Numerical and Analytical Study of Reinforced Concrete Beams with Ultra-High Performance Fiber Reinforced Concrete (UHPFRC)

Ingrid R. Irreño Palomo[1,2], Camila de Q. Moreira[1], Luiz C. De Almeida[1], Leandro M. Trautwein[1]

[1]*Dept. of Civil Engineering, University of Campinas - UNICAMP*
Rua Saturnino de Brito, 13083-889, São Paulo/Campinas, Brazil
Ingridpal2393@gmail.com, camilaqmoreira@hotmail.com
[2]*Dept. of Civil Engineering, Mackenzie Presbyterian University*
Avenida Brasil, 13073-148, São Paulo/Campinas, Brazil
ingrid.palomo@mackenzie.br

Abstract. Ultra-High Fiber Reinforced Concrete (UHPFRC) has recently been used to retrofit structures, presenting great mechanical behavior under flexural loads. This work used the data of a developed numerical study to develop a numerical and analytical methodology to obtain the ultimate bending moment (M_u) of a reinforced concrete (RC) beam retrofitted with UHPFRC subjected to monotonic load. The numerical model was developed in the commercial program ATENA 3D, using the Finite Elements Method (FEM). The numerical analysis considered the nonlinear material behavior to obtain closer values to the experimental test. The analytical methodology was based on the previous equations developed by other authors, with the modification of the tensile diagram of the UHPFRC. The results obtained from both methods presented good accuracy in the ultimate bending moment compared to the experimental one. Regarding the case study, the analytical and numerical models showed differences of 7.1% and 2.9%, respectively, and similarity in the load-displacement curve of the retrofit beam with UHPFRC. This work demonstrated the ability of the ATENA program to simulate the structural behavior of RC beams retrofitted with UHPFRC. Finally, it is possible to conclude that both methodologies could determine the ultimate bending moment for beams strengthened with UHPFRC.

Keywords: reinforced concrete, retrofit, UHPFRC, finite element, beam.

1 Introduction

The safety and economy of structures have been an essential topic for diverse researchers worldwide. With it, new materials such as Fiber Reinforced Concrete (FRC), High-Performance Fiber Reinforced Concrete (HPFRC), and Ultra-High Fiber Reinforced Concrete (UHPFRC) have been developed as alternatives for structural strengthening of old structures and for the construction of new ones. UHPFRC comprises steel fibers that improve the cracking behavior of concrete and increase its durability properties, load-bearing capacity, and safety [1,2]. UHPFRC also has a higher elasticity modulus and tensile strength than conventional concrete.

Numerical simulations of unretrofitted and retrofitted structural elements with UHPFRC is an essential approach to increasing the knowledge about the structural response of elements subjected to monotonic or seismic loading [3-6]. So far, diverse studies have used computational programs to simulate structural elements using the Finite Element Method (FEM) through the discretization of elements and the consideration of the nonlinear behavior of the material, such as ABAQUS, ATENA, ANSYS, and DIANA.

Safdar et al. [3] developed an experimental study composed of 4 Reinforced Concrete (RC) beams strengthened with UHPFRC in the tension and compression faces with different thicknesses. The authors observed that the addition of the UHPFRC jacket increased the bending capacity and delayed cracking, preventing the formation of macrocracks in the UHPFRC layer under service conditions, improving the capacity and durability of the beams.

Garg et al. [4] studied the structural behavior of beams deficient in shear (initially damaged) that were repaired with UHPFRC jackets, using retrofit configurations in a U-shape and in the tensile or compression faces. The authors observed the enhancement in the flexural performance of the beams, being the U-shape the best configuration to increase load capacity. This work also confirmed that an adequate preparation technique for the interface between the materials increased the duration for resisting high stresses in the structure.

Elsayed et al. [5] carried out an analytical and experimental study of RC columns strengthened with UHPFRC with eccentric load, in which parameters such as eccentricity ratio and thickness of the retrofit were evaluated. The results showed improvements in the axial load capacity, moment capacity, and stiffness of the columns.

