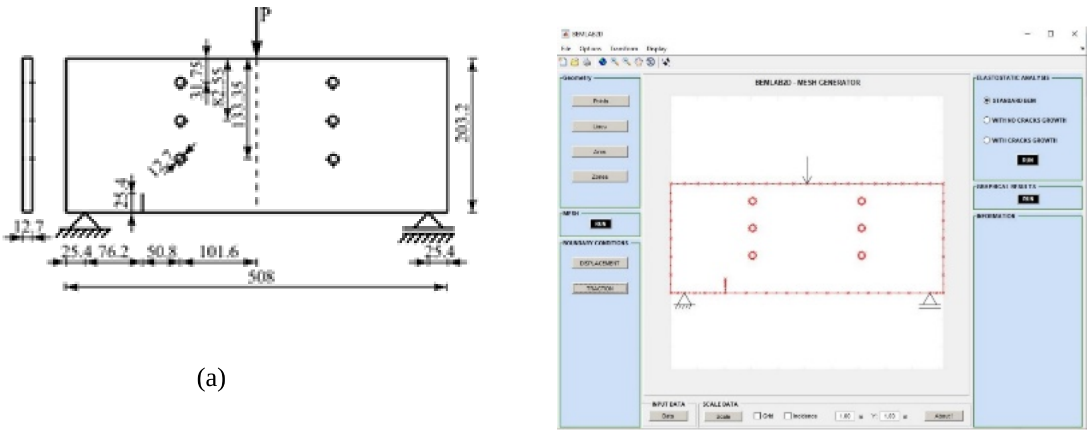
The analysis of structures with cracks presents challenges in modeling and calculating SIFs. For this purpose, the DBEM has been applied, showing advantages over the FEM, as it does not require continuous remeshing at each crack increment. In this work, we will use the BemCracker2D software, specifically designed for two-dimensional analysis for LEFM problems. Accompanied by the BemLab2D GUI, the program provides a user-friendly platform for conducting the analysis.

To explore the functionality of BemCracker2D, we will model two main applications with a focus on crack propagation analysis: SIFs x crack direction criteria (CASE 1); Linkup or coalescence of cracks (CASE 2); The idea is evaluate the efficiency and robustness of the program.

4 Results and Discussion

CASE 1 – SIFs x Crack Direction Criteria

This case, Moura and Gomes [23] investigated the behavior of a bi-supported beam with central loading and an initial notch in the lower left corner – as depicted in Figure 2 (a). The material of the beam under investigation was polymethylmethacrylate (PMMA), and it was subjected to cyclical loading with a maximum load of $5.7kN$ and a minimum of at least 10% of the maximum. The material properties included a modulus of elasticity $E = 3103MPa$ and a Poisson coefficient $\nu = 0.36$. The propagation analysis was performed using the MPS criterion, considering nine increments with a length of $12.7mm$, except for the last increment, which had a length of $6.35mm$ due to its proximity to the hole. The Figure 2 (b) shows the mesh made of boundary elements in BEMLAB2D GUI.



(a)

(b)

Figure 2. CASE 1: a) PMME Beam model; b) Boundary element mesh in BEMLAB2D GUI

During propagation, the values of K_Ie K_{II} were calculated using three criteria: Maximum Principal Stress Criterion (MPS), Maximum Energy Release Rate Criterion (MERR) and Strain Energy Density Criterion (SED) (Figure 3). When comparing the three curves of SIFs between each other, it is possible to perceive that there is almost no variation from one curve to another, since the propagation also occurs in a very similar way.

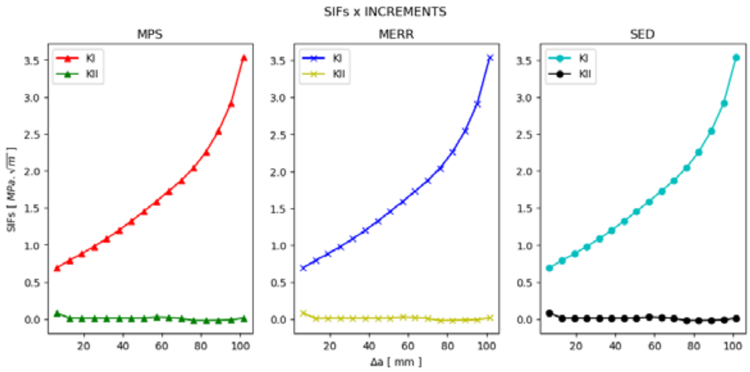
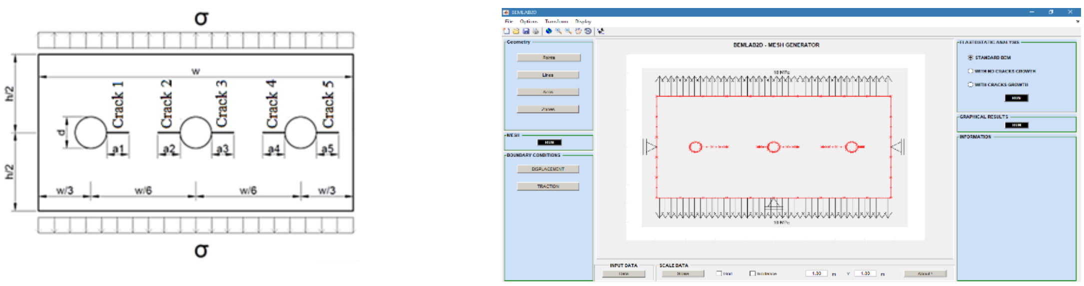


Figure 3. SIFs vs. Increments of the three direction criteria.