Therefore, this work aims to evaluate the structural response of RC beams retrofitted with UHPFRC through two methodologies and compared with results obtained in the literature [6-7]. The first one corresponds to a new analytical proposal based on one previously developed, which depends on the compression and tensile behavior of the materials and the geometry of the cross-section of elements. The second refers to a 3D numerical simulation via FEM through the computational program ATENA, considering the nonlinearity and cracking of the material.

2 Case study

This work uses one of the beams studied by Tsioulou [7] and Lampropoulos et al. [8], which analyzed the behavior of a RC beam retrofitted with UHPFRC subjected to a bending test of four points through numerical models. The RC beam had a cross-section of 150x250 mm² and a length of 2200 mm. The longitudinal reinforcement comprised 4 bars of 12 mm, two placed in the top face and 2 in the bottom. The transversal reinforcement was given by stirrups of 8 mm spaced each 100 mm with a yield strength of 500 MPa. The UHPFRC jacket had a thickness of 50 mm, and it was placed on the tension side of the beam, as shown in Fig. 1. The concrete had a compressive strength of 37.9 MPa, and the UHPFRC had a compressive strength of 164 MPa and an elasticity modulus of 57.5 GPa. The maximum and ultimate tensile strength of UHPFRC was 12 MPa and 4.5 MPa, respectively.

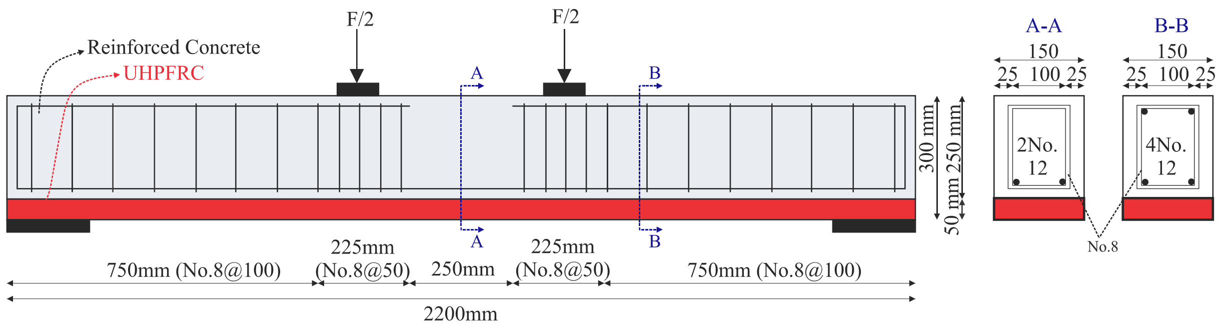

Figure 1. Geometrical and mechanical details of the RC beam retrofitted with UHPFRC.

3 Analytical model

Al-Osta et al. [9] developed an analytical methodology able to determine the ultimate bending moment (M_u) of beams retrofitted with UHPFRC, which is governed by geometrical and mechanical parameters of the concrete cross-section, reinforcement properties, and the UHPFRC jacket. Based on the previous methodology, in this work is presented a new proposal able to determine the value of the maximum moment of the beam retrofitted with UHPFRC, showing a modification in the shape of the tensile strain-stress diagram along the height of the section. The equilibrium of internal forces can be determined from Fig. 2. The neutral axis (x_c) of the section (when the axial force equals zero N_N), and the moment (M_u) are calculated, following Eq. (1) and (2), respectively.

$$N_N = 0 = C_c + C_{R1} - T_{R1} - T_{R2} - T_{R3} - T_{R4} - T_{R5} - C_{sc'} - C_{sc} \quad (1)$$

$$M_u = C_c\left(x_c - \frac{\beta_c x_c}{2}\right) + C_{R1}\left(x_c - \frac{\beta_R x_c}{2}\right) + T_{R1}\left(\frac{2y}{3}\right) + T_{R2}\left(\frac{H-\delta_R-x_c-y}{3} + y\right) + T_{R3}\left(\frac{H-\delta_R-x_c-y}{2} + y\right) + T_{R4}\left(\frac{\delta_R}{3} + (H-\delta_R-x_c)\right) + T_{R5}\left(\frac{\delta_R}{2} + (H-\delta_R-x_c)\right) + C_{sc}(H - x_c - \delta_R - d') + C_{sc'}(d' - x_c) \quad (2)$$

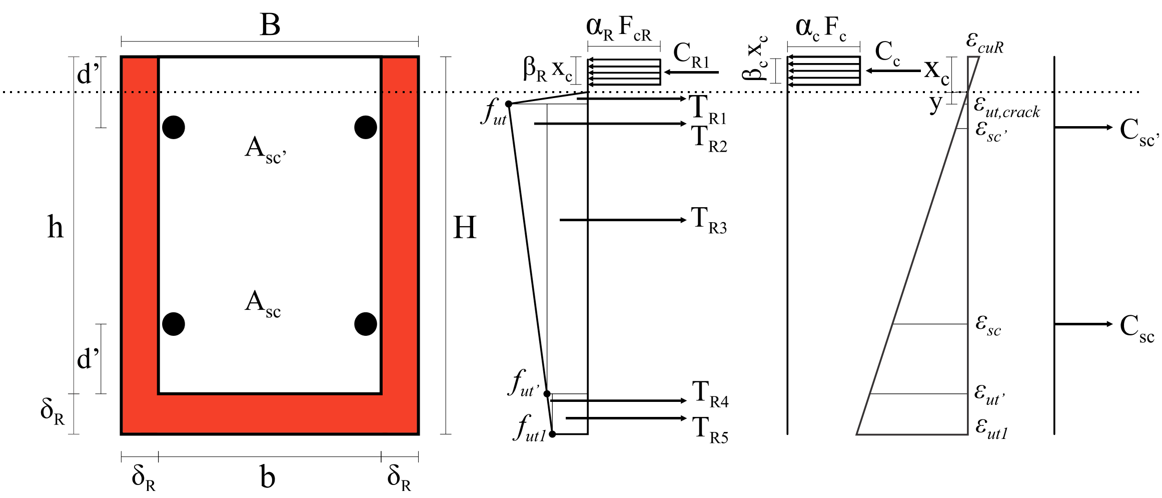

Figure 2. Cross-section forces used in the analytical model

where δ_R is the thickness of the UHPFRC, B and H are the width and depth of the retrofitted section, b and h are the width and height of the section, and d' is the original cover reinforcement. A_{sc}' and A_{sc} are the areas of the superior and inferior steel rebars with their respective strains (ε_{sc}', ε_{sc}). ε_{cuR} and $\varepsilon_{ut,crack}$ are the maximum and cracking strain of UHPFRC, ε_{ut}' and ε_{ut1} are the strains obtained by the stresses f_{ut}' and f_{ut1}, y is the distance from the neutral axis to the cracking strain, f_c and f_{cR} are the compressive strengths of the concrete and UHPFRC, respectively. The index β_R, β_c, α_R, and α_c can be determined following the design requirements [10-11]. C_{R1} and C_c are the compressive forces of the UHPFRC and concrete, respectively. The internal forces $T_{R1} - T_{R5}$ correspond to the tensile forces of the UHPFRC, and C_{sc}' and C_{sc} are the resulting forces of superior and inferior reinforcement. The before-mentioned parameters can be determined following Eq. (3)-(11).