CASE 2 – Linkup and SIFs in the Plate with three holes

The following case investigates the physical model represented in Figure 4 (a), which will be treated through the modeling and visualization process, to analyze the crack propagation increments. It is defined by a plate, with five cracks and three holes arrangement, subjected to multiple site damage (MSD) in a line of holes in the panel. The coalescence of the cracks is analyzed according to the swift criterion of the ligament due to Dugdale. The elastic constants of the plate are: $E = 73.1\ GPa$ and $\nu = 0.33$. The considered stress was $\sigma = 10$ Mpa and the yield stress $\sigma y = 0.345$ GPa. The equation of Paris is used for the analysis of fatigue with the parameters $C = 4.624 \times 10^{-12}$ and $m = 3.3$, and with the stress amplitude ratio of the load cycle $R = 2/3$. The plate boundary and the holes had 66 continuous quadratic elements. In the discretization of the crack elements eight quadratic discontinuous elements were modeled in each crack, whose ratio was 0.4, 0.3, 0.2 and 0.1, in that order, with a total of 106 elements and 212 nodes. The boundary element model is shown in Figure 4 (b).



a) b)

Figure 4. (a) Plate with three holes; (b) Boundary element model.

All results were obtained with the program BemCracker2D after being modeled in the BEMLAB2D GUI. Initially are presented the SIFs along of the 20 crack growth increments in Figure 5.

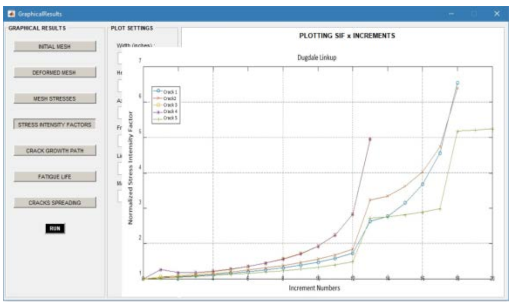
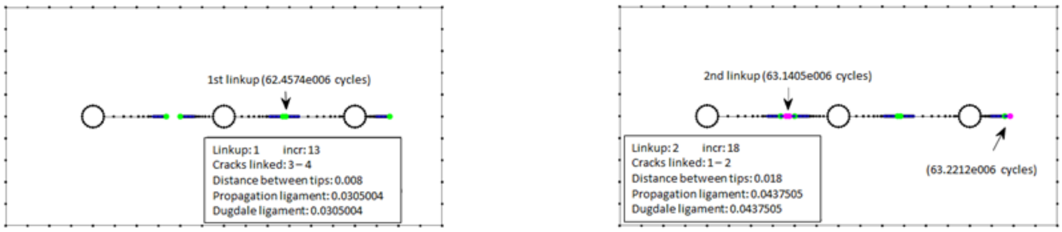


Figure 5. Stress Intensity Factors (SIF) x Increment.

To evaluate the linkup a post-processing was performed based on the results of SIF (K_I) and propagation path (crack advance in each increment) extracted from BemCracker2D.

In Figure 6 is observed that between the second and third hole, crack 3 and 4, a main crack increases. After 13 growth increments, in these two cracks, the first linkup of the panel occurs, with approximately $6{,}24 \times 10^7$ cycles, initiating a dominant crack with the cracks tip 2 and 5, as shown in Figure 6 (a).

The process continues and the dominant crack, with tip in the crack 2, grows faster. The velocity also increases at the tip of the crack 1, and when its plastic zones touch each other, the cracks linkup between the second and first holes. The second linkup, Figure 6 (b), of the plate occurred after $6{,}31 \times 10^7$ cycles, corresponding to 18 increments of crack growth. The result is a larger main crack than before, with a length from the left end of the first hole to the crack tip 5.



(a) (b)

Figure 6. Linkup: (a) 1st - cracks 3 and 4, (b) 2nd - cracks 1 and 2.

5 Conclusions

In Case 1, the BemCracker2D and BEMLAB2D GUI programs were utilized to perform two-dimensional crack propagation analysis and calculate Stress Intensity Factors (SIFs) using the J-Integral method. The boundary elements method was employed for the analysis and compared with results from previous studies [28], which were validated experimentally.

The modeling process through the BEMLAB2D GUI interface streamlined the procedure, allowing for efficient geometry input and automatic BEM mesh generation. The results obtained from BemCracker2D, including SIFs and crack propagation, showed negligible differences, thereby validating the accuracy of the program. Consequently, BemCracker2D proves to be a powerful tool for two-dimensional elastostatic calculations.

Finally, in case 2, the interaction between the programs used, BEMLAB2D and BemCracker2D, provides a versatile and automated analysis of the modeling processes and incremental analyzes, each with its functions. For the coalescence analysis of the adjacent cracks, the precision of the data provided by the programs was of vital importance for the study. The MSD scenario showed that the behavior of multiple fatigue cracks will have control over the failure in the analyzed structural element, when the process of propagation and linkup of the cracks will form an increasing main crack through its growth along the holes, until reaching a critical length.

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