$$C_c = \beta_c x_c f_c \alpha_c b \quad (3)$$

$$C_{R1} = 2\delta_R \alpha_R f_{cR} \beta_R x_c \quad (4)$$

$$T_{R1} = f_{ut} y \delta_R \quad (5)$$

$$T_{R2} = (f_{ut} - f_{ut'})(H - \delta_R - x_c - y)\delta_R \quad (6)$$

$$T_{R3} = f_{ut'}(H - \delta_R - x_c - y)2\delta_R \quad (7)$$

$$T_{R4} = \frac{f_{ut}' - f_{ut1}}{2}\delta_R B \quad (8)$$

$$T_{R5} = f_{ut1}\delta_R B \quad (9)$$

$$C_{sc} = \varepsilon_{sc} E_s A_{sc} \quad (10)$$

$$C_{sc'} = \varepsilon_{sc'} E_s A_{sc'} \quad (11)$$

4 Numerical model

The Finite Element Method (FEM) is a mathematical and numerical technique for solving problems involving differential and boundary equations. Essentially, FEM breaks down complex problems into smaller parts called "Finite Elements", allowing the analysis to be performed on each of them separately. This is possible due to the advancement of computers, which turned FEM into a valuable tool for analyzing experiments conducted in laboratories and analytical studies. It also enables the validation of these experiments through numerical simulations, generating results closer to the experimental ones. The numerical model of the retrofitted beam was developed in the ATENA software [12], and the pre-processing (geometry and mechanical properties) was developed through the GID software (version 16.0.1). The numerical model was simulated using the Modified Newton Raphson Method as nonlinear solver, with a maximum of 30 iterations. The convergence criteria used in the model were 1% for the displacement, residual, and absolute residual error and 0.1% for the energy error.

4.1 Material Properties

Concrete: The concrete was simulated through the fracture-plastic constitutive model using the material model *CC3DNonLinearCementitious2*. The compressive behavior of the concrete was controlled by the triaxial failure criterion of Menétrey-Willam [13], which considers the softening of the material that depends on its plastic displacement, following the recommendations of experimental tests from Van Mier [14]. The tensile behavior of the concrete was considered through of Hordijk softening function [15], governed by the opening crack of the material. The material was simulated with a model of fixed crack.

Reinforcement steel: The transversal (stirrups) and longitudinal beam reinforcement were simulated through the constitutive model *CCReinforcement* of truss finite elements through a multilinear strain-stress function, applied only for bars under compression stress conditions. This function is given by different parameters, such as the yield strength (f_y) and ultimate yield stress of the steel (f_{yu}), with its strain ε_y and ε_u, respectively.

UHPFRC: The *CC3DNonLinCementitious2user* material is essential for simulating UHPFRC due to its ability to represent the concrete stiffness reduction in the pos-peak behavior through the relationship between the transverse modulus of elasticity and the fracture strain. The parameters adopted were obtained from the experimental test of direct tension and flexure conducted by Lampropoulos et al. [8]. The authors provided a diagram for calibrating the tensile behavior, which enabled the performance input according to the test. However, the compressive behavior was determined according to the recommendation of Ouyang et al. [16], expressed as a simplified theory that represents the model without applying inverse analysis. The parameter mentioned before can be determined by Eq. 12.

$$\frac{\sigma_c}{f_c} = \frac{\varepsilon}{\varepsilon_o} \cdot \frac{n}{n-1+\left(\frac{\varepsilon}{\varepsilon_o}\right)^n} \tag{12}$$

where σ_c/f_c is the normalized stresses, $\varepsilon/\varepsilon_o$ is the normalized strain and n a factor dependent on the elastic modulus of the concrete, as shown in Eq. 13.

$$n = \frac{E_c\varepsilon_o}{E_c\varepsilon_o - f_c} \tag{13}$$

Interface: It is known that the perfect bond between the materials is not the best alternative to describe the real behavior due to the possible failures of the region that can significantly decrease the capacity load of the structure. For this reason, in the numerical model, the interaction between two materials was developed using an interface element with the model material *CC3DInterface* [12]. The geometry of the interface was represented by elements of volume that consider the contact surface to surface in the plane and, therefore, its relative and opening displacement. The cohesion was correlated with the experimental shear test developed by the authors [7]. The tensile strength and the friction coefficient were obtained from the literature [7,8]. The normal and shear stiffnesses were determined following the recommendations of the program [12]. However, to maintain the continuity of simulated element equilibrium after the rupture of the interface material, we adopted 1% for the minimum stiffness. The parameters used in the simulation are shown in Tab. 1.

Table 1. Properties of materials used in the numerical simulation

Parameter	Concrete	Reinforcement	Interface
Elastic modulus (MPa)	22000	-	57500
Compression strength (MPa)	39,5	-	164
Tensile strength (MPa)	2,6	-	11,5
Reduction factor of compression	0,8	-	-
Fracture energy (MN/m)	0,00011	-	-
Tension characteristic size (mm)	-	-	2
Tension localization onset	-	-	0.004
Compression characteristic size (mm)	-	-	2
Compression localization onset	-	-	-0.003
Normal stiffness (MN/m³)	-	$1.14 \cdot 10^6$	-
Tangent stiffness (MN/m³)	-	$1.14 \cdot 10^6$	-
Cohesion (MPa)	-	1.9	-
Friction coefficient	-	1.5	-
Tensile strength of the interface (MPa)	-	2.0	-

4.2 Boundary conditions and mesh size

The boundary conditions of the beam were modeled to represent the real behavior. For this, the displacement was restrained in all directions for the left bottom steel plate and in the directions *y* and *z* for the right bottom steel plate. A vertical load was also applied to the bottom face of the two steel plates using steps displacement of 0.1 mm (50 loading steps). All the steel plates had a thickness of 30 mm and were connected to the concrete through a master-slave interface. The model was simulated with a mesh of 37.5 mm x 50 mm of 2380 elements and 3586 nodes, as shown in Fig. 3.

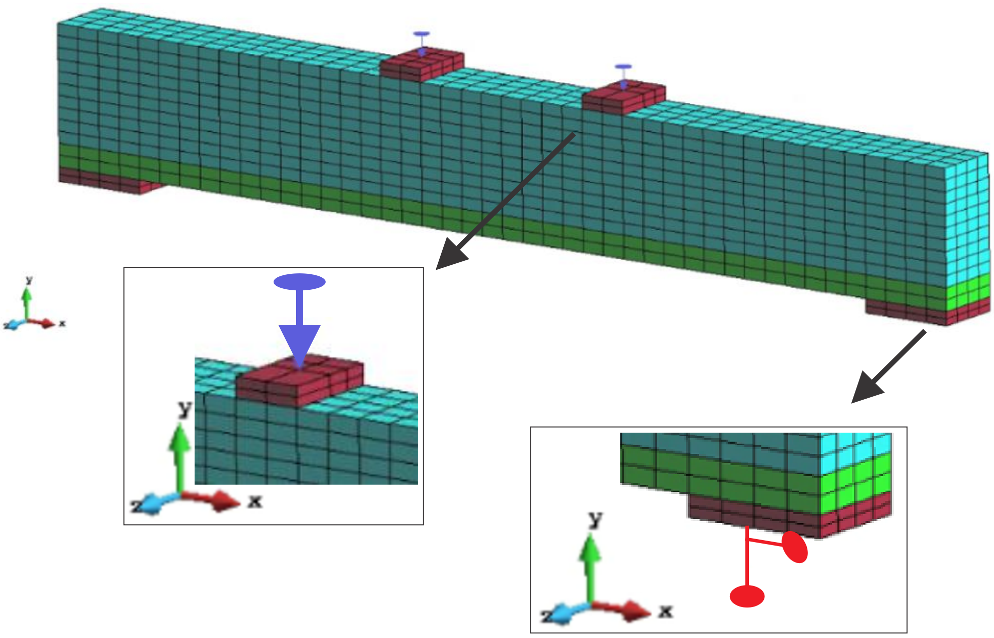

Figure 3. Mesh discretization of RC beam strengthened with UHPFRC

5 Results

The results obtained from both methodologies (analytical and numerical) showed good accuracy between the ultimate bending moment and the reference one. The analytical and numerical models resulted in an ultimate bending moment (M_u) of 39 kN.m and 43.2 kN.m, respectively, compared with the reference one of 42 kN.m, showing differences of 7.1% and 2.9%, respectively. Figure. 4 depicts the moment obtained in the numerical model to a load of 116 kN.

Additionally, the load-displacement curve obtained in the numerical model was compared with the one developed by Tsioulou et al. [6]. The numerical model reached a maximum load of 116 kN, while the reference case was 105 kN, resulting in a difference of 10.5%. It was observed that the parameters used to calibrate the numerical model presented a similar curve to the case study, as shown in Fig. 5. Thus, it is concluded that the methodology applied allows the numerical simulation of beams retrofitted with UHPFRC, however, is recommended develop more numerical models with aims to corroborate the compression function of the UHPFRC.

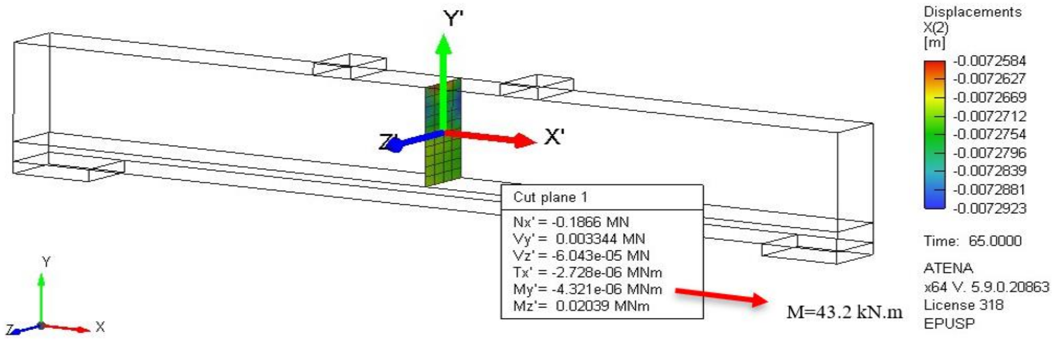

Figure 4. The ultimate bending moment of the RC beam retrofitted with UHPFRC

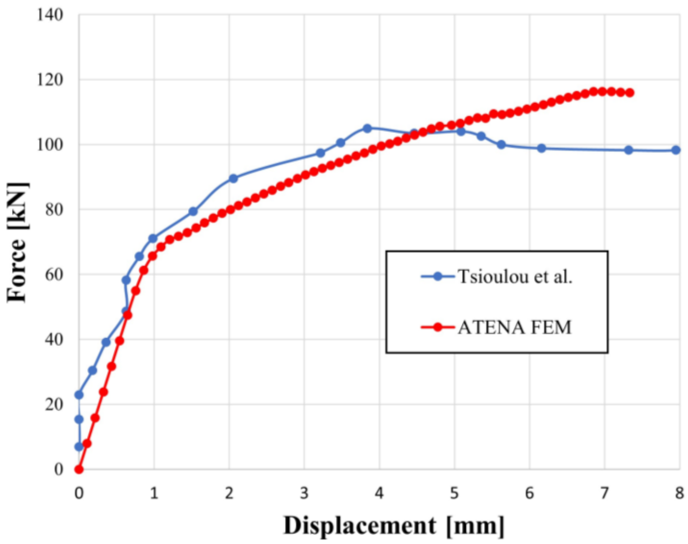

Figure 5. Comparison of diagram applied load versus applied displacement

6 Conclusions

This work provided a valuable understanding of the process of modeling RC beams retrofitted with new materials such as UHPFRC, where it was possible to observe that:

Using the Finite Element Method (FEM), the ATENA program can simulate the structural behavior of RC beams retrofitted with UHPFRC, considering the nonlinear and cracking behavior of the materials.

The constitutive model used to represent the compression behavior of the UHPFRC could be used as an initial hypothesis to describe its real behavior. However, it is necessary to highlight the need to develop more numerical models to corroborate the compression function of the UHPFRC.

The numerical and analytical model presented a maximum bending moment of 43.2 kN.m and 39 kN.m, respectively, showing low differences of 7.1% and 2.9% compared to the reference one.

The numerical simulation presented a good accuracy for the load-displacement curve with a maximum load of 116 kN, with a difference of 10.5% to the reference one (105 kN).

Finally, it was possible to conclude that the methodologies presented in this work allowed us to accurately determine the ultimate bending moment for RC beams strengthened with UHPFRC.

Acknowledgements. The authors greatly appreciate the financial support provided by the Brazilian National Council for Scientific and Technological Development (CNPq) (Grant 141517/2021-2).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors or has the permission of the owners to be included here.

References

[1] T. Matsumoto, P. Suthiwarapirak and T. Kanda, “Mechanism of multi cracking and fracture of DFRCCs under fatigue flexure”. *Journal of advanced concrete technology*, vol. 1, n. 3, pp. 299-306, 2003.
[2] P. Maca, R. Sovjak and T. Vavrinik, “Experimental investigation of mechanical Properties of UHPFRC”. *Procedia Engineering*, vol. 65, pp. 14-19, 2003.
[3] T. Matsumoto and K. Kakuma, “Flexural behavior of reinforced concrete beams repaired with ultra-high performance fiber reinforced concrete (UHPFRC)”. *Composite structures*, vol. 157, pp. 448-460, 2016.
[4] V. Garg, P. Bansal and R. Sharma, “Retrofitting of shear-deficient RC beams using UHP-FRC”. *Iranian Journal of Science and Technology, Transactions of Civil Engineerin*g, vol. 43, pp. 419-428, 2019.
[5] M. Elsayed, B. A. Tayeh, M. Abou Elmaatyand and Y. Aldahshoory, “Behaviour of RC columns strengthened with ultra-high performance fiber reinforced concrete (UHPFRC) under eccentric loading”. *Journal of Building Engineering*, vol. 47, pp. 103857, 2022.
[6] I. R. I. Palomo, J. D. J. Martínez, C. A. Benedetty, L. C. Almeida, L. M. Trautwein and P. A. Krahl, “Prediction of the ultimate capacity of reinforced concrete elements using nonlinear analysis methodologies”. *Revista IBRACON de Estruturas e Materiais*, vol. 17, pp. e17210, 2023.
[7] O. T. Tsioulou, A. P. Lampropoulos, and S. E. Dritsos, “Experimental investigation of interface behaviour of RC beams strengthened with concrete layers”. *Construction and Building Materials*, vol. 40, pp. 50-59, 2013.
[8] P. Lampropoulos, S. A. Paschalis, O. T. Tsioulou, and S. E. Dritsos, “Strengthening of reinforced concrete beams using ultra high performance fibre reinforced concrete (UHPFRC)”. *Engineering Structures*, vol. 106, pp. 370-384, 2016.
[9] M. A. Al-Osta, M. N. Isa, M. H. Baluch, and M. K. Rahman, “Flexural behavior of reinforced concrete beams strengthened with ultra-high performance fiber reinforced concrete”. *Construction and Building Materials*, vol. 134, pp. 279-296, 2017.
[10] Z. B. Haber, J. F. Munoz, I. De la Varga, and B. A. Graybeal, "Bond characterization of UHPC overlays for concrete bridge decks: Laboratory and field testing". *Construction and Building Materials*, vol. 190, pp. 1056-1068, 2018.
[11] Comité Euro-International du Béton, Fédération Internationale du Béton, “fib Model Code for Concrete Structures 2010”, *CEB-FIB 2010*, 2010.
[12] V. Červenka, L. Jendele, and J. Červenka, “ATENA Program Documentation Part 1: Theory”. *Červenka Consulting*, 2021.
[13] P. Menetrey and K. J. Willam, “Triaxial failure criterion for concrete and its generalization,” *ACI Struct. J*, vol. 92, no. 3, pp. 311–318, 1995, http://dx.doi.org/10.14359/1132.
[14] D. A. Hordijk, “Local approach to fatigue of concrete”, Ph.D. dissertation, Dept. Civil Eng., Delft Univ. Technol,1991.
[15] J. G. Van Mier, “Multiaxial strain-softening of concrete, part I: fracture”, *Mater. Struct.*, vol. 19, no. 111, 1986.
[116] X. Ouyang, Z. Wu, B. Shan, Q. Chen and C. Shi, “A critical review on compressive behavior and empirical constitutive models of concrete”. *Construction and Building Materials*, vol. 323, pp. 126572, 2022.